

Existence of positive weak solutions for (p, q) -Laplacian nonlinear systems

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Abstract. We mainly consider the existence of a positive weak solution of the following system

$$\begin{cases} -\Delta_p u + |u|^{p-2} u = \lambda[g(x)a(u) + c(x)f(v)], & \text{in } \Omega, \\ -\Delta_q v + |v|^{q-2} v = \mu[g(x)b(v) + c(x)h(u)], & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $p, q > 1$ and λ, μ are positive parameters, and $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$ and g, c are nonnegative and continuous functions and f, h, a, b are C^1 nondecreasing functions satisfying $a(0), b(0) \geq 0$. We have proved the existence of a positive weak solution for λ, μ large when

$$\lim_{x \rightarrow \infty} \frac{f[M(h(x))^{\frac{1}{q-1}}]}{x^{p-1}} = 0$$

for every $M > 0$.

Keywords. p -Laplacian systems; sub-supersolution; positive weak solutions.

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1. Introduction

In this work, we study the existence of a positive weak solution for the system

$$\begin{cases} -\Delta_p u + |u|^{p-2} u = \lambda[g(x)a(u) + c(x)f(v)], & \text{in } \Omega, \\ -\Delta_q v + |v|^{q-2} v = \mu[g(x)b(v) + c(x)h(u)], & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases} \quad (\text{P})$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $p, q > 1$ and λ, μ are positive parameters, and $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$.

In the literature many papers (see, for instance, [3–8, 11–13]) deal with nonlinear elliptic problems. Our motivation comes from [12], where the authors considered the existence of positive weak solutions for the system

$$\begin{cases} -\Delta_p u = \lambda f(v), & \text{in } \Omega, \\ -\Delta_p v = \lambda g(u), & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases} \quad (\text{I})$$

the first eigenfunction is used to construct the subsolution of p -Laplacian problems successfully. On the condition that λ, μ is large enough and

$$\lim_{x \rightarrow \infty} \frac{f[M(g(x))^{\frac{1}{p-1}}]}{x^{p-1}} = 0$$

for every $M > 0$, the authors give the existence of positive solutions for system (I).

In [4], the author considered the existence and nonexistence of positive weak solution to the following quasilinear elliptic system

$$\begin{cases} -\Delta_p u = \lambda f(u, v) = \lambda u^\alpha v^\gamma, & \text{in } \Omega, \\ -\Delta_q v = \lambda g(u, v) = \lambda u^\delta v^\beta, & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases} \quad (\text{II})$$

the first eigenfunction is used to construct the subsolution of system (II), the main results are as follows:

- (i) If $\alpha, \beta \geq 0, \gamma, \delta > 0, \theta = (p-1-\alpha)(q-1-\beta) - \gamma\delta > 0$, then system (II) has a positive weak solution for each $\lambda > 0$;
- (ii) If $\theta = 0$ and $p\gamma = q(p-1-\alpha)$, then there exists $\lambda_0 > 0$ such that for $0 < \lambda < \lambda_0$, then system (II) has no nontrivial nonnegative weak solution.

In this paper, we consider the existence of a positive solution of the system (P) based on the method of sub-supersolutions. First we give the following hypotheses:

(H₁) $\Omega \subset R^N$ is an open bounded domain with smooth boundary $\partial\Omega$.

(H₂) $f, h : [0, \infty] \rightarrow R^+ \cup \{0\}$ are C^1 , monotone functions such that

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} h(x) = +\infty$$

(H₃) $(f+h)(0)$ is not zero.

(H₄) For any positive constant M ,

$$\lim_{x \rightarrow \infty} \frac{f[M(h(x))^{\frac{1}{q-1}}]}{x^{p-1}} = 0.$$

(H₅) a, b are C^1 nondecreasing functions satisfying $a(0), b(0) \geq 0$, and

$$\lim_{x \rightarrow \infty} \frac{a(x)}{x^{p-1}} = \lim_{x \rightarrow \infty} \frac{b(x)}{x^{q-1}} = 0.$$

Let $W^{1,p}(\Omega) = \{u \in L^p(\Omega) : |\nabla u| \in L^p(\Omega)\}$ with the norm

$$\|u\|_{W^{1,p}(\Omega)} = \left(\int_{\Omega} (|\nabla u|^p + |u|^p) dx \right)^{\frac{1}{p}};$$

then $W^{1,p}(\Omega)$ is a Banach space. We denote by $W_0^{1,p}(\Omega)$ the closure of $C_0^\infty(\Omega)$ in $W^{1,p}(\Omega)$ [1, 9, 10].

DEFINITION 1.1

If $(u, v) \in (W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega))$, (u, v) is called a weak solution of the system (P), it satisfies

$$\begin{aligned} \int_{\Omega} |\nabla u|^{p-2} \nabla u \cdot \nabla \psi \, dx + \int_{\Omega} |u|^{p-2} u \psi \, dx \\ = \lambda \int_{\Omega} [g(x)a(u) + c(x)f(v)] \psi \, dx, \quad \forall \psi \in W_0^{1,p}(\Omega), \end{aligned}$$

and

$$\begin{aligned} \int_{\Omega} |\nabla v|^{q-2} \nabla v \cdot \nabla \varphi \, dx + \int_{\Omega} |v|^{q-2} v \varphi \, dx \\ = \mu \int_{\Omega} [g(x)b(v) + c(x)h(u)] \varphi \, dx, \quad \forall \varphi \in W_0^{1,q}(\Omega). \end{aligned}$$

2. Main results

Theorem 2.1. *Let H_1 – H_5 hold. Then (P) has one positive solution (u, v) .*

Proof. By using a method of [12], we shall establish Theorem 2.1 by constructing a subsolution (Φ_1, Φ_2) and supersolution (z_1, z_2) of (P), such that $\Phi_1 \leq z_1$ and $\Phi_2 \leq z_2$, i. e., (Φ_1, Φ_2) and (z_1, z_2) satisfies

$$\begin{cases} \int_{\Omega} |\nabla \Phi_1|^{p-2} \nabla \Phi_1 \cdot \nabla \psi \, dx + \int_{\Omega} |\Phi_1|^{p-2} \Phi_1 \psi \, dx \\ \quad \leq \lambda \int_{\Omega} [g(x)a(\Phi_1) + c(x)f(\Phi_2)] \psi \, dx, \\ \int_{\Omega} |\nabla \Phi_2|^{q-2} \nabla \Phi_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |\Phi_2|^{q-2} \Phi_2 \varphi \, dx \\ \quad \leq \mu \int_{\Omega} [g(x)b(\Phi_2) + c(x)h(\Phi_1)] \varphi \, dx, \end{cases}$$

and

$$\begin{cases} \int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla \psi \, dx + \int_{\Omega} |z_1|^{p-2} z_1 \psi \, dx \\ \quad \geq \lambda \int_{\Omega} [g(x)a(z_1) + c(x)f(z_2)] \psi \, dx, \\ \int_{\Omega} |\nabla z_2|^{q-2} \nabla z_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |z_2|^{q-2} z_2 \varphi \, dx \\ \quad \geq \mu \int_{\Omega} [g(x)b(z_2) + c(x)h(z_1)] \varphi \, dx, \end{cases}$$

for all $(\psi, \varphi) \in (W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega))$ with $\psi \geq 0$ and $\varphi \geq 0$. Also $(0, 0)$ is a subsolution of the system (P), and $(0, 0)$ is not a solution of the system (P) by (H_3) . We construct a supersolution of (P).

Let k_r be the solution of

$$\begin{cases} -\Delta_r k_r + |k_r|^{r-2} k_r = 1, & \text{in } \Omega, \\ k_r = 0, & \text{on } \partial\Omega, \end{cases}$$

for $r = p, q$.

Let

$$(z_1, z_2) = \left(\frac{C}{\eta} \lambda^{\frac{1}{p-1}} k_p, [th(C\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} k_q \right)$$

where $\eta = \|k_p\|_\infty$ and $\gamma = \|k_q\|_\infty$, t is a constant such that $t \geq 1 + \|c\|_\infty$, and $C > 0$ is a large number to be chosen later. We shall verify that (z_1, z_2) is a supersolution of (P). To this end, let $(\psi, \varphi) \in (W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega))$ with $\psi \geq 0$ and $\varphi \geq 0$. Then we have

$$\begin{aligned} & \int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla \psi \, dx + \int_{\Omega} |z_1|^{p-2} z_1 \psi \, dx \\ &= \lambda \left(\frac{C}{\eta} \right)^{p-1} \int_{\Omega} |\nabla k_p|^{p-2} \nabla k_p \cdot \nabla \psi \, dx + \lambda \left(\frac{C}{\eta} \right)^{p-1} \int_{\Omega} |k_p|^{p-2} k_p \psi \, dx \\ &= \lambda \left(\frac{C}{\eta} \right)^{p-1} \left(\int_{\Omega} |\nabla k_p|^{p-2} \nabla k_p \cdot \nabla \psi \, dx + \int_{\Omega} |k_p|^{p-2} k_p \psi \, dx \right) \\ &= \lambda \left(\frac{C}{\eta} \right)^{p-1} \int_{\Omega} \psi \, dx. \end{aligned}$$

By (H₄) and (H₅), we can choose C large enough such that

$$C^{p-1} \geq \eta^{p-1} \left[\|g\|_\infty a \left(C\lambda^{\frac{1}{p-1}} \right) + \|c\|_\infty f \left(t \left(C\lambda^{\frac{1}{p-1}} \right) \right)^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} \gamma \right]$$

and therefore

$$\begin{aligned} & \int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla \psi \, dx + \int_{\Omega} |z_1|^{p-2} z_1 \psi \, dx \\ & \geq \lambda \int_{\Omega} [\|g\|_\infty a(C\lambda^{\frac{1}{p-1}}) + \|c\|_\infty f((th(C\lambda^{\frac{1}{p-1}}))^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} \gamma)] \psi \, dx \\ & \geq \lambda \int_{\Omega} \left[g(x) a \left(C\lambda^{\frac{1}{p-1}} \frac{k_p}{\eta} \right) + c(x) f((th(C\lambda^{\frac{1}{p-1}}))^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} k_q) \right] \psi \, dx \\ & = \lambda \int_{\Omega} [g(x) a(z_1) + c(x) f(z_2)] \psi \, dx. \end{aligned}$$

Similarly, choose C large so that

$$\frac{\|g\|_\infty (b([th(C\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} \gamma))}{h(C\lambda^{\frac{1}{p-1}})} \leq 1,$$

then

$$\begin{aligned} & \int_{\Omega} |\nabla z_2|^{q-2} \nabla z_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |z_2|^{q-2} z_2 \varphi \, dx \\ &= t\mu h(C\lambda^{\frac{1}{p-1}}) \int_{\Omega} |\nabla k_q|^{q-2} \nabla k_q \cdot \nabla \varphi \, dx + t\mu h(C\lambda^{\frac{1}{p-1}}) \int_{\Omega} |k_q|^{q-2} k_q \varphi \, dx \end{aligned}$$

$$\begin{aligned}
&= t\mu h(C\lambda^{\frac{1}{p-1}}) \left(\int_{\Omega} |\nabla k_q|^{q-2} \nabla k_q \cdot \nabla \varphi \, dx + \int_{\Omega} |k_q|^{q-2} k_q \varphi \, dx \right) \\
&= [\mu h(C\lambda^{\frac{1}{p-1}}) + (t-1)\mu h(C\lambda^{\frac{1}{p-1}})] \int_{\Omega} \varphi \, dx \\
&\geq \mu \int_{\Omega} [\|g\|_{\infty} (b([th(C\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} \gamma)) + \|c\|_{\infty} h(C\lambda^{\frac{1}{p-1}})] \varphi \, dx \\
&\geq \mu \int_{\Omega} [g(x)b(z_2) + c(x)h(z_1)] \varphi \, dx,
\end{aligned}$$

i.e. (z_1, z_2) is a supersolution of (P) with $z_i \geq 0$ for C large, $i = 1, 2$. Thus, there exists a solution (u, v) of (P) with $0 \leq u \leq z_1$, $0 \leq v \leq z_2$. This completes the proof. \square

Now, we study the existence of a positive weak solution for the system

$$\begin{cases} -\Delta_p u + |u|^{p-2} u = \lambda a(x) f(u, v), & \text{in } \Omega, \\ -\Delta_q v + |v|^{q-2} v = \mu b(x) g(u, v), & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases} \quad (P')$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $p, q > 1$ and λ, μ are positive parameters, and $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$.

First we give the following hypotheses:

- (F₁) $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary $\partial\Omega$.
(F₂) $f, g \in C^1((0, \infty) \times (0, \infty)) \cap C([0, \infty) \times [0, \infty))$ be monotone functions such that $f_u, f_v, g_u, g_v \geq 0$ and

$$\lim_{u, v \rightarrow \infty} f(u, v) = \lim_{u, v \rightarrow \infty} g(u, v) = +\infty$$

- (F₃) $(f + g)(0, 0)$ is not zero.
(F₄) For any positive constant M ,

$$\lim_{x \rightarrow \infty} \frac{f(x, M[g(x, x)]^{\frac{1}{q-1}})}{x^{p-1}} = 0$$

- (F₅) $\lim_{x \rightarrow \infty} \frac{g(x, x)}{x^{q-1}} = 0$

Theorem 2.2. *Let F₁–F₅ hold. Then (P') has one positive solution (u, v) .*

Proof. We shall establish Theorem 2.2 by constructing a subsolution (Φ_1, Φ_2) and supersolution (z_1, z_2) of (P') , such that $\Phi_1 \leq z_1$ and $\Phi_2 \leq z_2$. That is, (Φ_1, Φ_2) and (z_1, z_2) satisfy

$$\begin{cases} \int_{\Omega} |\nabla \Phi_1|^{p-2} \nabla \Phi_1 \cdot \nabla \psi \, dx + \int_{\Omega} |\Phi_1|^{p-2} \Phi_1 \psi \, dx \leq \lambda \int_{\Omega} a(x) f(\Phi_1, \Phi_2) \psi \, dx, \\ \int_{\Omega} |\nabla \Phi_2|^{q-2} \nabla \Phi_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |\Phi_2|^{q-2} \Phi_2 \varphi \, dx \leq \mu \int_{\Omega} b(x) g(\Phi_1, \Phi_2) \varphi \, dx, \end{cases}$$

and

$$\begin{cases} \int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla \psi \, dx + \int_{\Omega} |z_1|^{p-2} z_1 \psi \, dx \geq \lambda \int_{\Omega} [a(x) f(z_1, z_2)] \psi \, dx, \\ \int_{\Omega} |\nabla z_2|^{q-2} \nabla z_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |z_2|^{q-2} z_2 \varphi \, dx \geq \mu \int_{\Omega} b(x) g(z_1, z_2) \varphi \, dx, \end{cases}$$

for all $(\psi, \varphi) \in (W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega))$ with $\psi \geq 0$ and $\varphi \geq 0$. $(0,0)$ is a subsolution of the system (P') , and $(0,0)$ is not a solution of the system (P') by (F_3) . We construct a supersolution of (P') .

Let k_r be the solution of

$$\begin{cases} -\Delta_r k_r + |k_r|^{r-2} k_r = 1, & \text{in } \Omega, \\ k_r = 0, & \text{on } \partial\Omega, \end{cases}$$

for $r = 1, 2$. Let

$$(z_1, z_2) = \left(\frac{C}{\eta} \lambda^{\frac{1}{p-1}} k_1, [g(C \lambda^{\frac{1}{p-1}}, C \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} k_2 \right),$$

where $\eta = \|k_1\|_{\infty}$ and $\gamma = \|k_2\|_{\infty}$, $C > 0$ is a large number to be chosen later. We shall verify that (z_1, z_2) is a supersolution of (P') . To this end, let $(\psi, \varphi) \in (W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega))$ with $\psi \geq 0$ and $\varphi \geq 0$. Then we have

$$\begin{aligned} & \int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla \psi \, dx + \int_{\Omega} |z_1|^{p-2} z_1 \psi \, dx \\ &= \lambda \left(\frac{C}{\eta} \right)^{p-1} \int_{\Omega} |\nabla k_1|^{p-2} \nabla k_1 \cdot \nabla \psi \, dx + \lambda \left(\frac{C}{\eta} \right)^{p-1} \int_{\Omega} |k_1|^{p-2} k_1 \psi \, dx \\ &= \lambda \left(\frac{C}{\eta} \right)^{p-1} \left(\int_{\Omega} |\nabla k_1|^{p-2} \nabla k_1 \cdot \nabla \psi \, dx + \int_{\Omega} |k_1|^{p-2} k_1 \psi \, dx \right) \\ &= \lambda \left(\frac{C}{\eta} \right)^{p-1} \int_{\Omega} \psi \, dx. \end{aligned}$$

By (F_4) , we can choose C large enough such that

$$C^{p-1} \geq \eta^{p-1} \|a\|_{\infty} f(C \lambda^{\frac{1}{p-1}}, [\mu^{\frac{1}{q-1}} \gamma] [g(C \lambda^{\frac{1}{p-1}}, C \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}})$$

and therefore

$$\begin{aligned} & \int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla \psi \, dx + \int_{\Omega} |z_1|^{p-2} z_1 \psi \, dx \\ & \geq \lambda \int_{\Omega} \|a\|_{\infty} f(C \lambda^{\frac{1}{p-1}}, [\mu^{\frac{1}{q-1}} \gamma] [g(C \lambda^{\frac{1}{p-1}}, C \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}) \psi \, dx \\ & \geq \lambda \int_{\Omega} a(x) f(C \lambda^{\frac{1}{p-1}} \frac{k_1}{\eta}, [\mu^{\frac{1}{q-1}} k_2] [g(C \lambda^{\frac{1}{p-1}}, C \lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}}) \psi \, dx \\ & = \lambda \int_{\Omega} a(x) f(z_1, z_2) \psi \, dx. \end{aligned}$$

Similarly, choose C large so that

$$\frac{\|b\|_{\infty} [g(C\lambda^{\frac{1}{p-1}}, [g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} \gamma)]}{g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}})} \leq 1,$$

then

$$\begin{aligned} & \int_{\Omega} |\nabla z_2|^{q-2} \nabla z_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |z_2|^{q-2} z_2 \varphi \, dx \\ &= \mu g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}}) \int_{\Omega} |\nabla k_2|^{q-2} \nabla k_2 \cdot \nabla \varphi \, dx \\ & \quad + \mu g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}}) \int_{\Omega} |k_2|^{q-2} k_2 \varphi \, dx \\ &= \mu g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}}) \left(\int_{\Omega} |\nabla k_2|^{q-2} \nabla k_2 \cdot \nabla \varphi \, dx + \int_{\Omega} |k_2|^{q-2} k_2 \varphi \, dx \right) \\ &= \mu g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}}) \int_{\Omega} \varphi \, dx \\ &\geq \mu \int_{\Omega} \|b\|_{\infty} g(C\lambda^{\frac{1}{p-1}} \frac{k_1}{\eta}, [g(C\lambda^{\frac{1}{p-1}}, C\lambda^{\frac{1}{p-1}})]^{\frac{1}{q-1}} \mu^{\frac{1}{q-1}} k_2) \varphi \, dx \\ &\geq \mu \int_{\Omega} b(x) g(z_1, z_2) \varphi \, dx. \end{aligned}$$

i.e. (z_1, z_2) is a supersolution of (P') with $z_i \geq 0$ for C large, $i = 1, 2$. Thus, there exists a solution (u, v) of (P') with $0 \leq u \leq z_1$, $0 \leq v \leq z_2$. This completes the proof. \square

3. Example

By using example of [2], in problem (P') , let $a(x) \equiv b(x) \equiv 1$. Then, consider the problem

$$\begin{cases} -\Delta_p u + |u|^{p-2} u = \lambda[v^{\alpha} + (uv)^{\beta} - 1], & \text{in } \Omega, \\ -\Delta_q v + |v|^{q-2} v = \mu[u^{\sigma} + (uv)^{\frac{\gamma}{2}} - 1], & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases} \quad (3.1)$$

where $\alpha, \beta, \sigma, \gamma$ are positive parameters. Then it is easy to see that (3.1) satisfies the hypotheses of Theorem 2.2 if $\max\{\sigma, \gamma\} \frac{\alpha}{q-1} < p-1$, $(\max\{\sigma, \gamma\} \frac{1}{q-1} + 1)\beta < p-1$ and $\max\{\sigma, \gamma\} < q-1$.

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References

- [1] Adams R A and Fournier J J F, Sobolov Spaces (2003) (Academic Press)
- [2] Ali J and Shivaji R, On positive solutions for a class of strongly coupled P -Laplacian systems, Electronic Journal of Differential Equations, Conference 16 (2007) pp. 29–34

- [3] Chaib K, Necessary and sufficient conditions of existence for a system involving the P -Laplacian ($1 < P < N$), *J. Differ. Equ.* **189** (2003) 513–523
- [4] Chen C H, On positive weak solutions for a class of quasilinear elliptic systems, *Nonlinear Anal.* **62** (2005) 751–756
- [5] Covei D-P, Existence and asymptotic behavior of positive solution to a quasilinear elliptic problem in \mathbb{R}^N , *Nonlinear Anal. TMA* **69** (2008) 2615–2622
- [6] Covei D-P, Large and entire large solution for a quasilinear problem, *Nonlinear Anal. TMA* **70** (2009) 1738–1745
- [7] Covei D-P, Existence and uniqueness of solutions for the Lane, Emden and Fowler type problem, *Nonlinear Anal. TMA* **72** (2010) 2684–2693
- [8] Covei D-P, Existence results for a quasilinear elliptic problem with a gradient term via shooting method, *Appl. Math. Comput.* **218** (2011) 4161–4168
- [9] Evans L C, *Partial Differential Equations* (1998) (American Mathematical Society)
- [10] Gua D J, *Nonlinear Functional Analysis* (2002) (Shandong Scientific and Technology Press)
- [11] Hai D D and Shivaji R, Existence and uniqueness of solutions for quasilinear elliptic systems, *Proc. Amer. Math. Soc.* **133**(1) (2005) 223–228
- [12] Hai D D and Shivaji R, An existence result on positive solutions of P -Laplacian systems, *Nonlinear Anal.* **56** (2004) 1007–1010
- [13] Song X, Wang W and Zhao P, Positive solutions of elliptic equations with nonlinear boudary conditions, *Nonlinear Anal.* (2007) 1–7

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