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# (3, 1)*-Choosability of graphs of nonnegative characteristic without intersecting short cycles

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**Abstract.** A graph $G$ is called $(k, d)^*$-choosable if for every list assignment $L$ satisfying $|L(v)| \geq k$ for all $v \in V(G)$, there is an $L$-coloring of $G$ such that each vertex of $G$ has at most $d$ neighbors colored with the same color as itself. In this paper, it is proved that every graph of nonnegative characteristic without intersecting $i$-cycles for all $i = 3, 4, 5$ is $(3, 1)^*$-choosable.

**Keywords.** Graph; defective choosability; characteristic; intersecting cycles.

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## 1. Introduction

Graphs considered in this paper are finite, simple and undirected. Let $G = (V, E, F)$ be a graph, where $V$, $E$ and $F$ denote the set of vertices, edges and faces of $G$, respectively. For the used but undefined terminologies and notations, we refer the reader to the book by Bondy and Murty [1].

Let $b$ be a positive integer. A proper $b$-set colouring of $G$ is an assignment of a set $B(v)$ of $b$ colors to each vertex $v$ such that, for each pair $\{u, v\}$ of adjacent vertices, $B(u)$ and $B(v)$ are disjoint. When $b = 1$, this specializes to the standard proper vertex colouring (see the definition in [13]). We say $G$ is $(k, b)$-choosable if for every assignment of a set $S(v)$ of at least $k$ colours to each vertex $v \in V(G)$, there is a proper $b$-set colouring $\{B(v) : v \in V(G)\}$ of $G$ such that $B(v) \subseteq S(v)$ for each $v$. Such a proper $b$-set colouring is said to be admissible by the collection $\{S(v) : v \in V(G)\}$.

The $b$-choice number $\mathrm{ch}(G, b)$ (also referred to as the list $b$-chromatic number) of $G$ is the minimum value of $k$ such that $G$ is $(k, b)$-choosable. When $b = 1$, $(k, 1)$-choosability is also known as $k$-choosability and the 1-choice number is also known simply as the choice number or the list chromatic number (when $b = 1$, we use the shorter notation $\mathrm{ch}(G)$ instead of $\mathrm{ch}(G, 1)$). If, in addition to setting $b = 1$, we also set $S(u) = S(v)$ for each pair $u, v$ of vertices, $(k, b)$-choosability specializes to $k$-colourability. Hence it follows that for any $b \geq 1$ and any graph $G$, $\mathrm{ch}(G, b) \geq \mathrm{ch}(G) \geq \chi(G)$, where $\chi(G)$ denotes the usual chromatic number of $G$.

The notion of $(k, b)$-choosability was introduced by Erdös *et al.* in [8], and its specialized version choice number was introduced earlier by Vizing in [14]. The notion of choice number has received a lot of attention recently and results have been obtained relating $\mathrm{ch}(G)$ to various other parameters.

In the above definition, while $S(u) = S(v)$, $|S(u)| = |S(v)| = k$, and $|B(u)| = |B(v)| = 1$, $B(u) \neq B(v)$ for any adjacent two vertices $u$ and $v$, then the $(k, b)$-choosability specializes to the proper $k$-coloring of $G$. Here we cited another coloring $(k, d)^*$-colorability, also called defective colorability, or improper colorability, which was introduced in 1986 by Cowen *et al.* [4]. A graph $G$ is said to be $d$-improper $k$-colorable, or simply, $(k, d)^*$-colorable, if the vertices of $G$ can be colored with $k$ colors in such a way that each vertex has at most $d$ neighbors receiving the same color as itself, that is $S(u) = S(v)$, $|S(u)| = |S(v)| = k$, and $|B(u)| = |B(v)| = 1$, and $|\{v|B(v) = B(u), v \in N_G(u)\}| \leq d$ for any vertex $u$ in the $(k, b)$-choosability. Here, we use $(k, d)^*$-coloring in order to stand apart from the definition of the $(k, d)$-choosable. Obviously, a $(k, 0)^*$-coloring is an ordinary proper $k$-coloring.

A list assignment of $G$ is a function $L$ that assigns a list $L(v)$ of colors to each vertex $v \in V(G)$. An $L$-coloring with impropriety $d$ for integer $d \geq 0$, or simply $(L, d)^*$-coloring, is a mapping $\phi$ that assigns a color $\phi(v) \in L(v)$ to each vertex $v \in V(G)$ such that $v$ has at most $d$ neighbors colored with $\phi(v)$. For integers $m \geq d \geq 0$, a graph is called $(m, d)^*$-choosable, if $G$ admits an $(L, d)^*$-coloring for every list assignment $L$ with $|L(v)| = m$ for all $v \in V(G)$. An $(m, 0)^*$-choosable graph is simply called $m$-choosable.

The notion of list improper coloring was introduced independently by Škrekovski [10] and Eaton and Hull [7]. They proved that every planar graph is $(3, 2)^*$-choosable and every outerplanar graph is $(2, 2)^*$-choosable. Škrekovski proved in [11] that every planar graph without 3-cycles is $(3, 1)^*$-choosable, and in [12] he proved that every planar graph $G$ is $(2, 1)^*$-choosable if its girth $g(G) \geq 9$, $(2, 2)^*$-choosable if $g(G) \geq 7$, $(2, 3)^*$-choosable if $g(G) \geq 6$, and $(2, 4)^*$-choosable if $g(G) \geq 5$. Lih *et al.* [9] proved that every planar graph without 4-cycles and $l$-cycles for some $l \in \{5, 6, 7\}$ is $(3, 1)^*$-choosable. Dong and Xu [6] showed that it is also true for some $l \in \{8, 9\}$. Cushing and Kierstead [5] constructively proved that every planar graph is $(4, 1)^*$-choosable which perfectly solved the last remaining question left open in [7, 10]. In [3], Chen and Raspaud proved that every planar graph without 4-cycles adjacent to 3- and 4-cycles is $(3, 1)^*$-choosable, as a corollary, every planar graph without 4-cycle is $(3, 1)^*$-choosable. Wang and Xu proved every planar graph without cycles of length 4 is $(3, 1)^*$-choosable in [15].

For other classes of graphs, Zhang [17] proved that every graph $G$ embeddable on the torus without 5- and 6-cycles is $(3, 1)^*$-choosable. Xu and Zhang [16] proved that every toroidal graph without adjacent triangles is $(4, 1)^*$-choosable.

Recall that the Euler characteristic of a surface is equal to $|V(G) - |E(G)| + |F(G)|$ for any graph $G$ that is 2-cell embedded in that surface. The Euclidean plane, the projective plane, the torus, and the Klein bottle are all the surfaces of nonnegative characteristic. For simplification, we call a graph of nonnegative characteristic an NC-graph. Chen *et al.* [2] proved that every graph embeddable in a surface of nonnegative characteristic without a 5-cycle with a chord or a 6-cycle with a chord is $(4, 1)^*$-choosable, and every graph embeddable in a surface of nonnegative characteristic without a $k$-cycle with a chord for all $k \in \{4, 5, 6\}$ is $(3, 1)^*$-choosable.

In [15], Wang and Xu conjectured every planar graph without intersecting triangles is $(3, 1)^*$-choosable. We consider this problem with a relaxed condition. In fact, this paper investigates improper choosability for graphs of nonnegative characteristic without intersecting short cycles.

We call two cycles intersecting if they share at least one common vertex or edge. Let $\mathcal{G}$ denote the family of graphs with nonnegative characteristic containing no intersecting 3-, 4- and 5-cycles. The main result is to show that every graph in $\mathcal{G}$ is (3, 1)*-choosable. In order to prove the main theorem, we use the technique of discharging to obtain several forbidden configurations for the graphs in $\mathcal{G}$ and state a theorem as follows:

**Theorem 1.** *For every graph $G \in \mathcal{G}$, one of the following must hold*:

(1) $\delta(G) < 3$.
(2) *$G$ contains two adjacent $3^-$-vertices.*
(3) *$G$ contains a $(4^-, 4^-, 4^-)$-face.*
(4) *$G$ contains an even $(3^-, 4^-, \ldots, 3^-, 4^-)$-$2n$-face, here $n \geq 2$.*

As a consequence of Theorem 1, we can prove the following theorem.

**Theorem 2.** *Every graph with nonnegative characteristic without intersecting $i$-cycles for all $i = 3, 4, 5$ is* $(3, 1)^*$*-choosable.*

## 2. Notation

We use $N_G(v)$ and $d_G(v)$ to denote the set and number of vertices adjacent to a vertex $v$, respectively, and use $\delta(G)$ to denote the minimum degree of $G$. A face of an embedded graph is said to be incident with all edges and vertices on its boundary. Two faces are adjacent if they share a common edge. The degree of a face $f$ of $G$, denoted also by $d_G(f)$, is the number of edges incident with it, where each cut-edge is counted twice. When no confusion may occur, we write $N(v)$, $d(v)$, $d(f)$ instead of $N_G(v)$, $d_G(v)$, $d_G(f)$. A $k$-vertex (or $k$-face) is a vertex (or face) of degree $k$, a $k^-$-vertex (or $k^-$-face) is a vertex (or face) of degree at most $k$, and a $k^+$-vertex (or $k^+$-face) is a vertex (or face) of degree at least $k$. For $f \in F(G)$, we write $f = [u_1 u_2 \cdots u_n]$ if $u_1$, $u_2$, ..., $u_n$ are the vertices lying clockwise on the boundary of $f$. An $n$-face $[u_1 u_2 u_3 \cdots u_n]$ is called an $(m_1, m_2, m_3, \ldots, m_n)$-face if $d(u_i) = m_i$ for $i = 1, 2, 3, \ldots, n$. A $k$-cycle is a cycle with $k$ edges. Two cycles are adjacent if they share at least one common edge. Two cycles or faces are intersecting if they share at least one common (boundary) vertex. A chord of a $k$-cycle ($k \geq 4$) is an edge joining two nonconsecutive vertices on $C$ and a chordal cycle is a cycle with a chord.

## 3. Proof of Theorem 1

In the proof of Theorem 1, we use the technique of discharging. In the beginning, each vertex $v$ is assigned a charge $\frac{2d(v)}{6} - 1$ and each face $f$ is assigned a charge $\frac{d(f)}{6} - 1$.

If $K$ is a finite cell complex, then its Euler characteristic is equal to the alternating sum of the Betti numbers of each dimension, that is $\chi(K) = \Sigma_k (-1)^k \beta_k(K)$. Using the Euler–Poincaré formula for the NC-graphs $|V(G)| - |E(G)| + |F(G)| \geq 0$ and the well-known hand-shaking relation $\sum_{v \in V(G)} d(v) = \sum_{f \in F(G)} d(f) = 2|E(G)|$, we have

$$\sum_{v \in V(G)} \left\{ \frac{2d(v)}{6} - 1 \right\} + \sum_{f \in F(G)} \left\{ \frac{d(f)}{6} - 1 \right\} \leq 0. \tag{1}$$

By the discharging rules stated in the following, we will redistribute the charges for the vertices and faces so that the total sum of the weights is kept constant while the transferring is in progress. However, once the transferring is finished, we get that the new charges are nonnegative, moreover, there exists some $x \in V(G) \cup F(G)$ such that $w'(x) > 0$, then

$$0 < \sum_{x \in V(G) \bigcup F(G)} w'(x) = \sum_{x \in V(G) \bigcup F(G)} w(x) = 0. \tag{2}$$

This contradiction completes the proof of Theorem 1.

Assume in the contrary that the theorem does not hold. Let $G$ be such a connected graph in $\mathcal{G}$. Let $w$ be a weight on $V(G) \cup F(G)$ by defining $w(v) = \frac{2d(v)}{6} - 1$ if $v \in V(G)$, and $w(f) = \frac{d(f)}{6} - 1$ if $f \in F(G)$. For two elements $x$ and $y$ of $V(G) \cup F(G)$, we use $\tau(x \to y)$ to denote the charge transferred from $x$ to $y$.

By the choice of $G$, we have

(O1) $\delta(G) \geq 3$;
(O2) Every 3-vertex is adjacent to only $4^+$-vertices;
(O3) $G$ contains no $(4^-, 4^-, 4^-)$-face;
(O4) $G$ contains no intersecting $i$-cycles for all $i \in \{3, 4, 5\}$;
(O5) $G$ contains no even $(3, 4, \dots, 3, 4)$-face.

Let $m_i(v)$ be the number of $i$-faces incident with $v$ and $n_j(v)$ be the number of $j$-vertices adjacent to $v$. Let $n_i(f)$ denote the number of $i$-vertices incident with $f$. We have as follows:

*Claim* 1. For each vertex $v \in V(G)$, $|m_i(v)| \leq 1$ for all $i \in \{3, 4, 5\}$.

*Claim* 2. For each face $f \in F(G)$, $n_3(f) \leq \lfloor \frac{d(f)}{2} \rfloor$.

Let $v$ be a $k$-vertex and $f$ be an $l$-face incident with $v$. The new charge function $w'(x)$ is obtained by the discharging rules given below:

(R1) For $k = 4$, $\tau(v \to f) = \frac{1}{18}$ if $l = 5$, $\frac{1}{9}$ if $l = 4$, and $\frac{1}{6}$ if $l = 3$.
(R2) For $k = 5, 6$, $\tau(v \to f) = \frac{1}{18}$ if $l = 5$, $\frac{2}{9}$ if $l = 4$, and $\frac{1}{3}$ if $l = 3$ and $f$ is a $(3, 4, 5^+)$-face, $\frac{1}{4}$ to other incident 3-faces.
(R3) For $k \geq 7$, $\tau(v \to f) = \frac{1}{18}$ if $l = 5$, $\frac{2}{9}$ if $l = 4$, and $\frac{1}{3}$ if $l = 3$.

We now verify that $w'(x) \geq 0$ for any $x \in V(G) \cup F(G)$.

Let $f$ be an $h$-face of $G$. The proof is divided into four cases according to the value of $h$.

*Case* 1: $h \geq 6$. Then $w'(f) = w(f) \geq 0$.

*Case* 2: $h = 5$. Then $n_3(f) \leq 2$ by Claim 2. So $n_{4^+}(f) \geq 3$, then $w'(f) \geq w(f) + 3 \cdot \frac{1}{18} = 0$ by (R1), (R2) and (R3).

*Case* 3: $h = 4$. Then $n_3(f) \leq 2$ by (O2), (O5) and Claim 2. While $n_3(f) = 1$, we have $w'(f) \geq w(f) + 3 \cdot \frac{1}{9} = 0$, if $n_3(f) = 2$, we have $n_{5^+}(f) \geq 1$, so $w'(f) \geq w(f) + 1 \cdot \frac{1}{9} + 1 \cdot \frac{2}{9} > 0$.

*Case* 4: $h = 3$. Then $w(f) = \frac{3}{6} - 1 = -\frac{1}{2}$. We write $f = [v_1v_2v_3]$. By Claim 2, we have $n_3(f) \leq 1$.

*Subcase* 4.1. If $n_3(f) = 0$, then $w'(f) \geq w(f) + 3 \cdot \frac{1}{6} = 0$.

*Subcase* 4.2. If $n_3(f) = 1$, then there must be a $5^+$-vertex incident with $f$ by (O2) and (O3). If $f$ is a $(3, 4, 5^+)$-face, then $w'(f) \geq w(f) + 1 \cdot \frac{1}{6} + 1 \cdot \frac{1}{3} = 0$ by (R1), (R2) and (R3). If $f$ is a $(3, 5^+, 5^+)$-face, then $w'(f) \geq w(f) + 2 \cdot \frac{1}{4} = 0$ by (R1), (R2) and (R3).

Let $v$ be a $k$-vertex of $G$. If $k = 3$, then $w'(v) \geq w(v) = 0$. If $k = 4$, then $w'(v) \geq w(v) - \frac{1}{18} - \frac{1}{9} - \frac{1}{6} = 0$ by Claim 1 and (R1). If $k \geq 5$, by Claim 1, (R2) and (R3), we have $w'(v) \geq w(v) - \frac{1}{18} - \frac{2}{9} - \frac{1}{3} = \frac{6d-29}{18} > 0$ by (R2) and (R3).

Now, we get that $w'(x) \geq 0$ for each $x \in V(G) \cup F(G)$. It follows that $0 \leq \sum_{x \in V(G) \cup F(G)} w'(x) = \sum_{x \in V(G) \cup F(G)} w(x) \leq 0$.

If $\sum_{x \in V(G) \cup F(G)} w'(x) > 0$, we are done. Assume that $\sum_{x \in V(G) \cup F(G)} w'(x) = 0$, so we have no $5^+$-vertices and $7^+$-faces by the above proof.

*Claim* 3. $G$ contains no 3-faces.

*Proof.* Let $G$ be a graph in $\mathcal{G}$. We have that $G$ contains no $5^+$-vertices, so the vertices incident with any 3-faces must be $4^-$-vertices, and we get it as $G$ contains no $(4^-, 4^-, 4^-)$-faces. □

*Claim* 4. $G$ contains no 4-vertices.

*Proof.* Let $v$ be a vertex in $V(G)$, then $m_3(f) + m_4(f) + m_5(f) \leq 2$ by Claim 3, so $w'(v) \geq w(v) - \frac{1}{18} - \frac{1}{9} = \frac{1}{6} > 0$.

By Claim 4, we have $d(v) = 3$ for every vertex in $V(G)$, this contradiction completes the proof of Theorem 1. □

## 4. Proof of Theorem 2

Suppose Theorem 2 is false. Let $G = (V, E)$ be a counterexample to Theorem 2 with the smallest $|V| + |E|$. Clearly $G$ is connected. Embedding $G$ into the surface with nonnegative characteristic, let $L = \{L(v) || L(v)| \geq 3$ for all $v \in V(G)\}$ be a list assignment such that $G$ has no $L$-coloring in the sense that every vertex has at most one neighbor colored the same color as itself.

For proving this theorem, we need the following lemma in [9].

*Lemma* 1 [9]. *Let G be a graph and* $d \geq 1$ *an integer. If G is not* $(k, d)^*$*-choosable but every subgraph of G with fewer vertices is, then the following facts hold*:

(*a*) $\delta(G) \geq k$;
(*b*) *If u*$\in$ *V(G) is a k-vertex and v is a neighbor of u, then* $d(v) \geq k + d$.

By the minimality of $G$, the following lemma is straightforward.
*Lemma* 2. *Let* $G = (V, E)$ *be a counterexample to Theorem* 2 *with the smallest* $|V| + |E|$. *Then we have*

(1) $\delta(G) \geq 3$;
(2) *G has no adjacent 3-vertices*;
(3) *There is no* (3, 4, 4)*-face*.

*Proof.* The proof of Claim (1) and (2) can be got by Lemma 1. Claim (3) goes as follows. Suppose to the contrary that $G$ contains a (3,4,4)-face $[uvw]$ such that $d(u) = 3$ and $d(v) = d(w) = 4$. By the minimality of $G$, $G$-$\{u, v, w\}$ has an $(L, 1)^*$-coloring $\phi$. Define $L'(x) = L(x) - A(x)$ for every $x \in \{u, v, w\}$, where $A(x)$ denotes the set of colors that $\phi$ assigns to the neighbors of $x$ in $G$-$\{u, v, w\}$. Thus $|L'(u)| \geq 2$, $|L'(v)| \geq 1$, and $|L'(w)| \geq 1$. An $(L', 1)^*$-coloring of the 3-circuit $uvwu$ can be constructed easily. Hence, $G$ is $(L, 1)^*$-colorable, this contradicts the choice of $G$.

By Theorem 1, $G$ must either contain a $(4^-, 4^-, 4^-)$-face or an even $(3, 4, \ldots, 3, 4)$-face. Now we show that both cases are impossible.

If $G$ contains a $(4^-, 4^-, 4^-)$-face, then it is a $(4, 4, 4)$-face by Lemma 2 (see Figure 1). Let $f = [x, y, z]$, and let $x', x'', y', y'', z', z''$ be the neighbors of $x, y, z$ respectively. Let $G'$=$G$-$xy$. Then we know $G'$ is $(3, 1)^*$-choosable, we denote the coloring of $G'$ by $\phi$. Now we will show that $\phi$ can be extended to the $(3, 1)^*$-coloring of the whole graph $G$.

If $\phi(x) \neq \phi(y)$, then $G$ itself is $(L, 1)^*$-colorable, so we get it. Otherwise, we assume that $\phi(x) = \phi(y)$. If $\phi(x) = \phi(y) \notin \{\phi(x'), \phi(x''), \phi(y'), \phi(y'')\}$, $\phi$ is also a $(L, 1)^*$-coloring of $G$. So we conclude $\phi(x) = \phi(y) \in \{\phi(x'), \phi(x''), \phi(y'), \phi(y'')\}$.

So, by symmetry, we can assume that $\phi(x) = \phi(y) = 1$, and $\phi(z) = 2$. Of course, $1 \in \{\phi(x'), \phi(x''), \phi(y'), \phi(y'')\}$, assume $\phi(x') = 1$, and the neighbors of $x'$ except $x$ are not colored 1.

Now we recolor the vertex $x$ with a color $L(x) \backslash \{\phi(x'), \phi(x'')\}$. Also, we color $y$ with a color $L(y) \backslash \{\phi(y'), \phi(y'')\}$, $z$ with a color $L(z) \backslash \{\phi(z'), \phi(z'')\}$ in the new coloring $\phi'$. Maybe the new color of $x, y, z$ is same as that colored in $\phi$. Note that if at most two of $x, y, z$ are arranged with the same color, then the resulting coloring is an $(L, 1)^*$-coloring and we are done. So we should suppose that $\phi'(x) = \phi'(y) = \phi'(z)$, moreover $\phi'(x) \neq 1$ because $\phi(x') = 1$. We further rearrange the color 1 to $x$ because $1 \in L(x)$. Then we get a $(L, 1)^*$-coloring of $G$. This contradicts the choice of $G$.

If $G$ contains an even $(3, 4, \ldots, 3, 4)$-face $f$. Let $L$ be a 3-list assignment of $G$, and $f = [v_1 v_2 \cdots v_{2k}]$ be such a $(3, 4, \ldots, \ldots, 3, 4)$-face.

By the assumption, there exists an $(L, 1)^*$-coloring $\phi$ of $G - V(f)$. Let $L'(v)$ be the color list of $v$ after removing the colors used by the neighbors of $v$ used in $\phi$. Thus $|L'(v_i)| \geq 2$, $|L'(v_{i+1})| \geq 1$, here $i \in \{1, 3, \ldots, 2k - 1\}$. Let $\phi'(v_{2k})$ be the remaining color from its color list for $k = 1, \ldots, n$, and $\phi'(v_j)$ with the color from $L'(v_j) \backslash \phi'(v_{j-1})$

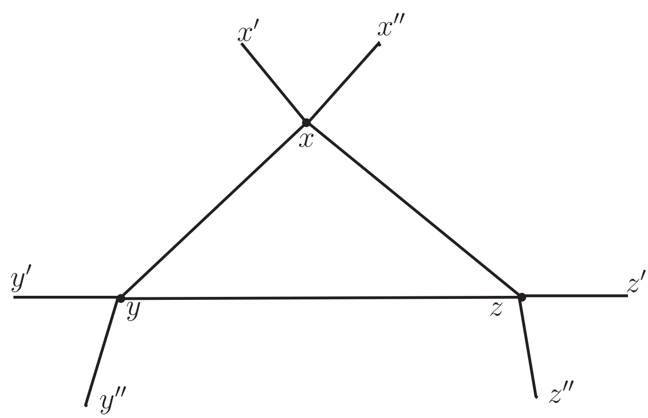


**Figure 1.** The (4, 4, 4)-face $f$.

for $j = 3, 5, 7, \ldots, 2k-1$ in order, finally color $v_1$ by one color of in $L'(v_1)\backslash\phi'(v_{2k})$. This coloring $\phi'$ of $V(f)$ combining with the coloring $\phi$ of $V(G) - V(f)$ gives a $(L, 1)$*-coloring of the graph $G$. This completes the proof of Theorem 2. □

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## References

[1] Bondy J A and Murty U S R, Graph theory (2008) (New York: Springer)
[2] Chen Y, Zhu W and Wang W, Improper choosability of graphs of nonnegative characteristic, *Comput. Math. Appl.* **56** (2008) 2073–2078
[3] Chen M and Raspaud A, On (3, 1)*-choosability of planar graphs without adjacent short cycles, *Discrete Appl. Math.* **162** (2014) 159–166
[4] Cowen L, Cowen R and Woodall D, Defective colorings of graphs in surfaces: partitions into subgraphs of bounded valency, *J. Graph Theory* **10** (1986) 187–195
[5] Cushing W and Kierstead H A, Planar graphs are 1-relaxed, 4-choosable, *European J. Combin.* **31** (2009) 1385–1397
[6] Dong W and Xu B, A note on list improper coloring of plane graphs, *Discrete Appl. Math.* **28** (2009) 433–436
[7] Eaton N and Hull T, Defective list colorings of planar graphs, *Bull. Inst. Comb. Appl* **25** (1999) 79–87
[8] Erdös P, Rubin A L and Taylor H, Choosability in graphs, *Congres. Numer.* **26** (1979) 125–157
[9] Lih K, Song Z, Wang W and Zhang K, A note on list improper coloring planar graphs, *Appl. Math. Lett.* **14** (2001) 269–273
[10] Škrekovski R, List improper colorings of planar graphs, *Comb. Prob. Comp.* **8** (1999) 293–299
[11] Škrekovski R, A Grötzsch-type theorem for list colorings with impropriety one, *Comb. Prob. Comp.* **8** (1999) 493–507
[12] Škrekovski R, List improper colorings of planar graphs with prescribed girth, *Discrete Math.* **214** (2000) 221–233
[13] Subramanian C R, List set colouring: bounds and algorithms, *Combin. Probab. Comput.* **16** (2007) 145–158
[14] Vizing V G, Coloring the vertices of a graph in prescribed colors, *Diskret. Analiz.* **29** (1976) 3–10
[15] Wang Y and Xu L, Improper choosability of planar graphs without 4-cycle, *Siam. J. Discrete Math.* **27(4)** (2013) 2029–2037
[16] Xu B and Zhang H, Every torodial graph without adjacent triangles is (4, 1)*-choosable, *Discrete Appl. Math.* **155** (2007) 74–78
[17] Zhang L, A (3, 1)*-choosable theorem on toroidal graphs, *Discrete Appl. Math.* **160** (2012) 332–338

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