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Heat transfer effects on dusty gas flow past a semi-infinite vertical plate

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Abstract:
The flow of a dusty gas past an impulsively started semi-infinite vertical plate is studied. The non-dimensional governing equations are solved by an implicit finite difference scheme of Crank-Nicolson method, which is fast convergent and unconditionally stable. Gas velocity, dust particle velocity, temperature, skin-friction and Nusselt numbers are calculated numerically for various parameters and are shown graphically. It is observed that an increase in mass concentration of dust leads to a decrease in the dusty gas velocity but an increase in skin-friction.
Key Words: Dusty gas. Heat transfer, Vertical plate, Finite difference.

Nomenclature

C_p	specific heat of the fluid at constant pressure
C_{pL}	specific heat of the particle at constant pressure
d	radius of the spherical particle
f	mass concentration of dust
g	acceleration due to gravity
Gr	Grashof number
h	non dimensional dust parameter
κ	thermal conductivity
K_1	Stokes resistance coefficient
K	nondimensional dust parameter
m	mass of a dust particle
N_0	number density of small dust particle
Nu_X	dimensionless local Nusselt number
$\overline{Nu}$	dimensionless average Nusselt number
Pr	Prandtl number

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T'	temperature
t	dimensionless time
u, v	velocity components of fluid in x, y-directions respectively
u_p, v_p	velocity components of dust particles in x, y-directions respectively
U, V	dimensionless velocity components of fluid in X, Y- directions respectively
U_p, V_p	dimensionless velocity components of dust particles in X, Y-directions respectively
x	spatial coordinate along the plate
X	dimensionless spatial coordinate along the plate
y	spatial coordinate normal to the plate
Y	dimensionless spatial coordinate normal to the plate

Greek symbols

α	thermal diffusivity
β	coefficient of volume expansion
λ	nondimensional parameter[λ=Pr(1+h)]
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	density
$\overline{\tau}$	average skin-friction
τ_X	dimensionless local skin-friction
$\overline{\tau}$	dimensionless average skin-friction

Subscripts

w	conditions at the wall
∞	conditions in the free stream

1 Introduction

Problem of two-dimensional free convection flow past a semi-infinite plate with different boundary conditions has attracted the attention of many researchers. However, in nature, the fluid in pure form is rarely available. Air and water contains impurities like dust particles and foreign bodies. This is connected with a wide range of natural occurring phenomena and practical applications. The study of the flow of dusty fluids is of practical importance, particularly through packed beds, sedimentation, environmental pollution, chemical reactors, combustion systems, pneumatic transport, and centifugal separation of particles.

Saffman[1] has formulated the basic equations for the flow of dusty fluid. Since then many researchers have discussed the problem of dusty fluid. Michael and Miller[2] studied the flow of a dusty gas past an impulsively started horizontal plate using the momentum equations given by saffman[3] and solved by Laplace transform technique. Micheal[3] considered the effect on the steady flow past a sphere of uniform upstream distribution of dust particles having a small relaxation time. Soundalgekar and Gokhale[4] studied the flow of a dusty gas past an impulsively started infinite vertical plate by employing an implicit finite difference technique.

An explicit finite difference solution of flow of a dusty gas past a uniformly accelerated horizontal plate in a viscous incompressible gas was presented by Das et al. [5]. Ganesan and Palani[6] studied numerical solution of Heat transfer effects on dusty gas flow past a semi-infinite inclined plate using an implicit finite difference method. Due to the importance of dusty viscous flows in other technological fields various studies have appeared in the literature. The effects of heat transfer on the flow of dusty gas past a semi-infinite isothermal vertical plate have not yet received the attention in the literature.

2 Mathematical Analysis

A transient, laminar, two-dimensional flow of a dusty gas past a semi-infinite isothermal vertical plate is considered. The X-axis is taken along the plate in the vertically upward direction and the Y-axis is taken normal to the plate. Initially, the plate and the dusty gas are assumed to be at the same temperature. The plate starts moving impulsively in the vertical direction with constant velocity u_0 against gravitational field and the temperature of the plate is also raised to T'_w. They are maintained at the same level for all time $t' > 0$.

In the present analysis, the following important assumptions are also made:

1. The dust particles are spherical in shape and are uniformly distributed.
2. Chemical reaction, mass transfer and radiation between the particles and fluid are not considered.
3. Interaction between particles themselves is not considered.
4. The buoyancy force is neglected.
5. The number density of dust particles is constant throughout the motion.
6. Viscous dissipation is negligible.

Under these assumptions, the governing equations of the flow based on Saffman model of dusty viscous incompressible fluid with usual Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T' - T'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} + \frac{k_1 n_0}{\rho}(u_p - u) \tag{2.2}$$

$$m\left(\frac{\partial u_p}{\partial t'} + u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \mathrm{K}_1(u - u_p) \tag{2.3}$$

$$\rho C_p\left(1 + \frac{m\mathrm{N}_0}{\rho}\frac{C_{pL}}{C_p}\right)\left(\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y}\right) = \kappa\frac{\partial^2 T'}{\partial y^2} \tag{2.4}$$

If the dust particles are spheres of radius d, then Stokes resistance coefficient $k_1 = 6\pi\mu d$, μ being the viscosity of the fluid.

The initial and boundary conditions are

$$\begin{array}{llllll} t' \leq 0: & u=0, & v=0, & u_p=0, & T'=T'_\infty & \\ t'>0: & u=u_0, & v=0, & u_p=0, & T'=T'_w, & \text{at } y=0 \\ & u=0, & u_p=0, & T'=T'_\infty, & \text{at } x=0 & \\ & u\to 0, & u_p\to 0, & T'\to T'_\infty, & \text{as } y\to\infty & \end{array} \tag{2.5}$$

On introducing the following non-dimensional quantities: ,

$$X=\frac{xu_0}{\nu},\quad Y=\frac{yu_0}{\nu},\quad U=\frac{u}{u_0},\quad V=\frac{v}{u_0},\quad U_p=\frac{u_p}{u_0},\quad V_p=\frac{u_p}{u_0},$$

$$t=\frac{t'u_0^2}{\nu},\quad T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{u_0^3},\ f=\frac{mn_0}{\rho} \tag{2.6}$$

$$Pr=\frac{\nu}{\alpha},\ h=\frac{fC_p}{C_{pL}},\ \lambda=Pr(1+h),\quad \kappa=\frac{\nu K_1}{mu_0^2}$$

Equations (1) to (4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y} = 0 \tag{2.7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y} = Gr\,T+\frac{\partial^2 U}{\partial Y^2}+f\kappa(U_p-U) \tag{2.8}$$

$$\frac{\partial U_p}{\partial t}+U_p\frac{\partial U_p}{\partial X}+V_p\frac{\partial U_p}{\partial Y} = \kappa(U-U_p) \tag{2.9}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y} = \frac{1}{\lambda}\frac{\partial^2 T}{\partial Y^2} \tag{2.10}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} t\leq 0: & U=0, & V=0, & U_p=0, & T=0 & \\ t>0: & U=1, & V=0, & U_p=0, & T=1 & \text{at } Y=0 \\ & U=0, & U_p=0, & T=0, & & \text{at } X=0 \\ & U\to 0, & U_p\to 0, & T\to 0 & & \text{as } Y\to\infty \end{array} \tag{2.11}$$

3 Numerical Technique

An implicit finite difference scheme of Crank-Nicolson type has been used to solve the governing non-dimensional equations (7) to (10) under the conditions (11). The finite difference equations corresponding to equations (7) to (10) are as follows:

$$\frac{\left[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}+U_{i,j-1}^{n+1}-U_{i-1,j-1}^{n+1}+U_{i,j-1}^{n}-U_{i-1,j-1}^{n}\right]}{4\triangle X} + \frac{\left[V_{i,j}^{n+1}-V_{i,j-1}^{n+1}+V_{i,j}^{n}-V_{i,j-1}^{n}\right]}{2\triangle Y}=0 \tag{3.1}$$

$$\frac{\left[U_{i,j}^{n+1}-U_{i,j}^{n}\right]}{\triangle t}+U_{i,j}^{n}\frac{\left[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}\right]}{2\triangle X}+V_{i,j}^{n}\frac{\left[U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}+U_{i,j+1}^{n}-U_{i,j-1}^{n}\right]}{4\triangle Y}$$
$$=\frac{Gr}{2}\left[T_{i,j}^{n+1}+T_{i,j}^{n}\right]+\frac{\left[U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}\right]}{2(\triangle Y)^{2}}$$
$$+f\,\kappa(U_{p(i,j)}^{n}-U_{i,j}^{n}) \tag{3.2}$$

$$\frac{\left[U_{p(i,j)}^{n+1}-U_{p(i,j)}^{n}\right]}{\triangle t}+U_{p(i,j)}^{n}\frac{\left[U_{p(i,j)}^{n+1}-U_{p(i-1,j)}^{n+1}+U_{p(i,j)}^{n}-U_{p(i-1,j)}^{n}\right]}{2\triangle X}$$
$$+V_{p(i,j)}^{n}\frac{\left[U_{p(i,j+1)}^{n+1}-U_{p(i,j-1)}^{n+1}+U_{p(i,j+1)}^{n}-U_{p(i,j-1)}^{n}\right]}{4\triangle Y}=\kappa(U_{i,j}^{n}-U_{p(i,j)}^{n}) \tag{3.3}$$

$$\frac{\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]}{\triangle t}+U_{i,j}^{n}\frac{\left[T_{i,j}^{n+1}-T_{i-1,j}^{n+1}+T_{i,j}^{n}-T_{i-1,j}^{n}\right]}{2\triangle X}+V_{i,j}^{n}\frac{\left[T_{i,j+1}^{n+1}-T_{i,j-1}^{n+1}+T_{i,j+1}^{n}-T_{i,j-1}^{n}\right]}{4\triangle Y}$$
$$=\frac{1}{\lambda}\frac{\left[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}\right]}{2(\triangle Y)^{2}} \tag{3.4}$$

Here the region of integration is considered as a rectangle with sides $X_{max}(=1)$ and $Y_{max}(=14)$, where Y_{max} corresponds to $Y=\infty$ which lies very well outside both the momentum and thermal boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (11) are satisfied with in the tolerance limit 10^{-5}. After experimenting with a few set of mesh sizes, the appropriate mesh sizes $\triangle X=0.05$, $\triangle Y=0.25$ with time step $\triangle t=0.01$ are considered for calculation.

The coefficients $U_{i,j}^{n}, U_{p(i,j)}^{n}, V_{i,j}^{n}$ and $V_{p(i,j)}^{n}$ appearing in the finite-difference equations are treated as constants in any one time step. Here i-designates the grid point along the X-direction, j along the Y-direction and k to the t-time. The values of U,V,U_p,V_p and T are known at all grid points at $t=0$ from the initial conditions.

The computations of U, V, U_p and T at time level $(n+1)$ using the values at previous time level (n) are carried out as follows: The finite difference Equation (15) at every internal nodal point on a particular i-level constitute a tridiagonal system of equations. Such a system of equations are solved by using Thomas algorithm as discussed in Carnahan *et al.*[7]. Thus, the values of T are found at every nodal point for a particular i at $(n+1)^{th}$ time level. Similarly, the values of U_p are calculated from Equation (14). Using the values of T and U_p at $(n+1)^{th}$ time level in the equation (13), the values of U at $(n+1)^{th}$ time level are found in a similar manner. Thus, the values of U_p, T and U are known on a particular i-level. Finally, the values of V is calculated explicitly using the Equation (12) at every nodal point on a particular i-level at $(n+1)^{th}$ time level. This process is repeated for various i-levels. Thus the values of T, U_p, U and V are known, at all grid points in the rectangular region at $(n+1)^{th}$ time level.

In a similar manner computations are repeated until the steady-state is reached. The steady-state solution is assumed to have been reached, when the absolute difference between the values of U, as well as temperature T and concentration U_p at two consecutive time steps are less than 10^{-5} at all grid points.

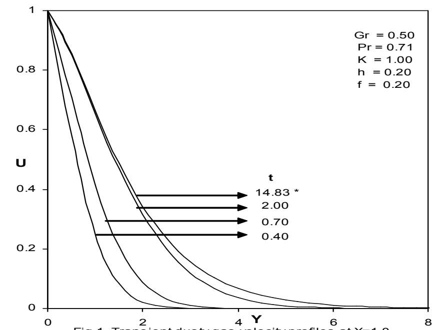

Fig.1. Transient dusty gas velocity profiles at X=1.0 (*-steady state)

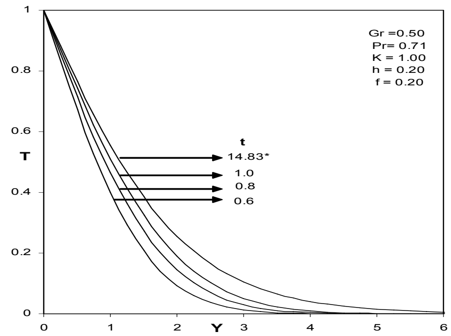

Fig.2. Transient temperature profiles at X = 1.0 (* -- steady state)

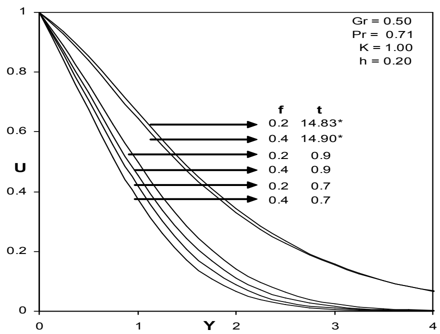

Fig.3. Transient dusty gas velocity profiles at X=1.0 for different f (*-steady state)

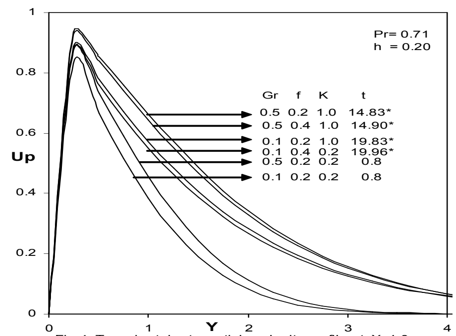

Fig.4. Transient dusty particle velocity profile at X=1.0 (*-steady state)

4 Results and Discussion

The velocity profiles for the dusty gas are shown in Fig.1. The velocity of the dusty gas increases with time. When t is small, the effects of dust particles are not significant. It is observed that due to the presence of dust particles in the fluid, time taken to reach the temporal maximum and steady state increases. However due to the presence of dust particles, the velocity of the gas decreases since these dust particles oppose the motion of the gas.

The temperature profiles for the fluid with dust particles are shown in Fig.2. It is observed that the temperature increases with time. Here time taken to reach the steady state increases since these dust particles carry away heat and the fluid gets cooled. The effects of the mass concentration of the dust f on the dusty gas velocity are shown in Fig.3. An increase of f leads to a fall in the gas velocity because these dust particles oppose the motion of the gas. It is observed that time taken to reach the steady state increases with f. Temporal maximum and steady state values for dusty particles velocity are shown in Fig.4. This figure shows the effect of f, K and Gr in dust particle velocity U_p. According to the numerical results, the velocity of the dust particles increases with an increasing value of K and Gr. The velocity of the dust particles decreases with increasing mass concentration of the dust. This is quite nature.

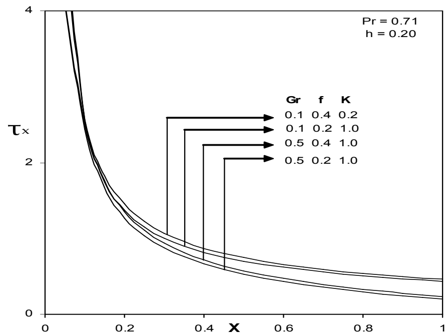

Fig.5. Local Skin-friction

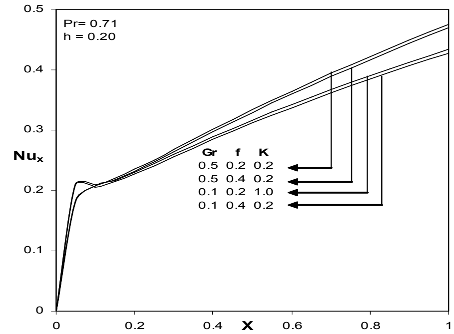

Fig.6. Local Nusselt number

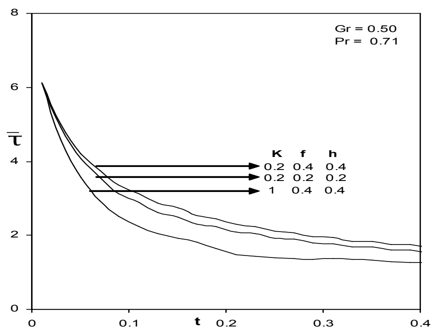

Fig.7. Average skin friction

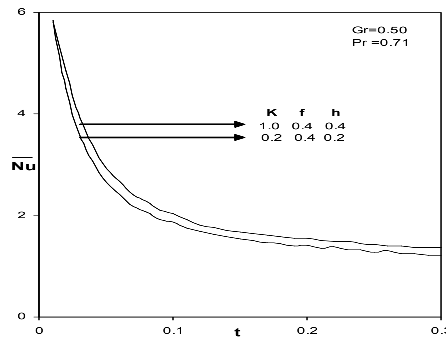

Fig.8. Average Nusselt number

Knowing the velocity and temperature field, it is customary to study the skin-friction and the Nusselt number. The local as well as average values of skin-friction and Nusselt number in dimensionless form are as follows:

$$\tau_X = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{4.1}$$

$$\overline{\tau} = -\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \tag{4.2}$$

$$Nu_X = -X\left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] \tag{4.3}$$

$$\overline{Nu} = -\int_0^1 \left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] dX \tag{4.4}$$

The derivatives involved in equations (16) to (19) are evaluated by using a five-point approximation formula and then the integrals are evaluated by Newton-Cotes closed integration formula.

The effects of f, K and Gr on local skin-friction are shown in Fig.5. The local wall shear stress increases as f increases. It is observed that local skin friction decreases as K increases. Local Nusselt number for different values of f, K and Gr are shown in Fig.6. It increases as X increases. It is observed that local Nusselt number increases by decreasing value of f and increasing value of Gr. The average values of skin-friction and Nusselt number are shown in figures 7 and 8 respectively. The average skin-friction decreases with increasing value of K and its increases by increasing value of f and h. The average Nusselt number increases with increasing value of K and h.

5 Conclusions

Finite difference study has been carried out for the dusty gas flow past a semi-infinite isothermal vertical plate. The dimensionless governing equations are solved by an implicit finite difference scheme of Crank-Nicolson method. Conclusions of the study are as follows:

1. Due to the presence of dust particles in the fluid, time taken to reach the temporal maximum and steady state increases.i.e.,the velocity of the gas decreases since these dust particles oppose the motion of the gas.
2. The temperature increases with time. The time taken to reach the steady state increases since these dust particles carry away heat and the fluid gets cooled.
3. The velocity of the dusty gas decreases with increasing mass concentration of the dust because these dust particles oppose the motion of the gas.
4. The velocity of the dust particles decreases with increasing mass concentration of the dust.
5. It is observed that local skin friction decreases as K increases and local Nusselt number increases by decreasing value of f and increasing value of Gr.
6. The average skin-friction decreases with increasing value of K and its increases by increasing value of f and h. The average Nusselt number increases with increasing value of K and h.

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