International Journal of Applied Mathematics and Computation
Volume 1(4),pp 194–204, 2009
http://ijamc.psit.in

Self Similar Solution of Strong Cylindrical Shock Wave in Magnetogasdynamics: Lagrangian Description

Mithilesh Singh, L. P. Singh and Akmal Husain

Department of Applied Mathematics, Institute of Technology, Banaras Hindu University, Varanasi-221005 India
Email: msingh.rs.apm@itbhu.ac.in

Abstract:
In the present paper the self similar solution of strong cylindrical shock wave in magnetogasdynamics using Langrangian mass co-ordinate has been studied. Analytic solutions of governing equations in closed form are obtained. The strong cylindrical shock wave generated by sudden line source explosion in an inhomogeneous medium of infinite electrical conductivity has been studied. The influence of specific heat ratio and magnetic field strength on flow variables for various cases is assessed. The general behaviour of velocity distribution remains unaffected. However, the density and pressure profiles are significantly affected in presence of magnetic field interaction.
Keywords: Cylindrical shock wave, self similar, magnetogas dynamics, Lagrangian description.

1 Introduction

In the present paper we analyze the self similar motion of strong cylindrical shock waves in magnetogasdynamics in Langrangian mass co-ordinate system. The well known Taylor-Sedov method of similarity solutions, which is based on dimensional considerations, have been used by many authors e.g., Sedov [1], Taylor [2], Roger [3], Sakurai [4] , Sharma et al. [5], to study the problem of strong explosion in different medium. Chisnell [12] provided an analytical description of converging shock waves by replacing the previous approach of numerical solutions of the ordinary differential equations by a theoretical study of the singular points of the differential equations. Different aspects of strong explosion theory were studied by many authors available in an excellent review [16]. The analytical solution of the corresponding self similar equations was also studied by Sedov [1] and Korobeinikov [6]. It is well known that one dimensional non stationary flows of the explosion type may be conveniently analyzed numerically in Langrangian variables in different medium. Misyuchenko et al. [13],Stepanov et.al. [14], [15], Zenkevich and Stepanov [16] presented the solution of the problem of a strong explosion in a perfect gas in Langrangian mass co-ordinates.

The present paper deals with the problem of strong cylindrical shock wave in presence of axial magnetic field in an inhomogeneous medium. Self similar solutions have been obtained

Corresponding author: **Mithilesh Singh**

in Lagrangian mass co-ordinate system. A new term total pressure in the paper refers to the sum of the gas dynamics pressure and magnetic pressure in the medium under consideration. A closed form solution of the governing system is obtained in terms of flow variables such as mass velocity, density, pressure, magnetic pressure and total pressure.

The effect of increasing values of specific heat ratio γ is to increase the values of flow variables. The effect of Alfvén Mach number on the density and pressure profiles show unique behaviour. For low Alfvén Mach number ($< \sqrt{60}$) the density and pressure profiles are reversed but for higher Alfvén Mach number ($\geq \sqrt{60}$) the profiles show similar behaviour as in non- magnetic case.

2 Governing equations

The basic equations describing the unsteady, one dimensional motion in magnetogasdynamics, where the applied magnetic field is orthogonal to the trajectories of the gas particle can be written in following form [6]

$$\rho_t + (u\rho)_x + \rho u x^{-1} = 0 \tag{2.1}$$

$$u_t + uu_x + \rho^{-1}(p_x + h_x) = 0 \tag{2.2}$$

$$(p\rho^{-\gamma})_t + u(p\rho^{\gamma})_x = 0, \tag{2.3}$$

$$h_t + uh_x + 2h(u_x + ux^{-1}) = 0, \tag{2.4}$$

where x is the cylindrical radial co-ordinate; t is the time; $\rho(x,t)$ is the density; $u(x,t)$ is the velocity; $p(x,t)$ is the pressure and $h(x,t)$ is the magnetic pressure and γ is the specific heat ratio. Subscripts denote partial differentiation unless stated otherwise.
The above basic equations are supplemented with an equation of state $p = \rho RT$, where R is the gas constant and T is the temperature.
In order to obtain the closed form solution in terms of flow variables behind the shock wave, the pressure is taken in the form [18]

$$p = \beta p^* \text{ and } 0 < \beta < 1 \tag{2.5}$$

where $p^* = p + h$, is the sum of gasdynamic pressure and magnetic pressure.
Combining (2.1)-(2.4) and using equation (2.5), the law of conservation of energy may be expressed as

$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon^* + \frac{u^2}{2}\right)\right] + \frac{1}{x}\frac{\partial}{\partial x}\left[x\rho u\left(\varepsilon^* + \frac{p^*}{\rho} + \frac{u^2}{2}\right)\right] = 0, \tag{2.6}$$

where ε^* is specific internal energy written as

$$\varepsilon^* = \frac{p^*}{\rho(\gamma^* - 1)} \tag{2.7}$$

with

$$\gamma^* = \beta\gamma + 2(1 - \beta) \tag{2.8}$$

Also, the combination of the (2.3) and (2.4) and using (2.5) and (2.8) yields

$$\frac{\partial}{\partial t}(p^*\rho^{-\gamma^*}) + u\frac{\partial}{\partial x}(p^*\rho^{-\gamma^*}) = 0 \tag{2.9}$$

expressing the constancy of the entropy of a gas particle which holds true behind the shock front.
Introducing the non-dimensional variable η defined as

$$\eta = \frac{x}{x_f} \text{ where } x_f = \alpha\left(\frac{E}{\rho_0}\right)^{\frac{1}{4}} t^{\frac{1}{2}} \tag{2.10}$$

where E is the explosion energy per unit length and ρ_0 is the density of the gas in an unperturbed region. The proportionality factor α is determined from the law of conservation of energy.
To describe the flow field behind the shock, the boundary conditions for the strong discontinuity at the shock may be written as [6]

$$x = x_f,\, u_f = \frac{2}{\gamma+1}C, \rho_f = \left(\frac{\gamma+1}{\gamma-1}\right)\rho_0, p_f = \frac{2}{\gamma+1}\rho_0 C^2, h_f = \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2\frac{1}{M_A^2}\rho_0 C^2, \tag{2.11}$$

where $M_A = C/c$ is the Alfvén Mach number with Alfvén speed $c = \sqrt{a^2+b^2}$, sound speed $a = (\gamma p/\rho)^{1/2}$, magneto sonic speed $b = (2h/\rho)^{1/2}$ and C is the wave velocity expressed in terms of the co-ordinates at the wave front as

$$C = \frac{dx_f}{dt} = \frac{x_f}{2t} \tag{2.12}$$

3 Lagrangian Equation of Strong Explosion

To determine the solution of the problem in Lagrangian variables we introduce the mass coordinate according to the relation

$$dm = \sigma\,\rho\,x\,dx, \tag{3.13}$$

where $\sigma = 2\pi$.
If m is reckoned off from the axis of symmetry, the mass coordinate of point with an Eulerian coordinate x is the mass of the gas in the domain [0, x]. In terms of mass coordinate, basic equations (2.1)-(2.4) reduces to the following form

$$\frac{du}{dt} + \sigma\,x\frac{dp^*}{dm} = 0, u = \frac{dx}{dt}, \tag{3.14}$$

$$\frac{d}{dt}\left(\varepsilon^* + \frac{u^2}{2}\right) + \sigma\frac{d}{dm}(xup^*) = 0, \tag{3.15}$$

$$\frac{d}{dt}\left(p^{*}\rho^{-\gamma^{*}}\right)=0, \tag{3.16}$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}$.
We introduce the mass coordinate of shock wave front. Since the wave moves in the medium with a constant density, using the transformation (3.13), (2.10) reduces to

$$m_f=\frac{\sigma\rho_0 x^2}{2}=\alpha^2\frac{\sigma}{2}\left(\rho_0 E\right)^{1/2}t. \tag{3.17}$$

Since the pressure of the gas a head of the shock wave can be neglected in strong shock limit, the law of conservation of total energy in the region of motion reduces to

$$E=\int_0^{m_f}\left(\varepsilon^{*}+\frac{u^2}{2}\right)dm. \tag{3.18}$$

Let us introduce a self similar mass coordinate

$$\xi=\frac{m}{m_f}. \tag{3.19}$$

The field variables describing the flow field can then be written in terms of the dimensionless functions of ξ as

$$\begin{aligned}
&x=x_f,\ u=\frac{2}{\gamma+1}CU\left(\xi\right),\rho=\frac{\gamma+1}{\gamma-1}\rho_0 G\left(\xi\right),\ p=\frac{2}{\gamma+1},\\
&h=\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2\frac{1}{M_A^2}\rho_0 C^2 H\left(\xi\right),\ p^{*}=\frac{2}{\gamma+1}\rho_0 C^2 P^{*}\left(\xi\right)
\end{aligned} \tag{3.20}$$

Introducing the specific volume $\rho^{-1}=\rho_0^{-1}\left(\gamma-1\right)V\left(\xi\right)/\left(\gamma+1\right)$and using this value and equation (3.20) in (3.14)-(3.16) yields

$$\frac{\gamma-1}{\gamma+1}V=2\eta\frac{d\eta}{d\xi}, \tag{3.21}$$

$$U+2\xi\frac{dU}{d\xi}=2\eta\frac{dP^{*}}{d\xi} \tag{3.22}$$

$$\frac{2}{\gamma+1}U=\eta-2\xi\frac{d\eta}{d\xi} \tag{3.23}$$

$$\frac{d}{d\xi}\left[\xi\left(\Gamma P^{*}V+U^2\right)\right]=\frac{d}{d\xi}\left(2\eta P^{*}U\right) \tag{3.24}$$

$$\frac{d}{d\xi}\left(\xi P^{*}V^{\gamma^{*}}\right)=0, \tag{3.25}$$

where

$$\Gamma = \frac{\gamma - 1}{\gamma^* - 1}.$$

The Boundary conditions for solutions of equation (3.21) - (3.24) may be written as

$$\xi = 0, U = 0, \xi = 1, \eta = 1, U = 1, V = 1, P = 1, H = \frac{1}{\beta} - 1, \tag{3.26}$$

where

$$\beta = \frac{\frac{2}{\gamma+1}}{\frac{2}{\gamma+1} + \frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)^2 \frac{1}{M_A^2}}. \tag{3.27}$$

4 Analysis and Solution

Integration (3.24) and (3.25) and using boundary conditions (3.26) yield two algebraic equations expressing the law of conservation of energy and constancy of the entropy of a gas particle behind the shock front:

$$\xi\left(\Gamma P^* V + U^2\right) = 2\eta P^* U, \xi P^* V^{\gamma^*} = 1 \tag{4.28}$$

To analyze the given system introduce dimensionless temperature$W$

$$W = \Gamma P^* V. \tag{4.29}$$

Using (4.29) in (3.24) and (3.25), we get the expressions for non-dimensional flow variablesP^*, $V$and η as

$$P^* = \frac{W}{\Gamma}\left(\frac{\xi\beta W}{\Gamma}\right)^{\frac{1}{\gamma^*-1}}, V = \left(\frac{\xi\beta W}{\Gamma}\right)^{\frac{-1}{\gamma^*-1}}, \eta = \frac{W + U^2}{2W^2 U\beta}\Gamma^{\frac{\gamma^*}{(\gamma^*-2)}}\left(\xi\beta W\right)^{\frac{\gamma^*-2}{\gamma^*-1}}. \tag{4.30}$$

Substituting the values evaluated in (4.30) into equations (3.21)-(3.23) we get a system of two ordinary differential equations given as

$$\xi\frac{dU}{d\xi} = \frac{a_3 b_2 - b_3 a_2}{a_1 b_2 - b_1 a_2}, \tag{4.31}$$

$$\xi\frac{dW}{d\xi} = \frac{a_1 b_3 - b_1 a_3}{a_1 b_2 - b_1 a_2}, \tag{4.32}$$

where

$$a_1 = 2\left[\Gamma(W+U^2)^2 + (\gamma-1)W(W-U^2)\right], a_2 = 2(\gamma-1)\Gamma U^3,$$

$$a_3 = (\gamma-1)WU(W+U^2), b_1 = 2(\gamma^*-1)WU, b_2 = -\gamma^*(W+U^2),$$

$$b_3 = W^2 + (2-\gamma^*)WU^2.$$

Combining equations (4.31) and (4.32) we get the following relation

$$\frac{dW}{dU} = \frac{A-B}{-(\gamma-1)\gamma^*\frac{\Gamma}{\beta}WU\left(\frac{\Gamma}{\beta}W+U^2\right)^2 - 2(\gamma-1)U^3\left[\left(\frac{\Gamma W}{\beta}\right)^2 + (2-\gamma^*)\frac{\Gamma}{\beta}WU^2\right]} \tag{4.33}$$

where

$$A = 2\left[\Gamma\left(\frac{\Gamma}{\beta}W+U^2\right)^2 + (\gamma-1)\frac{\Gamma}{\beta}W\left(\frac{\Gamma}{\beta}W-U^2\right)\right]\left[\left(\frac{\Gamma W}{\beta}\right)^2 + (2-\gamma^*)\frac{\Gamma}{\beta}WU^2\right]$$

and

$$B = 2(\gamma^*-1)(\gamma-1)\left(\frac{\Gamma W}{\beta}\right)^2 U^2\left(\frac{\Gamma}{\beta}W+U^2\right)$$

To simplify further the above equation, let us introduce a new variable z defined as

$$W = \frac{\Gamma}{\beta}U^2 z \tag{4.34}$$

Substituting the transformation defined by (4.34) in (4.33) we obtain

$$\frac{1}{U}\frac{dU}{dz} = \frac{-(\gamma-1)\frac{\Gamma}{\beta}\left[\gamma^*\left(\frac{\Gamma}{\beta}z+1\right)^2 + 2\left(\frac{\Gamma}{\beta}z+2-\gamma^*\right)\right]}{\left(\frac{\Gamma}{\beta}z+1\right)\left(\gamma^*\frac{\Gamma}{\beta}z+1\right)\left[2\gamma^*\frac{\Gamma}{\beta}z+2(2-\gamma^*)\right]}, \tag{4.35}$$

which may further be reduced to

$$\frac{1}{U}\frac{dU}{dz} = \frac{N_1}{\frac{\Gamma}{\beta}z+1} - \frac{N_2}{\gamma^*\frac{\Gamma}{\beta}z+1} - \frac{N_3}{2\gamma^*\frac{\Gamma}{\beta}z+2(2-\gamma^*)}, \tag{4.36}$$

where

$$N_1 = \frac{\Gamma^2}{2\beta},$$

$$N_2 = \frac{(\gamma - 1)\,\Gamma}{2\beta},$$

$$N_3 = \frac{\gamma^*\,(\gamma - 1)\,\Gamma}{(\gamma^* - 1)\,\beta}.$$

Integration of eqn. (4.36) with boundary condition $U = 1$ and$z = 1$ we get the following expression for velocity distribution

$$U = (R)^{\frac{N_1\beta}{\Gamma}}\,(S)^{\frac{N_2\beta}{\gamma^*\Gamma}}\,(T)^{\frac{N_3\beta}{2\gamma^*\Gamma}} \tag{4.37}$$

where

$$R = \frac{\frac{\Gamma}{\beta}z + 1}{\frac{\Gamma}{\beta} + 1},\quad S = \frac{\gamma^*\frac{\Gamma}{\beta} + 1}{\gamma^*\frac{\Gamma}{\beta}z + 1},\quad T = \frac{2\gamma^*\frac{\Gamma}{\beta} + 2\,(2 - \gamma^*)}{2\gamma^*\frac{\Gamma}{\beta}z + 2\,(2 - \gamma^*)}$$

and the transformation relation (4.34) reduces to

$$W = \frac{\Gamma}{\beta}z\,(R)^{\frac{2N_1\beta}{\Gamma}}\,(S)^{\frac{2N_2\beta}{\gamma^*\Gamma}}\,(T)^{\frac{N_3\beta}{\gamma^*\Gamma}} \tag{4.38}$$

Equation (4.37) and (4.38) represent the parametric solutions of Lagrangian strong shock equations in magnetogasdynamics for mass velocity and temperature. For further analysis we concentrate ourselves to the solution of eqn (4.31). Using the eqns (4.34) and (4.35) in (4.31)-(4.32) and simplifying we get the following differential equation relating z and ξ as

$$\frac{1}{\xi}\frac{d\xi}{dz} = \frac{-2\left[\left\{\left(\frac{\Gamma}{\beta}z + 1\right)^2 + (\gamma - 1)\left(\frac{\Gamma}{\beta}z\right)^2\right\} + (\gamma - 1)\right]}{z\left(\frac{\Gamma}{\beta}z + 1\right)\left(2\gamma^*\frac{\Gamma}{\beta}z + 2\,(2 - \gamma^*)\right)}, \tag{4.39}$$

which on further simplification we get

$$\frac{1}{\xi}\frac{d\xi}{dz} = \frac{L_1}{z} - \frac{L_2}{\frac{\Gamma}{\beta}z + 1} - \frac{L_3}{2\gamma^*\Gamma z + 2\,(2 - \gamma^*)}, \tag{4.40}$$

where

$$L_1 = \frac{\gamma + \Gamma - 1}{\gamma^* - 2},$$

$$L_2 = \frac{\Gamma^2}{\beta},$$

$$L_3 = \frac{2\Gamma}{\beta}\left(\Gamma + \gamma - 1 - \frac{\gamma^*\Gamma}{2} - \frac{\gamma^*\Gamma\,(2 + \gamma^*)}{2\,(2 - \gamma^*)}\right).$$

Integration of equation (4.40) with boundary conditions $\xi = 1$ at $z = 1$ yields

$$\xi = z^{L_1} \left(\frac{1}{R}\right)^{\frac{\beta L_2}{\Gamma}} (T)^{\frac{\beta L_3}{2\gamma^* \Gamma}} . \tag{4.41}$$

$$G = z^{\frac{L_1+1}{(\gamma^*-1)}} \left(\frac{1}{R}\right)^{\frac{\beta(-2N_1+L_2)}{\Gamma(\gamma^*-1)}} (S)^{\frac{2\beta N_2}{\gamma^*\Gamma(\gamma^*-1)}} (T)^{\frac{\beta(2N_3+L_3)}{2\gamma^*\Gamma(\gamma^*-1)}} , \tag{4.42}$$

$$P^* = \frac{1}{\beta} z^{\frac{L_1+\gamma^*}{(\gamma^*-1)}} \left(\frac{1}{R}\right)^{\frac{\beta(-2\gamma^* N_1+L_2)}{\Gamma(\gamma^*-1)}} (S)^{\frac{2\gamma^*\beta N_2}{\gamma^*\Gamma(\gamma^*-1)}} (T)^{\frac{\beta(2\gamma^* N_3+L_3)}{2\gamma^*\Gamma(\gamma^*-1)}} \tag{4.43}$$

Also the non-dimensional pressure, magnetic pressure and non-dimensional variable η may be written as

$$P = z^{\frac{L_1+\gamma^*}{(\gamma^*-1)}} \left(\frac{1}{R}\right)^{\frac{\beta(-2\gamma^* N_1+L_2)}{\Gamma(\gamma^*-1)}} (S)^{\frac{2\gamma^*\beta N_2}{\gamma^*\Gamma(\gamma^*-1)}} (T)^{\frac{\beta(2\gamma^* N_3+L_3)}{2\gamma^*\Gamma(\gamma^*-1)}} , \tag{4.44}$$

$$H = \left(\frac{1}{\beta} - 1\right) z^{\frac{(L_1+\gamma^*)}{(\gamma^*-1)}} \left(\frac{1}{R}\right)^{\frac{\beta(-2\gamma^* N_1+L_2)}{\Gamma(\gamma^*-1)}} (S)^{\frac{2\gamma^*\beta N_2}{\gamma^*\Gamma(\gamma^*-1)}} (T)^{\frac{\beta(2\gamma^* N_3+L_3)}{2\gamma^*\Gamma(\gamma^*-1)}} \tag{4.45}$$

$$\eta = \frac{\beta\left(\frac{\Gamma}{\beta}z+1\right)}{2z^2\Gamma^{\frac{\gamma^*}{(2-\gamma^*)(\gamma^*-1)}}} \left[z^{\frac{L_1+1}{(\gamma^*-1)}} \left(\frac{1}{R}\right)^{\left(\frac{\beta(-2N_1+L_2)}{\Gamma(\gamma^*-1)}\right)} (S)^{\frac{2\beta N_2}{\gamma^*\Gamma(\gamma^*-1)}} (T)^{\frac{\beta(2N_3+L_3)}{2\gamma^*\Gamma(\gamma^*-1)}} \right]^{(2-\gamma^*)} \tag{4.46}$$

Eqn. (4.37)-(4.38) and (4.41)-(4.46) completely determine the parametric solution of self similar strong shock equations in magnetogasdynamics. Also the parameter α appearing in (2.10) may be determined from the conservation of energy eqn. (3.18)

$$\alpha = \left[\frac{\pi}{2(\gamma+1)^2} \int_0^1 \left(\Gamma P^* V + U^2\right) d\xi\right]^{-\frac{1}{4}} . \tag{4.47}$$

5 Results and Discussions

Numerical computations have been performed to determine the values of dimensionless parameter α as a function of γ for different values of Alfvén Mach number M_A. The results of calculation have been plotted in fig.11. This quantity can be evaluated by making an assumption that entire mass of the gas involved in motion has the parameters corresponding to the shock front.
Also, we have $V = 1, \quad U = 1, \quad P = 1, P^* = \frac{1}{\beta}$.
In light of the above assumptions and the boundary conditions, eqn. (4.47) reduces to

$$\alpha = \left[\frac{\pi(2\Gamma+\beta)}{4\beta(\gamma+1)^2}\right]^{-\frac{1}{4}} . \tag{5.48}$$

For the case $\gamma = 2$, eqn. (2.8) yields $\gamma^* = 2$ and $\Gamma = 1$ for all values of β. Also for very large Alfvén Mach number i.e. $M_A \to \infty$, eqn. (3.27) yields $\beta = 1$ and$\Gamma = 1$. In both cases whole analysis reduces as in the non-magnetic case [15]. Numerical values of non-dimensional flow variables $U(\xi),\ G(\xi),\ P(\xi),\ H(\xi)\ ,\ P^*(\xi)$ and $\eta(\xi)$ have been computed using the relations (4.37) and (4.42)-(4.46) and corresponding profiles are presented in figs. 1 - 10 for various values of specific heat ratio, γ and Alfvén Mach number M_A. Fig.1 and fig.5 shows that the general behavior of velocity profiles are not greatly affected by the magnetic field strength for $\gamma = 1.4$ and 1.67. However, there is an increase in magnitude of mass velocity with an increase in magnetic field strength.

Further, fig.2 and fig.3 show that the general behaviour of density and pressure profiles for the value of $\gamma = 1.4$ are similar as in non-magnetic case for the Alfvén Mach numbers greater than the critical value where as for the Alfvén Mach numbers less than the critical value, the effect of magnetic field strength serves to completely reverse the trend of the profiles relative to what it would be in the absence of the magnetic field.

It may be noted that the higher values of Alfvén Mach number M_A corresponds to very weak ambient magnetic field and the corresponding flow is very close to that of the non-magnetic case Rosenau and Frankenthal[20]. Further, the lower values of Alfvén Mach number $M_A$corresponds to increasing magnetic field. For the values of M_A less than the critical value, there is magnetically dominated layer in the flow field behind the shock wave which causes very high increase in the gas pressure and the magnetic pressure and shows unique behaviour.

In case of $\gamma = 1.67$ the general behavior of density and pressure profiles remain same as in case of non-magnetic case for all values ofM_A. However, there is an increase in magnitude of density and pressure distribution with increasing magnetic field strength which can be seen from fig. 7 and fig.8. The profile for non-dimensional variable η is presented in fig. 9 and fig.10 for $\gamma = 1.4$ and 1.67 for various values of M_A and show an increasing trend with an increase in magnetic field strength and specific heat ratio γ.

6 Conclusions

In this paper the classical similarity solution for a strong cylindrical shock wave is extended in magnetogasdynamic regime. Analytic solution of governing equations in closed form in Langrangian mass coordinate system is obtained. The case when specific heat ratio $\gamma = 2$, which is of special interest in magnetogasdynamics, has been analyzed. The effect of magnetic field strength, which enters into the solution through Alfvén Mach number, on flow variables in disturbed medium is assessed. It was observed that when the Alfvén Mach number is less than a critical value, the distribution of flow variables show a unique behaviour which is demonstrated through fig.2 and fig.3.

Acknowledgments

Financial support from Institute of Technology, Banaras Hindu University, Varanasi, INDIA and UGC, INDIA is gratefully acknowledged.

References

[1] L. I. Sedov, Similarity and Dimensional Methods in Mechanics, Academic, New York, 1959.

[2] G. I. Taylor, Proc. Roy. Soc. 201, (1950) 159-174.

[3] M. H. Rogers, Astrophysics. J. 125, (1957), 478-493.

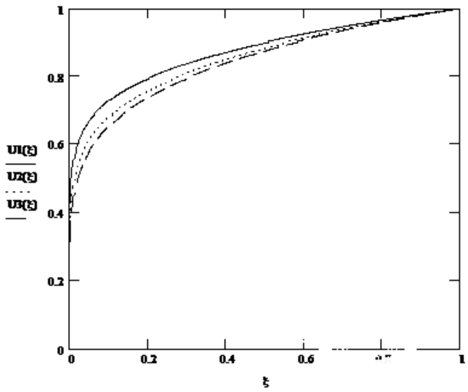

Figure 1: Self-similar profiles of dimensionless mass velocity $U1(\xi) - M_A = \sqrt{50}$ solidline; $U2(\xi) - M_A = 10$ dotted line; $U3(\xi) - M_A = 10\sqrt{10}$ dash line, for $\gamma = 1.4$.

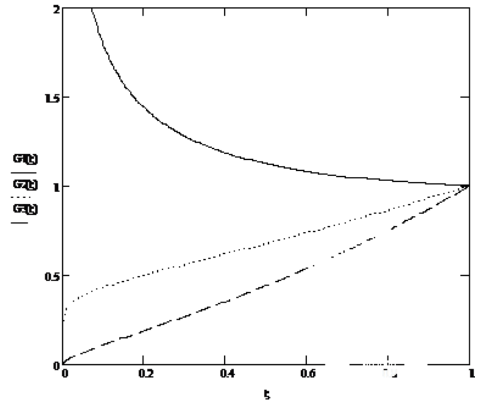

Figure 2: Self-similar profiles of dimensionless density $G1(\xi) - M_A = \sqrt{50}$ solid line; $G2(\xi) - M_A = 10$ dotted line; $G3(\xi) - M_A = 10\sqrt{10}$ dash line, for $\gamma = 1.4$.

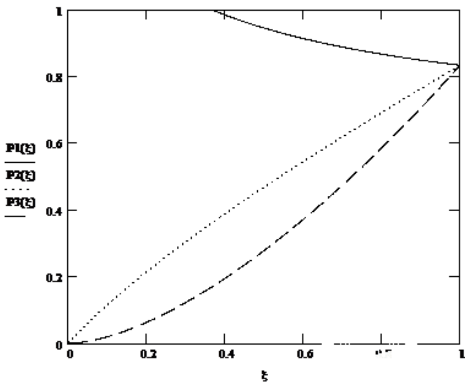

Figure 3: Self-similar profiles of dimensionless pressure flow variables $P1(\xi) - M_A = \sqrt{50}$ solid line; $P2(\xi) - M_A = 10$ dotted line; $P3(\xi) - M_A = 10\sqrt{10}$ dash line, for $\gamma = 1.4$.

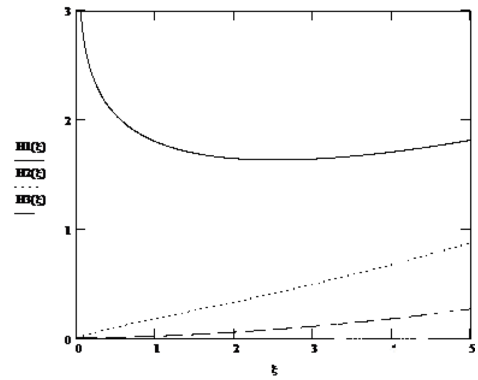

Figure 4: Self-similar profiles of dimensionless magnetic pressure $H1(\xi) - M_A = \sqrt{50}$ solid line; $H2(\xi) - M_A = 10$ dotted line; $H3(\xi) - M_A = 10\sqrt{10}$ dash line , for $\gamma = 1.4$.

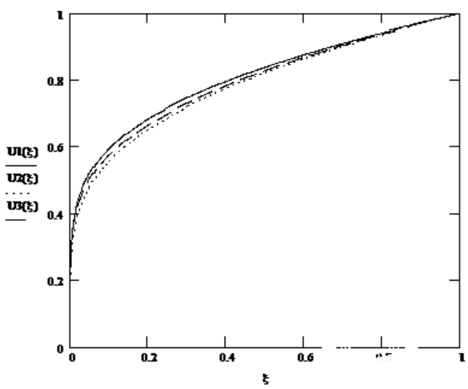

Figure 5: Self-similar profiles of dimensionless mass velocity $U1(\xi) - M_A = \sqrt{50}$ solid line; $U2(\xi) - M_A = 10$ dotted line ; $U3(\xi) - M_A = 10\sqrt{10}$dash line, for $\gamma = 1.67$.

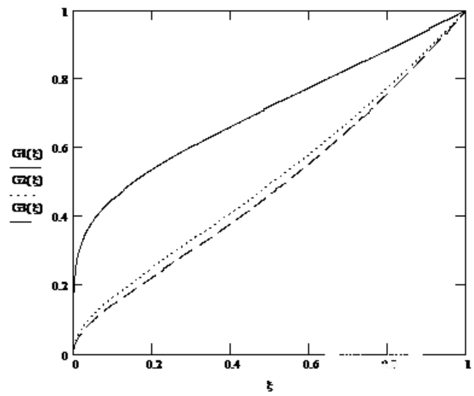

Figure 6: Self-similar profiles of dimensionless density $G1(\xi) - M_A = \sqrt{50}$ solid line; $G2(\xi) - M_A = 10$ dotted line; $G3(\xi) - M_A = 10\sqrt{10}$ dash line, for $\gamma = 1.67$.

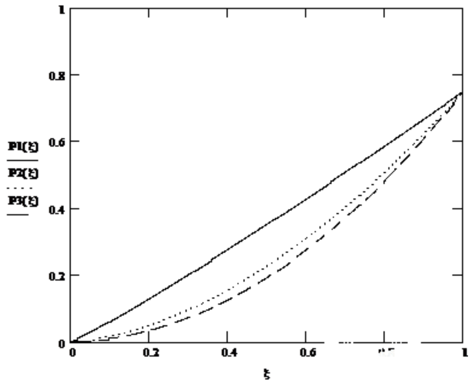

Figure 7: Self-similar profiles of dimensionless pressure $P1(\xi) - M_A = \sqrt{50}$solid line; $P2(\xi) - M_A = 10$dotted line;$P3(\xi) - M_A = 10\sqrt{10}$dash line, for $\gamma = 1.67$.

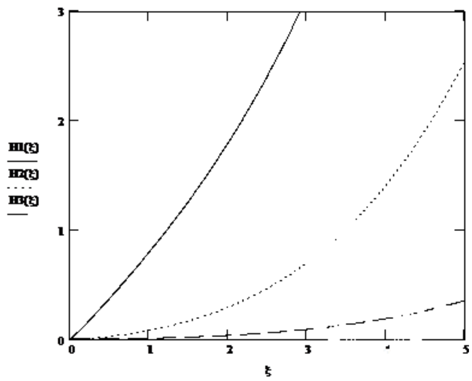

Figure 8: Self-similar profiles of dimensionless magnetic pressure $H1(\xi) - M_A = \sqrt{50}$solid line; $H2(\xi) - M_A = 10$ dotted line; $H3(\xi) - M_A = 10\sqrt{10}$ dash line, for $\gamma = 1.67$.

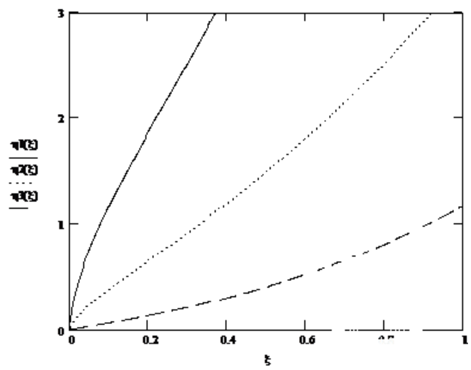

Figure 9: Dimensionless radial component $\eta 1(\xi) - M_A = \sqrt{10}$ solid line; $\eta 2(\xi) - M_A = 10$, dotted line; $\eta 3(\xi) - M_A = 10\sqrt{10}$ and dash line, for $\gamma = 1.4$.

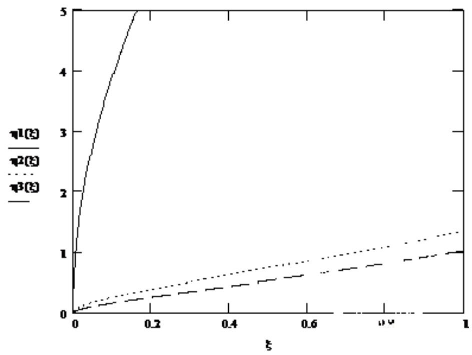

Figure 10: Dimensionless radial component $\eta 1(\xi) - M_A = \sqrt{10}$solid line; $\eta 2(\xi) - M_A = 10$,dotted line; $\eta 3(\xi) - M_A = 10\sqrt{10}$ and dash line, for $\gamma = 1.67$.

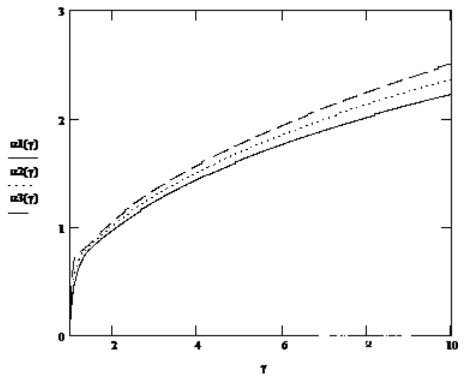

Figure 11: Dimensionless parameter $\alpha 1(\gamma) - M_A = \sqrt{10}$ solid line; $\alpha 2(\gamma) - M_A = 10$, dotted line; $\alpha 3(\gamma) - M_A = 10\sqrt{10}$ and dash line.

[4] A. Sakurai, J. Phys. Soc. Japan, 8 (1953), 662-669.

[5] L.P. Singh, V.D. Sharma and R. Ram, Phys. Fluids B, 1, (1989), 692-699.

[6] V. P. Korobeinikov, Problems in the Theory of Point Explosion in Gases, American Mathematical Society, Providence, RI, 1976.

[7] H. L. Brode, J. Appl. Phys., 26, (1955), 766-775.

[8] M. P. Friedman, J. Fluid. Mech., 11, (1961), 1-15.

[9] G. G. Bach and J. H. S. Lee, AIAA Journal, 8, (1970), 271-275.

[10] P. L. Sachdev, J. Fluid. Mech., 50, (1971), 669-674.

[11] P.L. Sachdev, J. Fluid. Mech., 52, (1972), 369-378.

[12] R. F. Chisnell, J. Fluid. Mech., 354, (1998) 357-375.

[13] N. I. Misyuchenko, G. S. Romanov, L. V. Rudak, K. L. Stepanov and A. V. Teterev, Izv. Akad. Nauk SSSR, Ser. Fizicheskaya, 55, No. 7, (1991), 1313-1321.

[14] K. L. Stepanov, B. N. Bazylev, and G. S. Romanv, Pisma Zh. Tekh. Fiz, 1, No. 6, (1975), 277-281.

[15] G. S. Romanov, B. N. Bazylev, and K. L. Stepanov, Dokl. Akad. Nauk BSSR, 22, No. 2, (1978), 138-141.

[16] S. M. Zenkevich and K. L. Stepanov, J. Eng. Phys. and Thermophysics, 80, No. 1, (2007), 89-96.

[17] Ya. B. Zeldovich and Yu. P. Raizer, Academic, New York, 1966.

[18] J.P. Vishwakarma and A.K. Yadav, Eur.Phys. J.B 34, (2003). 247-253

[19] G. B. Whitham, Linear and nonlinear waves, John Wiley & Sons, New York (1974).

[20] P. Rosenau and S. Frankenthal, Phys. Fluids 19, (1976) 1889-1899.