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# A comparison of Haar wavelet and Adomain decomposition method for solving one-dimensional reaction-diffusion equations

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**Abstract**:
Reaction-diffusion equations are fundamental in modeling several natural phenomena. In this paper, we develop an accurate and efficient Haar wavelet scheme for solving well-known one-dimensional reaction-diffusion equation. The power of this manageable method is confirmed. An attempt is made to combine the advantages of the ADM and Haar wavelets. The obtained numerical results have been validated against a closed form analytical solution as well as ADM results. Good agreement with the exact solution has been observed. Moreover the use of Haar wavelets is found to be accurate, simple, fast, flexible, convenient, small computation costs and computationally attractive.

**Keywords:** Haar wavelets, one-dimensional reaction-diffusion equation, Adomain decomposition method, computationally attractive.

## 1 Introduction

Many reaction–diffusion problems in biology and chemistry are modeled by partial differential equations (PDEs). These problems have been extensively studied in the literature and their numerical solution can be accurately computed provided the diffusion coefficients, reaction excitations, initial and boundary data are given in a deterministic way. However, modeling real-life reaction–diffusion systems is complicated by the high heterogeneity of the diffusion process combined with insufficient information characterizing the kinetic reactions. An example concerns the spatio-temporal pattern formation in cell metabolism where the intact living cell is based on a highly complex spatial organization of its constituents. The reactants mediating, and processed by the chemical pathways

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of cell are heterogeneously distributed through the cytoplasm and cell membranes. The diffusion of reactant species among localized reaction regions within the cell is therefore a central feature of biochemistry. For more details, we refer to [21] and further references are cited therein.

Reaction-diffusion equations are used to simulate a variety of different phenomena, from physics and engineering [3] to mathematical biology [18]. In the last decades, there have been great advances in the development of finite difference, finite element, spectral techniques, adaptive and non-adaptive algorithms and finite volume methods for the partial differential equations, especially for those of the advection-diffusion-reaction type. Reaction-diffusion equations also lead to many other interesting phenomena, such as, pulse splitting and shedding, reactions and competitions in excitable systems, and stability issues. Stable schemes for one-dimensional reaction-diffusion equation have demonstrated by Joao Teixeira [22]. Examples of this type of applications include numerical weather prediction and climate models [4, 26] where the time step and grid sizes are imposed from large-scale flow considerations, atmosperic chemistry models [13] or reactive flows in engineering [19]. J.I. Ramos [20] used a finite volume method for one-dimensional reaction–diffusion problems. Krishnan et. al [14] established Bifurcation analysis of nonlinear reaction–diffusion problems using wavelet-based reduction techniques.

This paper is devoted to study the linear single one-dimensional kinetic reaction-diffusion

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial U}{\partial x}\right) - \lambda U, \tag{1.1}$$

where $t$ and $x$ denote the time and spatial coordinate, respectively, $U$is the dependent variable and $k$ is a constant diffusion coefficient.

The Adomain decomposition method (ADM) is a creative and effective method for exactly solving functional equations of various kinds. It is important to note that a large amount of research work has been devoted to the application of the ADM to a wide class of linear and nonlinear, ordinary or partial differential equations [1,2,25]. The decomposition method provides the solution as an infinite series in which each term can be easily determined. The rapid convergence of the series obtained by this method is thoroughly discussed by Cherruault et al. in [7] and the references therein. Wazwaz [24] used the Adomain decomposition method for a reliable treatment of the Bratu- type equations.

It is somewhat surprising that among different solution techniques the wavelet method has not attained much attention. We found only one paper [23] in which the wavelet method is applied for solving singularly perturbed reaction-diffusion problems; for this purpose the cubic spline adaptive wavelet functions are used. Lepik [15,16,17] had solved higher order as well as nonlinear ODEs and some nonlinear evolution equations by Haar wavelet method. There are two possibilities for getting out of this situation. One way is to regularize the Haar wavelets with interpolating splines (e.g. B-splines or Deslaurier-Dabuc interpolating wavelets). This approach has been applied by Cattani [5], but the regularization process considerably complicates the solution and the main advantage of the Haar wavelets-the simplicity gets to some extent lost. Hariharan et al.[10,11] had solved linear and nonlinear PDEs.

Among the different wavelet families mathematically most simple are the Haar wavelets [9]. Due to the simplicity the Haar wavelets are very effective for solving ordinary differential

and partial differential equations. Therefore the idea, to apply Haar wavelet technique also for solving one-dimensional linear reaction-diffusion problem, arises. This is the main aim of the present paper. The method with far less degrees of freedom and with smaller CPU time provides better solutions than classical ones. The accuracy and effectiveness of the method are analyzed; the results obtained are compared with the results of other authors (using classical numerical techniques) and with the exact solution, evaluating the error.

We introduce a Haar wavelet method for solving one-dimensional linear reaction-diffusion equations, which will exhibit several advantageous features:

1. Very high accuracy fast transformation and possibility of implementation of fast algorithms compared with other known methods.

2. The simplicity and small computation costs, resulting from the sparsity of the transform matrices and the small number of significant wavelet coefficients.

3. The method is also very convenient for solving the boundary value problems, since the boundary conditions are taken care of automatically.

The paper is organized as follows. For completeness sake the Haar wavelet method is presented in Section 2. Function approximation is presented in Section 3. Adomain decomposition method (ADM) for one-dimensional linear kinetic reaction-diffusion equation in Section 4. The method of solution the PDE is proposed in Section 5. Some numerical examples are presented in Section 6.Concluding remarks are given in Section 7.

## 2 Haar wavelets

The set of Haar functions is defined as a group of square waves with magnitude $\pm 1$ some intervals and zero elsewhere

$$h_i(t) = \begin{cases} 1, & \text{for } t \in \left[\frac{k}{m}, \frac{k+0.5}{m}\right) \\ -1, & \text{for } t \in \left[\frac{k+0.5}{m}, \frac{k+1}{m}\right) \\ 0, & \text{elsewhere} \end{cases} \tag{2.1}$$

Integer $m = 2^j \quad (j = 0, 1, 2...J)$ indicates the level of the wavelet; $k = 0, 1, 2, .....m - 1$ is the translation parameter. Maximal level of resolution is J. The index $i$ is calculated according the formula$i = m + k + 1$; in the case of minimal values. $m = 1, k = 0$we have $i = 2$, the maximal value of $i$ is $i = 2M = 2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for which$h_1 \equiv 1 \ \ in \ \ [0, 1]$. Let us define the collocation points $t_l = (l - 0.5)/2M, \quad (l = 1, 2....2M)$ and discretise the Haar function $h_i(t)$; in this way we get the coefficient matrix$H(i, l) = (h_i(t_l))$, which has the dimension$2M \times 2M$. The operational matrix of integration P, which is a 2M square matrix, is defined by the equation

$$\left.\begin{array}{l} (PH)_{il} = \int_0^{t_l} h_i(t)\, dt \\ (QH)_{il} = \int_0^{t_l} dt \int_0^t h_i(t)\, dt \end{array}\right\} \tag{2.2}$$

The elements of the matrices H, P and Q can be evaluated according to (2.1) and (2.2).

$$H_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad P_2 = \frac{1}{4}\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \quad P_4 = \frac{1}{16}\begin{bmatrix} 8 & -4 & -2 & -2 \\ 4 & 0 & -2 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

$$H_4^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 2 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 2 \\ 1 & -1 & 0 & -2 \end{bmatrix}$$

$$P_8 = \frac{1}{64}\begin{bmatrix} 32 & -16 & -8 & -8 & -4 & -4 & -4 & -4 \\ 16 & 0 & -8 & 8 & -4 & -4 & 4 & 4 \\ 4 & 4 & 0 & 0 & -4 & 4 & 0 & 0 \\ 4 & 4 & 0 & 0 & -4 & 4 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & -2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Chen and Hsiao [6] showed that the following matrix equation for calculating the matrix $P$of order $m$ holds

$$P_{(m)} = \frac{1}{2m}\begin{pmatrix} 2mP_{(m/2)} & -H_{(m/2)} \\ H^{-1}_{(m/2)} & 0 \end{pmatrix}$$

where $O$ is a null matrix of order $\frac{m}{2} \times \frac{m}{2}$,

$$H_m X_m \underline{\underline{\Delta}} [h_m(t_0)h_m(t_1)...h_m(t_{m-1})] \tag{2.3}$$

and $\frac{i}{m} \leq t < i + \frac{1}{m}$ and $H^{-1}_{mxm} = \frac{1}{m} H^T_{mxm}\, diag(r)$
It should be noted that calculations for $P_{(m)}$ and $H_{(m)}$must be carried out only once; after that they will be applicable for solving whatever differential equations.
The higher order operational matrices $Q, R$ and $S$can be introduced as

$$\int_0^t PH(t)\,dt \cong QH(t), \quad \int_0^t QH(t)\,dt \cong RH(t), \quad \int_0^t RH(t)\,dt \cong SH(t). \tag{2.4}$$

It can be verified that the higher order operational matrices $Q, R$ and $S$ as satisfy the boundary conditions.

$$QH|_{t=0} = [0,0,...,0]^T, \quad RH|_{t=0} = [0,0,...,0]^T, \quad SH|_{t=0} = [0,0,...,0]^T,$$
$$QH|_{t=1} = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{256}, ..., \underbrace{\frac{1}{2^{2m}}, ..., \frac{1}{2^{2m}}}_{m/2}\right]^T \tag{2.5}$$

where $m$ denotes the dimension of the square matrices $P, Q, R$ and $S$. Since $H$ and $H^{-1}$contain many zeros, this phenomenon makes the Haar transform must faster than the Fourier transform, and it is even faster than the Walsh transform. This is one of the reason for rapid convergence of the Haar wavelet series [6, 10,12,27].

## 3 Function approximation

Any function $y(x) \in L^2[0,1)$ can be decomposed as

$$y(x) = \sum_{n=0}^{\infty} c_n h_n(x) \tag{3.1}$$

where the coefficients $c_n$are determined by

$$c_n = 2^j \int_0^1 y(x) h_n(x) dx \tag{3.2}$$

Where $n = 2^j + k,\ j \geq 0,\ 0 \leq k < 2^j$. Specially $c_0 = \int_0^1 y(x)dx$.

The series expansion of $y(x)$contains an infinite terms. If $y(x)$is piecewise constant by itself, or may be approximated as piecewise constant during each subinterval, then $y(x)$ will be terminated at finite terms, that is

$$y(x) = \sum_{n=0}^{m-1} c_n h_n(x) = c_{(m)}^T h_{(m)}(x) \tag{3.3}$$

Where the coefficients $c_{(m)}^T$and the Haar function vector $h_{(m)}(x)$are defined as

$$c_{(m)}^T = [c_0, c_1, ...., c_{m-1}]$$

and $h_{(m)}(x) = [h_0(x), h_1(x), ....., h_{m-1}(x)]^T$ where 'T' means transpose and $m = 2^j$.

## 4 The adomain decomposition method (ADM) for solving one-dimensional reaction-diffusion equation

We consider the linear kinetic one-dimensional reaction-diffusion equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial u}{\partial x}\right) - \lambda u, \tag{4.1}$$

with the initial and boundary conditions, where the notations $L_t = \frac{\partial}{\partial t}$ and $L_x = \frac{\partial^2}{\partial x^2}$ symbolize the linear differential operators. We assume the integration inverse operators $L_t^{-1}$ and $L_x^{-1}$ exist, and they are defined as $L_t^{-1} = \int_0^t (.)\, dt$ and $L_x^{-1} = \int_0^x \int_0^x (.)\, dx\, dx$, respectively. Therefore, we can write the solutions in $t$ and $x$ directions as [1].

$$u(x,t) = u(x,0) + L_t^{-1}\left[k L_x\left(u(x,t)\right) + \varphi(u)\right] \tag{4.2}$$

$$u(x,t) = u(0,t) + xu_x(0,t) + L_x^{-1}\left[kL_t(u(x,t)) - \varphi(u)\right] \tag{4.3}$$

respectively, where $\varphi(u) = -\lambda u$. By ADM [1] one can write the solution in series form as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{4.4}$$

To find the solutions in $t$ and $x$ directions, one solves the recursive relations

$$u_0 = u(x,0), \quad u_{n+1} = L_t^{-1}\left[kL_x(u_n) + B_n\right], \quad n \geq 0, \tag{4.5}$$

$$u_0 = u(0,t) + xu_x(0,t), \quad u_{n+1} = L_x^{-1}\left[kL_t(u_n) - B_n\right], \quad n \geq 0. \tag{4.6}$$

respectively, where the Adomain polynomials are [ ]

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\varphi\left(\sum_{n=0}^{\infty}\lambda^n u_n\right)\right]_{\lambda=0}, \quad n \geq 0 \tag{4.7}$$

We obtain the first few Adomain polynomials for as, $\varphi(u) = -\lambda u$ as

$$\begin{aligned} B_0 &= -\lambda u_0 \\ B_1 &= -\lambda u_1 \\ B_2 &= -\lambda u_2 \end{aligned}$$

and so on. The convergence of the decomposition series (4.4) is studied in [8]. Then $u(x,t)$ is the particular exact solution and $\varphi_n(x,t)$ is the partial sum

$$\varphi_n(x,t) = \sum_{k=0}^{\infty} u_k(x,t), \ n \geq 0 \tag{4.8}$$

As it is clear from (4.4) and (4.8)

$$u(x,t) = \lim_{n\to\infty} \varphi_n(x,t). \tag{4.9}$$

# 5 Method of solution

Consider the linear kinetic one-dimensional reaction-diffusion equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right) - \lambda u, \tag{5.1}$$

with the initial condition $u(x,0) = f(x), \quad 0 \le x \le 1.$

$$u(0,t) = g_0(t), \quad u(1,t) = g_1(t), \quad 0 < t \le T$$

Let us divide the interval (0,1] into N equal parts of length $\Delta t = (0,1]/N$ and denote $t_s = (s-1)\Delta t, \;\; s = 1, 2, .....N$. We assume that $\dot{u}''(x,t)$can be expanded interms of Haar wavelets as formula

$$\dot{u}''(x,t) = \sum_{n=0}^{m-1} c_s(n) h_n(x) = c_{(m)}^T h_{(m)}(x) \tag{5.2}$$

where . and $'$ means differentiation with respect to $t$ and $x$ respectively, the row vector $c_{(m)}^T$ is constant in the subinterval $t \in (t_s, t_{s+1}]$

Integrating formula (5.2) with respect to $t$ from $t_s$ to t and twice with respect to $x$ from 0 to $x$, we obtain

$$u''(x,t) = (t - t_s) c_{(m)}^T h_{(m)}(x) + u''(x, t_s) \tag{5.3}$$

$$\begin{aligned} u(x,t) = (t - t_s) c_{(m)}^T Q_{(m)} h_{(m)}(x) + u(x,t_s) - u(0,t_s) \\ + x[u'(0,t) - u'(0,t_s)] + u(0,t) \end{aligned} \tag{5.4}$$

$$\dot{u}(x,t) = c_{(m)}^T Q_{(m)} h_{(m)}(x) + x\dot{u'}(0,t) + \dot{u}(0,t) \tag{5.5}$$

By the boundary conditions, we obtain

$$\begin{aligned} u(0,t_s) &= g_0(t_s), \qquad u(1,t_s) = g_1(t_s) \\ \dot{u}(0,t) &= g_0^{'}(t), \qquad \dot{u}(1,t) = g_1^{'}(t) \end{aligned}$$

Putting $x = 1$in formulae (5.4) and (5.5), we have

$$u'(0,t) - u'(0,t_s) = -(t - t_s) c_{(m)}^T Q_{(m)} h_{(m)}(x) + g_1(t) - g_0(t) - g_1(t_s) + g_0(t_s) \tag{5.6}$$

$$\dot{u}'(0,t) = g_1'(t) - c_{(m)}^T Q_{(m)} h_{(m)}(x) - g_0^{'}(t) \tag{5.7}$$

Substituting formulae (5.6) and (5.7) into formulae (5.3)-(5.5), and discretizising the results by assuming $x \to x_l, \;\; t \to t_{s+1}$ we obtain

$$u''(x_l, t_{s+1}) = (t_{s+1} - t_s) c_{(m)}^T h_{(m)}(x_l) + u''(x_l, t_s) \tag{5.8}$$

$$\begin{aligned} u(x_l, t_{s+1}) = (t_{s+1} - t_s) c_{(m)}^T Q_{(m)} h_{(m)}(x_l) + u(x_l, t_s) - g_0(t_s) + g_0(t_{s+1}) \\ + x_l[-(t_{s+1} - t_s) c_{(m)}^T P_{(m)} f + g_l(t_{s+1}) - g_0(t_{s+1}) - g_1(t_s) + g_0(t_s)] \end{aligned}$$

(5.9)

$$\dot{u}(x_l, t_{s+1}) = c^T_{(m)} Q_{(m)} h_{(m)}(x) + g_0'(t_{s+1}) + x_l[-c^T_{(m)} P_{(m)} f + g_1'(t_{s+1}) - g_0'(t_{s+1})] \quad (5.10)$$

Where the vector $f$ is defined as

$$f = [1, \underbrace{0, ....., 0}_{(m-1)\ elements}]^T$$

In the following the scheme

$$\dot{u}(x_l, t_{s+1}) = u''(x_l, t_{s+1}) - \lambda u(x_l, t_{s+1}) \quad (5.11)$$

which leads us from the time layer $t_s$ to $t_{s+1}$ is used.

Substituting equations (5.8)-(5.10) into the equation (5.11), we gain

$$\begin{aligned} &c^T_{(m)} Q_{(m)} h_{(m)}(x_l) + x_l[-c^T_{(m)} P_{(m)} f + g_1'(t_{s+1}) - g_0'(t_{s+1})] + g_0'(t_{s+1}) \\ &= u''(x_l, t_{s+1}) - \lambda u(x_l, t_{s+1}) \end{aligned} \quad (5.12)$$

From formula (5.12) the wavelet coefficients $c^T_{(m)}$can be successively calculated.
In the following section we provide couple of examples and calculate the absolute errors by using the formula $E_W = |u_{exact} - u_{Haar}|$ and $\delta_{ex} = \|u(x,t) - u_{ex}(x,t)\| / 2M$.

# 6 Applications and results

**Example 6.1.** If we take $k = 1$ *and* $\lambda = -1$ in the equation (5.1), we obtain the linear heat equation, namely

$$u_t = u_{xx} + u \quad (6.1)$$

We impose the initial condition

$$u(x, 0) = \cos(\pi x) \quad (6.2)$$

and boundary conditions

$$u(0, t) = e^{(1-\pi^2)t}, \quad u_x(0, t) = 0. \quad (6.3)$$

To obtain the solution in $t$ direction, we use the recursive relation (4.5) by simply taking $u_0 = \cos(\pi x)$. In this case the Adomain polynomials are $B_0 = u_{0,}\, B_1 = u_1$, $B_2 = u_2$, and so on. Therefore, we have

$$u_1 = \left(1-\pi^2\right) t\cos\left(\pi x\right), \quad u_2 = \frac{1}{2!}\left(1-\pi^2\right)^2 t^2\cos\left(\pi x\right),$$

$$u_3 = \frac{1}{3!}\left(1-\pi^2\right)^3 t^3\cos\left(\pi x\right),$$

and so on, in this manner the rest of the components of the series (4.4) have been calculated using Mathcad7. Putting these individual terms in (4.4) one gets the exact solution

$$\begin{aligned} u\left(x,t\right) &= \cos\left(\pi x\right) + \left(1-\pi^2\right) t\cos\left(\pi x\right) + \tfrac{1}{2!}\left(1-\pi^2\right)^2 t^2\cos\left(\pi x\right) \\ &\quad + \tfrac{1}{3!}\left(1-\pi^2\right)^3 t^3\cos\left(\pi x\right) + ... = e^{\left(1-\pi^2\right)t}\cos\left(\pi x\right), \end{aligned} \tag{6.4}$$

which can be verified through substitution.

Similarly, to obtain the solution in $x$ direction, we use the recursive relation (4.6) by taking $u_0 = e^{\left(1-\pi^2\right)t}$, where the $B_n$'s are the same as above. We therefore have

$$u_1 = -\frac{\left(\pi x\right)^2}{2!}e^{\left(1-\pi^2\right)t}, \quad u_2 = \frac{\left(\pi x\right)^4}{4!}e^{\left(1-\pi^2\right)t}, \quad u_3 = -\frac{\left(\pi x\right)^6}{6!}e^{\left(1-\pi^2\right)t},$$

and so on, in this manner the rest of the components of the series (4.4) have been calculated. From the decomposition series (4.4), we gain obtain the exact solution

$$u\left(x,t\right) = e^{\left(1-\pi^2\right)t}\cos\left(\pi x\right). \tag{6.5}$$

In the following the scheme

$$\dot{u}(x_l, t_{s+1}) = u''(x_l, t_{s+1}) + u(x_l, t_{s+1})$$

which leads us from the time layer $t_s$ to $t_{s+1}$ is used.

$$\begin{aligned} &c^T_{(m)}Q_{(m)}h_{(m)}(x_l) + x_l[-c^T_{(m)}P_{(m)}\cos\left(\pi x\right) + g_1'(t_{s+1}) - g_0'(t_{s+1})] + g_0'(t_{s+1}) \\ &= u''(x_l, t_{s+1}) + u(x_l, t_{s+1}) \end{aligned} \tag{6.6}$$

From formula (6.6) the wavelet coefficients $c^T_{(m)}$can be successively calculated.
This process is started with

$$\begin{aligned} u\left(x_l, t_s\right) &= -\pi\sin\left(\pi x\right) \\ u'\left(x_l, t_s\right) &= -\pi^2\cos\left(\pi x\right) \\ u''\left(x_l, t_s\right) &= \pi^3\sin\left(\pi x\right) \end{aligned}$$

Using Adomain decomposition method, the exact solution in a closed form is given by $u(x,t) = e^{\left(1-\pi^2\right)t}\cos\left(\pi x\right)$ can be compared with the Haar solution. The accuracy of the

results is estimated by the error function. In the case of error estimates, if the exact solution of the problem $u = u(x,t)$ is known we shall calculate the differences $\Delta_{ex}(l) = u(x_l, t_{s+1}) - u_{ex}(x_l, t_{s+1})$, $l = 1, 2, ..., 2M$ and we define the error estimates as $\delta_{ex} = \max_l |\Delta_{ex}(l)|$ (local estimate) or $\delta_{ex} = \frac{1}{2M} \|u(x,t) - u_{ex}(x,t)\|$ (global estimate). The convergence of Haar method is fast and it's accuracy is height, as numerical examples show error [15].
Computer simulation was carried out in the cases $m = 32$ and $m = 64$, the computed results were compared with the exact solution, more accurate results can be obtained by using a larger $m$ (See Fig.1). The method with far less degrees of freedom and with a smaller CPU time provides better solutions than classical ones.

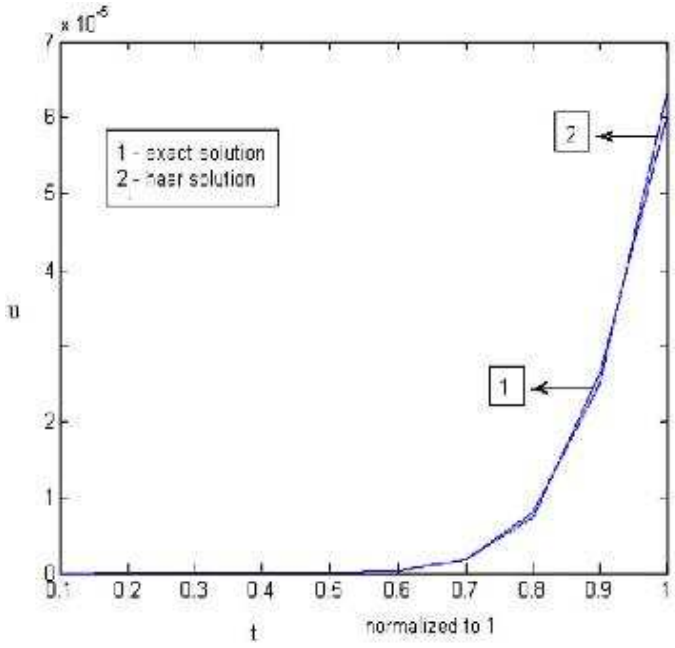


Figure 1: Comparison between exact and Haar solutions $x$=5 and $k$=12.5

**Example 6.2** Consider the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (1-u), \qquad -1 < x < 1, \;\; t > 0$$

with the data $u(-1,t) = u(1,t) = 0$
and the initial condition $u(x,0) = 0$
The exact solution of the model problem is given by

$$u(x,t) = 1 - \frac{\cosh x}{\cosh 1} - \frac{16}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \cos[(2n-1)(\pi x/2)]}{(2n-1)\left[(2n-1)^2 \pi^2 + 4\right]} \exp\left\{ - \left[ 1 + (2n-1)^2 \frac{\pi^2}{4} \right] t \right\}$$

All the numerical experiments presented in this section were computed in double precision with some MATLAB codes on a personal computer System Vostro 1400 Processor x86 Family 6 Model 15 Stepping 13 Genuine Intel ˜1596 Mhz.

## 7 Conclusion and future work

The goal to obtain exact and Haar solutions for one-dimensional reaction-diffusion problems has been achieved. The theoretical elegance of the Haar wavelet approach can be appreciated from the simple mathematical relations and their compact derivations and proofs. It has been well demonstrated that while applying the nice properties of Haar wavelets, the partial differential equations can be solved conveniently and accurately by using Haar wavelet method systematically.

In the present paper only linear equations are considered, but the method are applicable also for nonlinear systems. The main advantages of the presented method are its simplicity and small computation costs: it is due to the sparcity of the transform matrices and to the small number of significant wavelet coefficients. An authentic conclusion can be drawn from the numerical results that the Haar wavelet method provides more accurate numerical solutions than Adomian's decomposition method. In our opinion the Haar wavelet method is wholly competitive in comparison with the classical methods.

Future work will involve the extension of the scheme to two and three dimensions and to the advection-diffusion equation. Extending the scheme to higher dimensions and to the advection-reaction-diffusion equations.

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