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MECHANICS

A setup of systemic description of fluid motion

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Abstract. The systemic description of fluid motion sets the description to a certain system of interlinked node theories. The goals of the systemic description are systematization of the node theories and interconnecting links and explanation of common features of formulations of the node theories and links by similarities of their position in the system organization.

Key words: fluid mechanics, stochastic fields, turbulence.

1. INTRODUCTION

The multiplicity of problems associated with the description of fluid motion calls fourth a variety of particular descriptions focusing on different aspects of motion and related to different branches of science (mechanics, statistics, mathematics, informatics, synergetics, the theories of chaos, etc.). The formulated descriptions can be aggregated into groups differing in the complementary nature of the constituted medium properties. These properties, simultaneous presence of which from different groups is excluded, ascribe a sense of complementarity to the theories belonging to different groups. The systemic description [1,2] considers the theories of every particular group as different versions of a single node theory and treats the medium motion through a set of interlinked node theories. The links ascribe a sense of system of theories to the set. The main task of the systemic description is to reveal the node theories' systemic properties, formulate the links between the node theories, and identify invariance properties of the systemic descriptions reflecting the fluid motion in different details by using different sets of node theories.

Section 2 explains the essence of the systemic description on an example of an arbitrary object behaving stochastically. Section 3 particularizes the approach for the description of fluid motion accounting for molecular, viscous, and turbulent levels of the motion organization. Some systemic aspects of formulations of the conventional turbulence mechanics and of the turbulence mechanics discussed in [2–5] are commented. Section 4 discusses two expansions of the systemic descriptions discussed in Section 3. The first expansion makes use of the hierarchic structure of turbulence and the other is supposed to explain the systemic essence of certain commonness observed in formulations of descriptions of fluid motion and of some physical fields [6–9].

2. SYSTEMIC DESCRIPTION OF OBJECTS BEHAVING STOCHASTICALLY

2.1. Elementary level of systemic description

Consider an arbitrary object behaving stochastically. The description of such object [1] presumes the determination of its particular states, the probability distribution of the particular states, and the conditions of the probability distribution formation. The conditions are specified as determining the object's average state. Let $\boldsymbol{A}$ denote the set of quantities the values of which fix the object's average state, let $\boldsymbol{a}$ denote the set of quantities the values of which fix the object's particular states, and let $f = f(\boldsymbol{a}/\boldsymbol{A})$ denote the probability distribution determined on a family of the object's particular states formed under the conditions determined as the object's average state. We shall call $\boldsymbol{a}$, $\boldsymbol{A}$, and f the *codes* and the quantities determining them the *signs*. Each code determines a particular description of the object called the *node theory* (denoted by the same notation as the codes on which they are set up).

The codes and the node theories corresponding to them are organized according to the *code grid* in Fig. 1. The *determination levels* differ in their predictability horizons in describing the object's behaviour, the *structural levels* differ in the presentation of the object's structure, and the *information coding levels* characterize the type of presentation of information on the object's state. Besides comprising three node theories $\boldsymbol{a}$, $\boldsymbol{A}$, and f the code grid in Fig. 1 comprises also three *recoding theories* $\boldsymbol{a} \oplus \boldsymbol{A}$, $\boldsymbol{a} \oplus f$, and $\boldsymbol{A} \oplus f$ (expressed in Fig. 1 by lines connecting the node theories) linking the pairs of node theories. The recoding theories are formulated by expressions connecting the codes and the velocities of the change of the object's states on the codes.

The theory $\boldsymbol{a} \oplus \boldsymbol{A}$ treats an object's behaviour in a *phase space* V_a of the code $\boldsymbol{a}$ as presented by a phase trajectory characterized as a stochastic function of time. It determines $\boldsymbol{A}$ and $\partial \boldsymbol{A}/\partial t$ through $\boldsymbol{a}$ and $\partial \boldsymbol{a}/\partial t$ as $\overline{\boldsymbol{A}^*}$ and

$$\frac{\partial \boldsymbol{A}}{\partial t} = \overline{\frac{\partial \boldsymbol{A}^*}{\partial \boldsymbol{a}} \frac{\mathrm{d}\boldsymbol{a}}{\mathrm{d}t}}, \tag{1}$$

where $\boldsymbol{A}^* = \boldsymbol{A}^*(\boldsymbol{a})$ is a set of functions of $\boldsymbol{a}$ and the overbar denotes averaging over time intervals $\left[t - \frac{T}{2}, t + \frac{T}{2}\right]$, or, equivalently, statistical averaging with the probability distribution $p = p(\boldsymbol{a}, t)$ determined for the same time intervals by applying the Poincaré sections technique. The dependence $\boldsymbol{A}^* = \boldsymbol{A}^*(\boldsymbol{a})$, the duration of the time interval, and the specification of the integral on the right side of (1) as expressed through the code $\boldsymbol{A}$ (the closure problem) cannot be deduced from the theory $\boldsymbol{a}$. The situation characterizes the theory $\boldsymbol{a} \oplus \boldsymbol{A}$ as harmonizing the formulations of node theories $\boldsymbol{a}$ and $\boldsymbol{A}$ but not as a deduction of theory $\boldsymbol{A}$ from theory $\boldsymbol{a}$.

The theory $\boldsymbol{a} \oplus f$ treats the object's behaviour in the phase space V_a as presented by an infinite amount of virtual phase trajectories formed due to the split of any fixed phase trajectory in time. The object's states after the split are treated as random events not predicted by the object's state before the split. Due to the ongoing sequence of splitting the number of the object's possible states increases unboundedly in time and forms a statistical ensemble characterized by the probability distribution f. For a steady ensemble we have $\mathrm{d}f/\mathrm{d}t = 0$ or

$$\frac{\partial f}{\partial t} = -\frac{\partial f}{\partial \boldsymbol{a}} \frac{\mathrm{d}\boldsymbol{a}}{\mathrm{d}t}. \tag{2}$$

The theory $\boldsymbol{a} \oplus f$ (similar to $\boldsymbol{a} \oplus \boldsymbol{A}$) just harmonizes the formulations of the node theories $\boldsymbol{a}$ and f but does not deduce theory f from theory $\boldsymbol{a}$. The latter requires additional information about the formation conditions of f, formulated in terms of $\boldsymbol{A}$.

The theory $\boldsymbol{A} \oplus f$ specifies f as $f = f(\boldsymbol{a}/\boldsymbol{A})$ and determines $\boldsymbol{A}$ and $\partial \boldsymbol{A}/\partial t$ through f and $\partial f/\partial t$ as

$$\boldsymbol{A} = \int \boldsymbol{A}^\circ(\boldsymbol{a}) f(\boldsymbol{a}/\boldsymbol{A}) \mathrm{d}\boldsymbol{a}$$

and

$$\frac{\partial \boldsymbol{A}}{\partial t} = \int \boldsymbol{A}^\circ \frac{\partial f}{\partial t} \mathrm{d}\boldsymbol{a}, \tag{3}$$

where $\boldsymbol{A}^\circ(\boldsymbol{a})$ is a set of functions on signs of code $\boldsymbol{a}$. The expression $f = f(\boldsymbol{a}/\boldsymbol{A})$ interrelates specifications of f and $\boldsymbol{A}$: the code $\boldsymbol{A}$ characterizes not only the properties of the probability distribution f in averaged terms but also the conditions under which these properties are formed.

On the systemic description level the formulations of recoding theories $\boldsymbol{a} \oplus \boldsymbol{A}$, $\boldsymbol{a} \oplus f$, and $\boldsymbol{A} \oplus f$ are harmonized by declaring

$$\boldsymbol{A}^\circ = \boldsymbol{A}^* \text{ and } p(\boldsymbol{a}, t) = f(\boldsymbol{a}/\boldsymbol{A}). \tag{4}$$

Using (4) it is easy to be convinced that each of the relations (1)–(3) can be presented as a corollary of the other two.

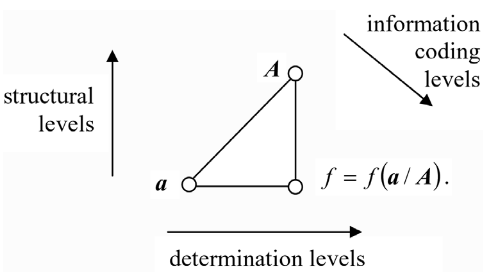

Fig. 1. The code grid of an elementary level of systemic description. The description comprises three particular descriptions specified by codes $\boldsymbol{a}$, $\boldsymbol{A}$, and f. The descriptions are classified as belonging to different combinations of structural, determination, and information coding levels of the description.

2.2. Systemic description of higher order

In general [1], the systemic description may contain more than two structural, determination, and/or information coding levels. A code grid of such systemic description is presented in Fig. 2. Unlike the elementary level of the systemic description containing three node theories, systemic descriptions contain in general $(N+1)(N+2)/2$ node theories, where N is called the range of the systemic description. (For the elementary level of systemic description $N = 1$.)

Let us define the operations of expansion and reduction of the systemic description correspondingly as

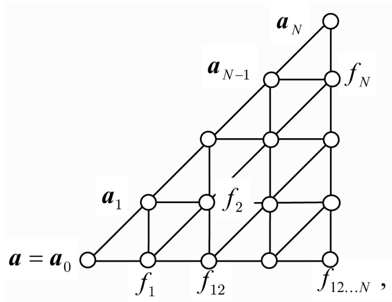

Fig. 2. The code grid of a systemic description with an arbitrary number of structural levels; $\boldsymbol{a}_n\,(n = 0, 1, \ldots, N)$ denote the codes of the lowest information coding levels and $f_n = f\,(\boldsymbol{a}_{n-1}/\boldsymbol{a}_n), \ldots, f_{m\ldots n} = f\,(f_{m\ldots n-1}/\boldsymbol{a}_n)$ $(m < n = 1, \ldots, N)$ denote the probability distributions specifying the codes of the higher information coding levels.

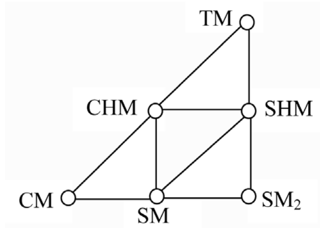

Fig. 3. A code grid for a systemic description of fluid motions embracing classical mechanics (CM), statistical mechanics (SM), classical hydromechanics (CHM), statistical hydromechanics (SHM), turbulence mechanics (TM), and the statistical theory of second order denoted as SM$_2$.

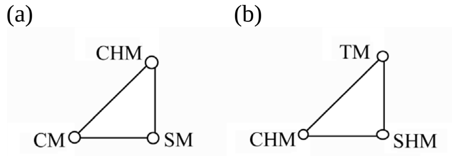

Fig. 4. Examples of reductions of a systemic description with the code grid in Fig. 3 down to the elementary level of the systemic description comprising CM, SM, and CHM (a) and CHM, SHM, and TM (b).

an increase and a decrease of the range of the systemic description. In particular, the systemic description corresponding to the code grid in Fig. 2 can be reduced to any of the node theories of the lowest information coding level, to the elementary level of systemic descriptions presented by the code grid in Fig. 1, with $\boldsymbol{a}$ and $\boldsymbol{A}$ identified with arbitrary $\boldsymbol{a}_n$ and $\boldsymbol{a}_m$, where $n < m = 1, ..., N$ etc. All systemic descriptions of the same object are considered as equivalent in the metatheoretical sense. The equivalency is formulated as the property of *metatheoretical invariance* stating the reducibility of any particular systemic description of the object to some other by the reduction and/or the expansion operations of the systemic description. The metatheoretical invariance comprises all particular systemic descriptions of the same object into a unique description.

3. SYSTEMIC DESCRIPTION OF FLUID MOTION

Let us specify now the introduced systemic description for the description of fluid motion. Figure 3 presents a code grid of the specification of range 2 including (as the node theories) canonical mechanics (CM), classical hydromechanics (CHM), turbulence mechanics (TM), statistical mechanics (SM), statistical hydromechanics (SHM), and an additional statistical theory of the second order denoted as SM$_2$, required to make the systemic description complete. In addition to the nine node theories, the systemic description corresponding to the code grid in Fig. 3 embraces also nine recoding theories. Let us treat the situation presented in Fig. 3 together with the situations in Fig. 4 presenting two different reductions of the situation in Fig. 3 down to the elementary level of the systemic description.

Consider first the recoding theories $\mathrm{CM} \oplus \mathrm{SM}$ and $\mathrm{CHM} \oplus \mathrm{SHM}$ coinciding by the type with $\boldsymbol{a} \oplus f$ discussed in 2.1. The splitting of the phase trajectories within $\mathrm{CM} \oplus \mathrm{SM}$ is explained by the analytical integrals of energy, momentum, moment of momentum, and centre of gravity forming only part of the integrals of the medium motion within the CM. The additional integrals are, as a rule, non-algebraic functions from canonical variables behaving irregularly in space and time. At the time instant when these integrals become essential the medium state is characterized as instable and the listed ten classical integrals become insufficient to determine the motion uniquely. One single phase trajectory is followed after this time instant by a bunch of (virtual) phase trajectories (the Poincaré explosion). Every phase trajectory in this bunch is possible and independent of the others, which founds the probabilistic description of the medium behaviour within SM by the probability distribution formed in the course of an unbounded increase of the number of Poincaré explosions in time. The situation with $\mathrm{CHM} \oplus \mathrm{SHM}$ is similar. In this case the probability distribution characterizing the motion within SHM is formed in the course of an unbounded sequence of bifurcations of realizations of medium behaviour described within CHM (which are analogues to the Poincaré explosions of the medium behaviour on the canonical structural level). Let us emphasize that similarly to $\boldsymbol{a} \oplus f$, discussed in 2.1, $\mathrm{CM} \oplus \mathrm{SM}$ and $\mathrm{CHM} \oplus \mathrm{SHM}$ only found the formation of the probability distributions retaining their specific properties, formed under the conditions formulated as the medium average states determined within CHM and TM, correspondingly, unspecified.

The situation with the recoding theories $\mathrm{CM} \oplus \mathrm{CHM}$ and $\mathrm{CHM} \oplus \mathrm{TM}$ is analogous to the theory $\boldsymbol{a} \oplus \boldsymbol{A}$ and the situation with the recoding theories $\mathrm{SM} \oplus \mathrm{CHM}$ and $\mathrm{SHM} \oplus \mathrm{TM}$ is analogous to the theory $\boldsymbol{A} \oplus f$ discussed in 2.1. The first analogy declares an impossibility to treat CHM and TM as deduced from CM and CHM, respectively, and the second analogy declares a coupling of statistical properties of the medium expressed in terms of SM and SHM with the medium average properties fixed in terms of CHM and TM, respectively.

Everything said above with respect to the node theories embraced by the systemic descriptions corresponding to the code grids in Figs 3 and 4 spreads equally to all particular realizations of the node theories. Consider, for example, two formulations of TM specified as the conventional formulation of TM (henceforth, conventional turbulence mechanics or CTM) and the formulation of TM as discussed in [2–5] (henceforth, the theory of rotationally anisotropic turbulence or the RAT theory). The formulations are founded on two different classical turbulence conceptions – the Reynolds conception [10], which disregards the effects of the preferred orientation of eddy rotation, and the Richardson–Kolmogorov conception [11,12] just stressing the preferred rotation orientation as an essential attribute of the large-scale eddies immediately interacting with the average flow. The RAT theory formalizes the difference by defining quantity $\boldsymbol{M} = \langle \boldsymbol{v}' \times \boldsymbol{R} \rangle$, where $\boldsymbol{v}'$ is the velocity fluctuation, $\boldsymbol{R}$ denotes curvature radius of the $\boldsymbol{v}'$ streamline, and the angular brackets denote statistical averaging. Quantity $\boldsymbol{M}$ has the physical sense of the density of moment of momentum per unit mass and a dynamic measure of the preferred orientation of eddy rotation. In terms of $\boldsymbol{M}$ the CTM and the RAT theories become specified as treating the situations with $\boldsymbol{M} \equiv 0$ and $\boldsymbol{M} \neq 0$, respectively. Within SHM the difference results in two formulations of SHM. The first formulation specifies the probability distribution (determining the medium state within SHM) as depending on the velocity fluctuations only while the second formulation considers $\boldsymbol{R}$ included to the set of arguments of the probability distribution. The inclusion means distinguishing the velocity fluctuations at flow field points (in addition to their magnitude and direction, as it is assumed within the setup of CTM) by the curvature of the velocity fluctuation streamlines passing the points.

4. EXPANSIONS OF THE INITIAL SYSTEMIC DESCRIPTION

The systemic description corresponding to the code grid in Fig. 3 can be not only reduced but also expanded. Figures 5 and 6 represent the code grids of two examples of expansion. The systemic description founded on the code grid shown in Fig. 5 accounts for the multi-scale

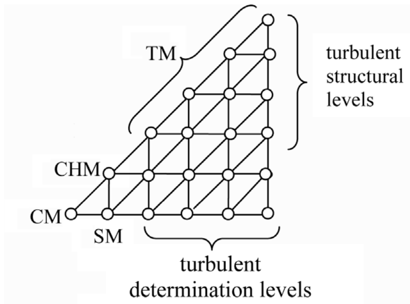

Fig. 5. The code grid of an expansion of the systemic description with the code grid in Fig. 3 to account for the multi-scale structure of turbulence.

structure of turbulence (Fig. 5) and the systemic description founded on the code grid in Fig. 6 accounts for the physical structural levels of the medium, represented in Fig. 6 by electrodynamics (ED) (the Maxwell's theory), quantum mechanics (QM) (formulated in the form of the Schrödinger equation disregarding the spin effects), the statistical theory of strong interactions (SSI), and a theory denoted as SI, although not formulated yet.

Let us consider the systemic description founded on the code grid in Fig. 6 in a greater detail. The node theories of the mechanical levels of the systemic description presented in the code grid in Fig. 6 have their counterparts on the physical levels located symmetrically to the CM as to a common theoretical origin of the setups of descriptions of the medium on the mechanical and on the physical structural levels. The theories specified as counterparts have also some similarities in their mathematical formulations. The similarities in mathematical formulations of CHM and ED [6] and of SM and QM represent the most familiar examples. Unlike the con-

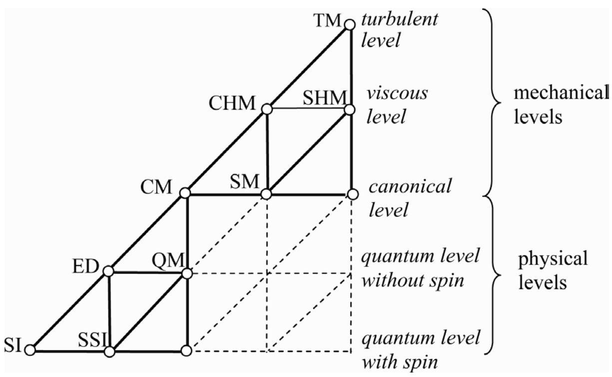

Fig. 6. The code grid for a conjoint systemic description of fluid motion and of some physical fields. Besides the abbreviations specifying the node theories included into the systemic description explained in the caption of Fig. 3 the following abbreviations are used: ED – electrodynamics; QM – quantum mechanics; SI – phenomenological theory of strong interaction; SSI – statistical theory of strong interaction.

sidered pairs of theories describing the fields without spin or its mechanical equivalent, spin appears to play a crucial role within SHM and SSI. (The role of spin within SHM is performed by $\boldsymbol{M}$.) Let us list some other similarities in the formulation of SHM and SSI (in addition to the essentiality of spin in both cases). Firstly, SHM allows a mathematical formulation in the form of an equation resembling the Schwinger equation of the theory of fields with a strong interaction [7] (with the Reynolds number in the role of the measure of the interaction 'strength'; the similarity is the closest for the systems of photon-type particles, i.e. of the quantums of the electromagnetic field). Secondly, the characteristic of the turbulent motion field scaling property is treated as a property common to all fields with a strong interaction [8]. Thirdly, consideration of transmutations as a physical form of motion, declared in [9] in the context of relativistic quantum theory of fields with a strong interaction, is inherent also to the understanding of turbulence in the sense expressed by the Richardson–Kolmogorov turbulence conception.

5. CONCLUSIONS

The discussed systemic description of fluid motion presents a meta-level of the fluid motion description. On this description level the fluid motion is described through a certain system(s) of interlinked particular theories with divided competence. Divided competence means that each particular theory embraced by the systemic description solves tasks that cannot be solved by the others. Classical hydromechanics (CHM), statistical hydromechanics (SHM), and turbulence mechanics (TM) are examples of this kind of particular theories.

The discussed systemic description was induced by the need to oppose the position considering the turbulence description reducible to integration of equations of CHM, to demonstrate the coupling of formulations of the turbulent medium statistical and average descriptions realized within SHM and TM and to found the fundamentality of the specification of the turbulence quality prior to any setup of turbulence description. The need rises in the context of explaining the methodological background of the theory of rotationally anisotropic turbulence introducing to the turbulence problems discussion a novelty not fitting into the familiar frames of turbulence treatment.

One specific feature of the discussed systemic description is the enrichment of the characterization of particular theories embraced by the systemic description with their systemic properties not revealing whether these theories are treated outside the system(s) they belong to. The coupling of the characterization of medium properties within SHM and TM is an example of this.

An essential property of the formulated systemic description is its ability to be reduced and/or expanded, explained respectively as a decrease or an increase of the number of structural levels of the medium simultaneously accounted for in the systemic description setup. (All systemic descriptions reducible to each other by the operations of expansion and/or reduction are considered as equivalent in the meta-theoretical sense.) As an example, the systemic description comprising CHM, SHM, and TM can be expanded to embrace also classical mechanics (CM) and statistical mechanics (SM). The expansion founds, in particular, the analogy between the recording theory $\mathrm{CHM} \oplus \mathrm{SHM}$, linking CHM and SHM, and the recoding theory $\mathrm{CM} \oplus \mathrm{SM}$, linking CM and SM. It follows from the analogy that SHM cannot be replaced by methods of direct numerical simulation founded on CHM. Similarly to the expansion of the systemic description comprising CHM, SHM, and TM accounting for the medium molecular level, the expansion can be also realized to account for the levels reflecting the medium properties in terms of physical fields.

Finally, the discussed systemic description is an alternative to the point of view treating a science field achieving its perfection as a universal theory from which all other theories of the field follow as special cases. Unlike this point of view, the systemic approach considers the theoretical perfection achievable on the systemic description level. Each particular theory embraced by the systemic description treats an object or a phenomenon under investigation within its own competence while a complete description is achieved by using the competence of all particular theories organized by the systemic description. The discussed setup of the systemic description of fluid motion is an example.

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Vedelike liikumise süsteemkirjeldus

Jaak Heinloo

On formuleeritud vedelike liikumise süsteemkirjelduse struktuur ja põhimõtted. Süsteemkirjeldus delegeerib vedelike liikumise kirjelduse omavahel seotud, kuid vastandlikke omadusi (diskreetsus ja pidevus, determineeritus ja juhuslikkus) eeldatavate teooriate kogule. Süsteemkirjelduse eesmärgiks on selles osalevate teooriate (kui teooriate süsteemi elementide) süsteemsete omaduste väljaselgitamine ja nende süstematiseerimine nimetatud süsteemsete omaduste sarnasuste ning erinevuste alusel.