Journal of Remanufacturing
a SpringerOpen Journal

RESEARCH Open Access

Analysis of the quantities of the remanufacturing plan of perfect cost

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Abstract

Background: The remanufacturing plan of perfect cost makes reference to the remanufacturing plan of an optimal solution of the economic lot-sizing problem with remanufacturing (ELSR). In this paper, we address the problem of determining the quantities of the remanufacturing plan of perfect cost in an independent way.

Results: Assuming that the periods where remanufacturing is carried out are known in advance and certain other assumptions on the costs, we can show that the total remanufacturing quantity of a remanufacturing plan of perfect cost can be determined separately and in a time-effective way.

Conclusions: We consider that the theoretical results obtained in this paper contribute to a deeper knowledge of the characteristics of the ELSR optimal solutions. Thus, the results obtained can be used to develop an effective algorithm for solving the ELSR.

Keywords: Remanufacturing, Economic lot-sizing problem, Inventory control, Reverse logistics

Background

We consider a single item economic lot-sizing problem where the demand can also be satisfied by remanufacturing used items backed to the origin. This problem is commonly known in the literature as the economic lot-sizing problem with remanufacturing (ELSR) and refers to the problem of determining the quantities to produce, remanufacture, and dispose in each period over a finite planning horizon in order to meet the demand requirements of a single item on time, minimizing the involved costs. Used products returned by the customers are available at each period for remanufacturing. In addition, we consider that the returns can be disposed off, e.g., when there is an overstock of used products. This kind of problem has been receiving an increasing academic attention in recent years as the industry has been involved with the recovery of used products. This has been the result of governmental and social pressures as well as economic opportunities. Remanufacturing can be defined as the recovery process of returned products after which it is warranted that the remanufactured products offer the same quality and functionality than those newly manufactured [1]. Remanufacturing tasks often involve disassembly, cleaning, testing, part replacement, and reassembling operations. Products that are remanufactured include automotive parts, engines, tires, aviation equipment, cameras, medical instruments, furniture, toner cartridges, copiers, computers, and telecommunication equipment. Among the recovery options, the remanufacturing offers benefits for all of the parties involved. We refer the readers to de Brito and Dekker [2], Guide [3], Gungor and Gupta [4], and Hormozi [5] for the detailed descriptions about the remanufacturing benefits.

This paper is focused on the analysis of the quantities of the remanufacturing plan of an optimal solution of the ELSR that we refer as the remanufacturing plan of perfect cost. The remanufacturing plan plays a key role in the ELSR resolution since both the optimal production plan and the optimal final disposal plan can be determined separately and in an effective time way if the remanufacturing plan is known [6]. Thus, we can say that solving the ELSR reduces to the problem of finding the remanufacturing plan of perfect cost, i.e., the remanufacturing plan of an optimal solution of the ELSR. We note that the problem of finding a remanufacturing plan of perfect cost is NP-hard, since it is equivalent to the

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ELSR which is a known NP-hard problem even under stationary cost structures [7-9]. Considering this difficulty, we tackle the problem of determining the quantities of a remanufacturing plan of perfect cost under the assumption that the periods where the remanufacturing is carried out are known in advance. This can occur in practice if cores, parts, machinery, or workers are only available in certain periods within the planning horizon. We also assume certain constrains on the costs that can be fulfilled in the real life, such as non-speculative motives or that the costs related to used items are at most equal to those related to new items. In addition, we provide a constraint on the costs which makes it more profitable to maximize the remanufacturing quantity in those periods where remanufacturing is allowed. This can be fulfilled in practice if the unit cost of producing is much greater than other unit costs of the problem, or in those cases for which the inventory holding costs can be neglected.

The rest of the paper is organized as follows: the 'Background' section finishes with a short literature review, followed by the problem formulation. The 'Results' section is devoted to the analysis of the quantities of the remanufacturing plan of perfect cost with fixed periods for remanufacturing. In the 'Discussion' section, we present a numerical example along with a discussion about the effectiveness of the theoretical results obtained. The 'Conclusions' section concludes the paper with possible directions for future research.

Literature review

According to our best knowledge, Richter and Sombrutzki [10] and Richter and Weber [11] are the first to analyze the ELSR. They consider the particular case that the returns in the first period are sufficient to satisfy the total demand over the planning horizon. Golany et al. [7] suggest a network flow formulation for the ELSR and provide an exact algorithm of $O(T^3)$ time for the case of linear cost functions. They also show that the ELSR is a NP-hard problem for the case of general concave cost functions. Yang et al. [8] show the same result of complexity for the case of stationary concave cost functions and provide a heuristic procedure of $O(T^4)$ time for the ELSR. van den Heuvel [9] shows that ELSR is NP-hard for the case of the setup and unit costs for the activities and unit costs for holding inventory, even in the case that they are stationary. Teunter et al. [12] consider the ELSR with joint setup costs for the production and remanufacturing activities and suggest an $O(T^4)$ time algorithm based on a dynamic programming approach. Also, several heuristic for the problem are provided. Piñeyro and Viera [6] suggest and compare several inventory policies for the ELSR using a divide-and-conquer approach and a tabu search based on procedure. They also show the key role that the remanufacturing plan plays in the ELSR resolution and introduce the concept of the remanufacturing plan of perfect cost. Piñeyro and Viera [13] consider the ELSR with different demand streams for new and remanufactured items where, in addition, substitution is allowed for remanufactured items, but not vice versa. Recently, Nenes et al. [14] provide an analysis of the ELSR taking into account the quality of the returns, and Helmrich et al. [15] provide and compare different mathematical formulations for the ELSR with separate and joint setup costs for the activities.

Problem formulation

Figure 1 below shows a sketch of the flow of items for the inventory system that represents the lot-sizing problem that we are facing.

We consider a lot-sizing problem for which the demand and return values are known in advance for each period over the finite planning horizon. The demand for serviceable items must be satisfied on time, i.e., backlogging demand is not allowed. Infinite capacity for producing, remanufacturing, and disposing is assumed with zero lead times. The inventory level of both used and serviceable items is determined after all activities were carried out. Setup and unit costs are incurred for producing, remanufacturing, or disposing, and unit costs for carrying ending positive inventory from one period to the next. Finally, we assume that the initial inventory levels of both used and serviceable items are zero, and the demand is positive for each period in the planning horizon. The objective is to determine the amounts to produce, remanufacture, and dispose for each one of the periods in the planning horizon such that all demand requirements are satisfied on time, and the sum of all the involved costs is minimized. We refer to this problem as the ELSR, and it can be modeled as the following mixed integer linear programming (MILP) problem:

$$\min \sum_{t=1}^{T} \left\{ K_t^p \delta_t^p + c_t^p p_t + K_t^r \delta_t^r + c_t^r r_t + K_t^d \delta_t^d + c_t^d d_t + h_t^s y_t^s + h_t^u y_t^u \right\} \quad (1)$$

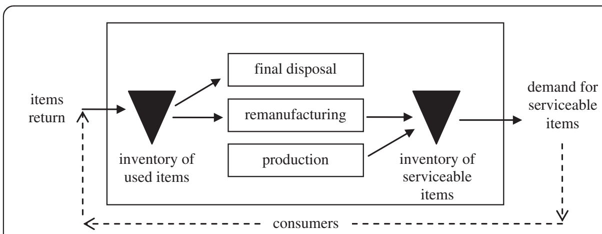

Figure 1 Flow of items in the system.

subject to

$$y_t^s = y_{t-1}^s + p_t + r_t - D_t \qquad \forall t = 1, 2, \ldots, T \tag{2}$$

$$y_t^u = y_{t-1}^u - r_t + R_t \qquad \forall t = 1, 2, \ldots, T \tag{3}$$

$$M\delta_t^p \geq p_t \qquad \forall t = 1, 2, \ldots, T \tag{4}$$

$$M\delta_t^r \geq r_t \qquad \forall t = 1, 2, \ldots, T \tag{5}$$

$$M\delta_t^d \geq d_t \qquad \forall t = 1, 2, \ldots, T \tag{6}$$

$$y_0^s = y_0^u = 0 \tag{7}$$

$$\delta_t^p, \delta_t^r, \delta_t^d \in \{0, 1\} \quad p_t, r_t, d_t, y_t^s, y_t^u \geq 0 \quad \forall t = 1, 2, \ldots, T. \tag{8}$$

In models (1) to (8), the parameters T, D_t, and R_t denote the length of the planning horizon, demand, and return values in periods $t = 1,\ldots,T$ respectively: K_t^p, K_t^r, K_t^d, c_t^p, c_t^r and c_t^d the setup and unit costs for production, remanufacturing, and final disposing in periods $t = 1,\ldots, T$, respectively; h_t^s and h_t^u, the unit cost of holding inventory for serviceable and used products in periods $t = 1,\ldots,T$, respectively; and M, a number at least as large as $\max\{D_{1T}, R_{1T}\}$, where D_{ij} and R_{ij} are the accumulative demand and returns between periods i and j, with $1 \leq i \leq j \leq T$. The decision variables p_t, r_t, and d_t denote the number of units produced, remanufactured, and disposed in periods $t = 1,\ldots,T$, respectively; δ_t^p, δ_t^r and δ_t^d, binary variables equal to 1 if production, remanufacturing, or disposing is carried out in periods $t = 1,\ldots,T$, or 0 otherwise, respectively; and y_t^s and y_t^u, the inventory level during periods $t = 1,\ldots,T$, for serviceable and used items, respectively.

Constraints (2) and (3) are the inventory equilibrium equations for serviceable and used items, respectively. Constraints (4) to (6) indicate that a setup is made whenever an activity is carried out in a period for a positive quantity. Constraint (7) states that the initial inventory level for both serviceable and used items is zero. Finally, the set of possible values for each decision variable is specified by constraint (8).

The ELSR as modeled above is a NP-hard problem [9]. As we mentioned earlier, solving the ELSR is equivalent to find a remanufacturing plan of perfect cost, i.e., the remanufacturing plan of an optimal solution of the ELSR. In the following section, we analyze this last problem assuming that the periods for which the quantity of remanufacturing is positive are known in advance.

Results

Fixed periods for remanufacturing

In this section, we tackle the problem of determining the quantities of the remanufacturing plan of perfect cost under the assumption that the periods with positive remanufacturing (periods for which the quantity of remanufacturing is greater than zero) are known in advance. We begin considering the particular case of only one positive-remanufacturing period and then we consider the case of more than one period. To conduct the analysis, we resort to certain assumptions on the costs as well as on the number of the available returns in the periods fixed. The first assumption that we introduce below is about the costs related to the used items.

Definition 1. We say that the costs of the returns are at most equal to the costs of the new items when the expressions below are fulfilled by the cost components:

$$K_i^r \leq K_j^p, \tag{9.1}$$

$$c_i^r \leq c_j^p, \tag{9.2}$$

$$h_i^u \leq h_j^s, \tag{9.3}$$

for any couple of periods i and j in $1,\ldots,T$.

Expressions (9.1) and (9.2) state the fact that the remanufacturing of used items is economically preferred to the production of new items. This can happen in practice due to the saving of energy and raw material of the remanufacturing activity. Expression (9.3) is fulfilled as it is assumed that the value is added to the used items in order to make them serviceable. In addition, we assume that the setup costs are at least equal to the unit costs for each activity, i.e, $K_t^p \geq c_t^p \geq 0$, $K_t^r \geq c_t^r \geq 0$, and $K_t^d \geq c_t^d \geq 0$, for each period $t = 1,\ldots,T$.

The single-period case

Consider an ELSR instance of T periods with only one period i fixed as positive-remanufacturing period, i.e., $r_i > 0$, with $1 \leq i \leq T$, and $r_t = 0$ for all t with $1 \leq i \leq T$ and $t \neq i$. The objective is to determine the optimal remanufacturing quantity Q_i^r of the period i, with $0 < Q_i^r \leq y_{i-1}^u + R_i$ and $y_{i-1}^u + R_i > 0$.

First, consider the case that the number of available returns in period i are at most equal to the demand of the period, i.e., $y_{i-1}^u + R_i \leq D_i$. Then, by (9), the optimal remanufacturing quantity must be equal to all of the available returns, i.e., $Q_i^r = y_{i-1}^u + R_i > 0$. On the other hand, for the case that $y_{i-1}^u + R_i > D_i$, we must determine the last period j within the planning horizon for which it is more profitable to meet at least one unit of its demand by remanufacturing in period i. Assume first that the number of available returns is sufficient to exactly meet the accumulative demand from the current period i to certain future period k, i.e., $y_{i-1}^u + R_i = D_{ik}$, with $1 \leq i \leq k \leq T$. Then, the optimal remanufacturing quantity of period i is $Q_i^r = D_{ij}$, with j as the last period for which

$c_i^r D_j + \sum_{t=i}^{j-1} h_t^s D_j \leq K_j^p + c_j^p D_j + \sum_{t=i}^{T} h_t^u D_j$ is fulfilled, with $1 \leq i \leq j \leq k \leq T$ and $D_j > 0$. However, we note that in general the number of available returns in period i is sufficient to meet only a portion of the demand of a certain future period k, i.e., $y_{i-1}^u + R_i = D_{i(k-1)} + \alpha < D_{ik}$, with $1 \leq i \leq k \leq T$ and $\alpha \geq 1$. Without loss of generality, let us assume that it is profitable to remanufacture in period i at least the needed quantity to cover the demand requirements from i to $(k-1)$, i.e., $D_{i(k-1)} \leq Q_i^r < D_{ik}$. This means that at least one unit of the demand requirement of period k is satisfied by means of the production of new items in a certain period t with $1 \leq t \leq k$. If $1 \leq t \leq i$, then there must be that $Q_i^r = D_{i(k-1)} + \alpha$, since $c_i^r \leq c_t^p + \sum_{\tau=t}^{i-1} h_\tau^s$ is true by (9). In the case of $i < t \leq k$, we have that $Q_i^r = D_{i(k-1)} + \alpha$ only if the condition $c_i^r + \sum_{t=i}^{t-1} h_t^s \leq c_t^p$ is fulfilled, otherwise $Q_i^r = D_{i(k-1)}$. This last condition can be relaxed by $c_i^r + \sum_{t=i}^{j-1} h_t^s \leq c_j^p + \sum_{t=i}^{T} h_t^u$ in the case that final disposal of used items is not considered, which is supported by economic as well as ecological reasons. Teunter et al. [12] point out that disposing option 'does not lead to a considerable cost reduction unless the remanufacturable return rate as a percentage of the demand rate is unrealistically high (above 90%) and the demand rate is very small (less than 10 per year)'. We resume the reasoning above by means of the following assumption about the profitability of maximizing the remanufacturing quantity in a certain period.

Definition 2. Given two periods i and k of an ELSR instance of T periods, with $1 \leq i \leq k \leq T$, such that $r_i > 0$, we say that it is profitable to maximize the remanufacturing quantity of period i if the expression

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \leq c_j^p \tag{10.1}$$

is fulfilled for each period j, with $1 \leq i \leq j \leq k \leq T$, or

$$c_i^r + \sum_{t=i}^{j-1} h_t^s \leq c_j^p + \sum_{t=i}^{T} h_t^u \tag{10.2}$$

in the case that the final disposal of used items is not considered.

Thus, if Definition 2 is fulfilled for any couple of periods i and j in $1,\ldots,T$, with $i \leq j$, we can assure that the optimal remanufacturing quantity Q_i^r f a single period i fixed as positive-remanufacturing period is the minimum between the amount of available returns and the accumulative demand from the current period and the end of the planning horizon, i.e., to remanufacture as much as possible. We note that for a given instance, it is sufficient that Definition 2 is fulfilled between the period fixed as positive-remanufacturing period and the last one for which at least a portion of its demand is attainable by remanufacturing in the period fixed. On the other hand, if Definition 2 is not fulfilled, it is unlikely that we can determine the optimal remanufacturing quantity of a certain period without knowing the periods where production is carried out since, in the case that the available returns in period i are only sufficient to partially meet the accumulative demand to certain future period k, we need to know if the rest of the demand of period k is produced either in the same period or in a previous one.

Real situations where Definition 2 is fulfilled include cases where holding costs of both used and serviceable items are similar or negligible, very low remanufacturing costs, as well as instances with few periods. We also note that the problem of finding the optimal positive-remanufacturing period for an ELSR instance for which it is profitable to remanufacture as much as possible at any period can be solved in $O(T^3)$ time since we must consider T different periods, and the corresponding optimal production and final dispose plans can be obtained in $O(T^2)$ by means of a Wagner-Whitin algorithm type [16].

The multi-period case

We now consider the problem of finding the remanufacturing quantities of a remanufacturing plan of perfect cost with at least two periods fixed as positive-remanufacturing periods. We first note that the amount to be remanufactured in a certain period depends in part of the remanufactured quantity in previous periods as well as affects the amount to be remanufactured in future periods. Then, it may not be possible to determine efficiently the optimal remanufacturing quantity for each period, even under the assumptions introduced in the previous section. In view of this difficulty, we focus on the problem of determining the total quantity of a remanufacturing plan of perfect cost. Before we tackle this problem, we provide a result about the form of the remanufacturing plan of perfect cost for a particular case.

Proposition 1. Consider an ELSR instance for which the number of available returns in a certain period i fixed as a positive-remanufacturing period is sufficient to fully cover the demand until the end of the planning horizon, i.e., $R_i + y^u_{i-1} \geq D_{iT}$, $r_i > 0$, with $1 \leq i \leq T$. If the optimal solution set is not empty, there is at least one optimal solution for which the total remaining demand from period i is satisfied only by remanufacturing from period i onwards, i.e., $r_{iT} = D_{iT}$, with $r_{ij} = \sum_{t=i}^{j} r_t, 1 \leq i \leq j \leq T$.

Proof. Let us consider an optimal solution of the ELSR with $r_i > 0$, $R_i + y^u_{i-1} \geq D_{iT}$, and $r_{iT} < D_{iT}$. Then, the quantity $(D_{iT} - r_{iT}) > 0$ is satisfied by means of the production of new items. We can determine a new solution with $r_{iT} = D_{iT}$ from the current solution as follows: First, for each period t with $i \leq t \leq T$ and $p_t > 0$, we replace the entire production in t by remanufacturing, i.e., $r_t \leftarrow p_t$, $p_t \leftarrow 0$. Note that the replacement operation is possible as we are assuming the returns are sufficient. Second, while $r_{iT} < D_{iT}$, take the last period t with $p_t > 0$ and $1 \leq t < i$, and transfer units of the production of period t to the remanufacturing of period i, until $r_{iT} = D_{iT}$ or $p_t = 0$. By (9), the cost of the new solution is at most equal to the cost of the original. Therefore, there must be an optimal solution of the ELSR for which $r_{iT} = D_{iT}$, if $r_i > 0$ and $R_i + y^u_{i-1} \geq D_{iT}$ is are complied.

Proposition 1 helps us to identify the form of a remanufacturing plan of perfect cost for the ELSR in the particular case that the number of available returns in a period fixed as positive-remanufacturing period is sufficient to meet all the remaining demand until the end of the planning horizon. We must note that if the amount of available returns in a certain period is sufficient to meet all the remaining demand but the period is not fixed as a positive-remanufacturing period, we cannot ensure the result above unless the period under consideration is the first one (see [10]).

We consider now the problem in general sense, i.e., no kind of relationship is assumed between the returns and the demand values. First, we provide the following definitions about the costs and the quantities of remanufacturing.

Definition 3. We say that the remanufacturing costs are non-speculative with respect to the transfer when they satisfy the following expressions:

$$K^r_i + c^r_i + \sum_{t=i}^{j-1} h^s_t - \sum_{t=i}^{j-1} h^u_t \quad \geq \; K^r_j + c^r_j + \sum_{t=i}^{j-1} h^u_t \tag{11.1}$$

$$c^r_i + \sum_{t=i}^{j-1} h^s_t - \sum_{t=i}^{j-1} h^u_t \quad \geq \quad c^r_j + \sum_{t=i}^{j-1} h^u_t \tag{11.2}$$

for any couple of period i and j in $1,\ldots,T$.

Expression (11.1) states that it is profitable to transfer the entire remanufacturing quantity from a certain period to other future period that was inactive, while (11.2) states that it is profitable to transfer forward at least one unit between two periods with positive remanufacturing. We note that the expressions given in (11) are fulfilled in different settings of practical interest, e.g., when all the costs involved are stationary or they do not increase over time.

Definition 4. Given an ELSR instance with a set of periods fixed as positive remanufacturing periods and a feasible remanufacturing plan r, we define the *upper bound of remanufacturing* of a certain period i to the quantity $u_i = 0$ if $r_i = 0$ and $u_i = \min(R_i + y^u_i, D_{i(j-1)})$ if $r_i > 0$, where j is either the next positive-remanufacturing period within the planning horizon, or $(T + 1)$ if i is the last positive-remanufacturing period, i.e., $r_t = 0$ for all periods t in $(i + 1), \ldots, T$.

Proposition 2. Given an ELSR instance, there is at least one optimal solution for which the remanufacturing quantity of each period is at most equal to its upper bound of remanufacturing, i.e., $0 \leq r_t \leq u_t$, for all periods $t = 1, \ldots, T$.

Proof. Without loss of generality, consider an optimal solution of an ELSR instance with only one period i for which $r_i > u_i = \min(R_i + y^u_i, D_{i(j-1)})$ and $r_j > 0$ with $1 \leq i \leq j \leq T$. First, we note that the case $u_i = R_i + y^u_i$ is not feasible since the remanufacturing quantity is greater than the amount of available returns. Now, consider the case that $u_i = D_{i(j-1)}$. Then, by (11), we can obtain a new solution with at most the same cost than the original by transferring remanufactured units from period i to the consecutive period j with $r_j > 0$, until $r_i = D_{i(j-1)}$ in the new solution. Therefore, an optimal solution for the same ELSR instance for which $r_t \leq u_t$ can be obtained, for all periods $t = 1, \ldots, T$.

Proposition 2 states that the remanufacturing quantity of a certain period is upper-bounded by the minimum between the number of available returns and the accumulative demand until the period preceding the next period with positive remanufacturing. We note that the upper bound value of certain period depends on the remanufacturing quantities of the previous periods. In addition, it may not be possible to determine how close or how far to its upper bound is the remanufacturing quantity of a certain period in an optimal solution of the ELSR. Despite these facts, the upper bound of remanufacturing allows us to determine the total remanufacturing quantity of a remanufacturing plan of perfect cost, as we show in the following proposition:

Proposition 3. Consider an ELSR instance with a set of periods F fixed as positive-remanufacturing periods such as for any pair of consecutive periods i and j of F, the Definition 2 is fulfilled for any pair of meaningful periods, i.e., pairs (i, t) with $i \in F$ and t the last period before j for which at least a portion of its demand is attainable by remanufacturing in i with $i \le t < j$. Then, consider the remanufacturing plan $\bar{r}$ obtained by remanufacturing in each period the amount given by the upper bound of remanufacturing applied in ascending order, i.e., $\bar{r}_t = u_t$ assuming that $\bar{r}_1 = u_1, \bar{r}_2 = u_2, \ldots, \bar{r}_{(t-1)} = u_{(t-1)}$, for all periods $t = 1, \ldots, T$. Then, there is an optimal solution with a remanufacturing plan r^* for which $r^*_{1T} = \bar{r}_{1T}$ where $r^*_{ij} = \sum_{t=i}^{j} r^*_t$ and $\bar{r}_{ij} = \sum_{t=i}^{j} \bar{r}_t$, with $1 \le i \le j \le T$.

Proof. We note that by Proposition 2 and Definition 4, there must be that $r^*_{1T} \le \bar{r}_{1T}$. Without loss of generality, let us assume that $r^*_{1T} = \bar{r}_{1T} - 1$. Then, there exists a period i, with $1 \le i \le T$, for which $0 < r^*_i = \bar{r}_i - 1$, $r^*_{1(i-1)} = \bar{r}_{1(i-1)}$, and $y^{*u}_{(i-1)} = \bar{y}^{u}_{(i-1)}$. This means that the upper bound of remanufacturing of period i is the same for both remanufacturing plans under consideration, with $0 < r^*_i < u_i = \bar{r}_i$. We also note that $y^{*u}_t \ge 1$ is fulfilled for all periods $t = \mathrm{i}, \ldots, T$. Therefore, we can obtain a new feasible solution for the same ELSR instance with at most the same cost by increasing the remanufacturing in period i in one unit, i.e., $r^*_i \leftarrow r^*_i + 1 = \bar{r}_i$, without affecting the remanufacturing of the future periods and in the meantime by reducing the production of a certain period j in $1, \ldots, T$. This new solution fulfills that $r^*_{1T} = \bar{r}_{1T}$, and this cost is at most the same than the cost of the original optimal solution as we are assuming that maximizing the remanufacturing quantity of the periods with positive remanufacturing is profitable according to Definition 2.

Proposition 3 states that in order to determine a remanufacturing plan of perfect cost for an ELSR instance with certain periods fixed as positive-remanufacturing periods, we only need to explore those remanufacturing plans for which the total remanufacturing quantity is equal to the sum of the upper bounds of remanufacturing. These values can be determined efficiently (linear time) by applying Definition 4 period by period, beginning with the first period fixed as positive-remanufacturing period. We show the usefulness of Proposition 3 through the following numeric example.

Discussion

A numerical example

Consider an ELSR instance with T = 5, a demand vector D = (5,3,6,4,5) and a return vector R = (3,2,2,2,3), where the periods 2, 4, and 5 are fixed as positive remanufacturing periods. The cost values are as follows: $K^p_t = 200$, $c^p_t = 20$, $K^r_t = 150$, $c^r_t = 15$, $K^d_t = 100$, $c^d_t = 10$, $h^s_t = 5$, and $h^u_t = 2$, with $1 \le t \le 5$. Note that the remanufacturing is profitable according to Definition 2 for all the meaningful pair of periods, i.e., (2,3), (4,4), and (5,5). Applying Definition 4, we have that the total remanufacturing quantity is 12 since the upper bounds of remanufacturing obtained sequentially are $u = (0, 5, 0, 4, 3)$. Table 1 below provides the candidate remanufacturing plans that we must consider in order to determine the remanufacturing plan of perfect cost for the ELSR instance.

Table 1 Candidate remanufacturing plans

t	*D*	*R*	*r*				
1	5	3	0	0	0	0	*0*
2	3	2	5	5	5	4	*3*
3	6	2	0	0	0	0	*0*
4	4	2	4	3	2	3	*4*
5	5	3	3	4	5	5	*5*

These candidate plans were obtained by assigning to each period the maximum quantity according to its upper bound and then transferring unit by unit from period 4 to period 5, and from period 2 to period 4. The last column of Table 1 in italics corresponds to the remanufacturing plan of perfect cost. The corresponding production and final dispose plans of the optimal solution are $p = (11, 0, 0, 0)$ and $d = (0, 0, 0, 0)$, respectively.

Effectiveness of the upper bound of remanufacturing

In [6], a basic tabu search (BTS) based on procedure was suggested and evaluated for the ELSR. The procedure receives among other parameters, an initial (0,1) T-tuple, where a value of 1 in position t indicates that remanufacturing is allowed to be positive in period t; otherwise, it must be zero. The procedure explores different remanufacturing plans by means of swapping the periods where remanufacturing can be positive. The remanufacturing quantity of each period i fixed as positive-remanufacturing period is equal to the minimum between the number of available returns in i and the accumulative demand from i to the period preceding the next period j with positive remanufacturing, i.e., the upper bound of remanufacturing of Definition 4. The BTS procedure was tested for a wide range of return-demand relationships, cost settings, and planning horizon lengths of 5, 10, and 15 periods. For all of the tested cases, the BTS showed a very good behavior (less than 2% of average gap between the cost of the solution obtained from BTS and the cost of the optimal solution), finding in many instances the optimal solution.

The good performance observed for the BTS procedure can be explained in part by the theoretical results provided in this paper about the quantities of the remanufacturing plan of perfect cost, at least for those cases where the conditions of Definitions 1 to 3 are fulfilled. In this sense, we note that for the numeric experiments of the BTS procedure of Piñeyro and Viera [6], it is assumed that the costs of the returns are at most equal to the costs of the new items according to expression (9) of Definition 1. In addition, horizon planning lengths of 5, 10, and 15 are used; thus, it can be assumed that the conditions of Definitions 2 and 3 are fulfilled in many of the tested instances. On the other hand, for those cases where the conditions are not fulfilled, it may be that the upper bound of remanufacturing is not a good option which in turn explains why the BTS procedure is not able to achieve high-quality solutions for some of the tested instances, e.g., when the positive-remanufacturing periods are widely separated or the holding costs of serviceable items are relatively greater.

Conclusions

In this paper, we have addressed the problem of determining the quantities of the remanufacturing plan of perfect cost for the ELSR assuming that the periods where remanufacturing is carried out are known in advance and that it is profitable to remanufacture as much as possible in a period fixed as positive remanufacturing period. Thus, we are able to determine the optimal remanufacturing quantity for the particular case of only one period fixed as positive-remanufacturing period. We also note that the problem of finding the optimal period for remanufacturing can be solved in $O(T^3)$ time. For the general case of more than one period fixed as positive-remanufacturing period, we note that it may not be possible to determine the optimal remanufacturing quantity for each one of them in an effective time way. Nevertheless, we show that the total remanufacturing quantity of an optimal solution can be determined as the sum of the upper bounds of remanufacturing, assuming also that the remanufacturing costs are non-speculative respect to the transfer, i.e., remanufacturing occurs as late as possible. The upper bounds of remanufacturing can be computed period by period in a linear time way as the minimum between the number of available returns and the accumulative demand from the current period to the period preceding the next period with positive remanufacturing. The theoretical results obtained about the quantities of a remanufacturing plan of perfect cost serve to explain the effectiveness of the tabu search based on procedure suggested in [6] for the ELSR.

Future research

More attention should be placed in the future on the problem of determining the quantities of the plan of perfect cost in an independent way by relaxing some of the assumptions imposed in this paper. More specifically, in identifying situations in which it is desirable to maximize the remanufacturing, even the condition of Definition 2 is not fulfilled. In addition, the problem of determining the periods with positive remanufacturing should be tackled. In this sense, we can resort to the useful remanufacturing problem (URP) introduced in [6]. The URP refers to the problem of determining the useful remanufacturing plan that minimizes the involved costs and maximizes the use of the returns. Then, we can assume that the positive periods of a useful remanufacturing plan are close to the positive periods of a remanufacturing plan of perfect cost. We may include also different demand streams for new and remanufactured items, as in [13].

Methods

This research provides theoretical contributions for solving the ELSR which are based on the study of the mathematical properties of the problem under certain particular assumptions.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

PP carried out the research presented in this paper as part of his Ph.D. OV supervised the research and corrected the draft versions of the paper. All authors read and approved the final manuscript.

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Acknowledgements

This work was supported by PEDECIBA, Uruguay. The authors thank the anonymous referees for their suggestions.

Received: 20 March 2012 Accepted: 24 September 2012
Published: 13 October 2012

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doi:10.1186/2210-4690-2-3
Cite this article as: Piñeyro and Viera: **Analysis of the quantities of the remanufacturing plan of perfect cost.** *Journal of Remanufacturing* 2012 **2**:3.