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Annihilating power values of co-commutators with generalized derivations

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Abstract. Let R be a prime ring with its Utumi ring of quotient U, H and G be two generalized derivations of R and L a noncentral Lie ideal of R. Suppose that there exists $0 \neq a \in R$ such that $a(H(u)u - uG(u))^n = 0$ for all $u \in L$, where $n \geq 1$ is a fixed integer. Then there exist $b', c' \in U$ such that $H(x) = b'x + xc'$, $G(x) = c'x$ for all $x \in R$ with $ab' = 0$, unless R satisfies s_4, the standard identity in four variables.

Keywords. Prime ring; derivation; generalized derivation; extended centroid; Utumi quotient ring.

1. Introduction

Let R be an associative ring with center $Z(R)$. For $x, y \in R$, the commutator of x, y is denoted by $[x, y]$ and defined by $[x, y] = xy - yx$. By d we mean a derivation of R. An additive mapping F from R to R is called a generalized derivation if there exists a derivation d from R to R such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$.

Throughout this paper, R will always present a prime ring with center $Z(R)$, extended centroid C and U its Utumi quotient ring. A well-known result proved by Posner [23], states that if the commutators $[d(x), x] \in Z(R)$ for all $x \in R$, then either $d = 0$ or R is commutative. Then the result of Posner was generalized in many directions by a number of authors. Posner's theorem was extended to Lie ideals in prime rings by Lee [20] and then by Lanski [16]. Carini and De Filippis [6] studied a more generalized situation considering power central values. They proved that if char $(R) \neq 2$ and $[d(x), x]^n \in Z(R)$ for all $x \in L$, where L is a noncentral Lie ideal of R and $n \geq 1$ a fixed integer, then $d = 0$ or R satisfies s_4. In [25], Wang and You removed the assumption of char $(R) \neq 2$.

Furthermore, De Filippis [10] studied the left annihilator of power values of commutators with derivations. He proved that if char $(R) \neq 2$, $0 \neq d$ and $a \in R$ such that $a[d(x), x]^n \in Z(R)$ for all $x \in L$, where L is a noncentral Lie ideal of R and $n \geq 1$ a fixed integer, then either $a = 0$ or R satisfies s_4. In this result, Wang [24] removed the assumption of char $(R) \neq 2$.

Recently, De Filippis [9] studied a situation replacing d with a generalized derivation g of R. More precisely, he proved the following:

Let R be a prime ring of characteristic $\neq 2$ with right quotient ring U and extended centroid C, $g \neq 0$ a generalized derivation of R, L a noncentral Lie ideal of R and

$n \geq 1$. *If* $[g(u),u]^n = 0$ *for all* $u \in L$, *then there exists an element* $a \in C$ *such that* $g(x) = ax$ *for all* $x \in R$, *unless* R *satisfies* S_4 *and there exists an element* $b \in U$ *such that* $g(x) = bx + xb$ *for all* $x \in R$.

Notice that in this result of De Filippis, assumption of char $(R) \neq 2$ is existing.

On the other hand, many authors generalized Posner's theorem by considering two derivations. In [4], Brešar proved that if d and δ are two derivations of R such that $d(x)x - x\delta(x) \in Z(R)$ for all $x \in R$, then either $d = \delta = 0$ or R is commutative. Later, Lee and Wong [21] considered the situation $d(x)x - x\delta(x) \in Z(R)$ for all x in some noncentral Lie ideal L of R and they proved that either $d = \delta = 0$ or R satisfies s_4. In these results, there are no restrictions on characteristic. Recently, Argac and De Filippis [1] studied the situation considering power values. They obtained the following result:

Let R *be a prime ring with* char $(R) \neq 2$, L *a non-central Lie ideal of* R, d, δ *non-zero derivations of* R, $n \geq 1$ *a fixed integer. If* $(d(x)x - x\delta(x))^n = 0$ *for all* $x \in L$, *then either* $d = \delta = 0$ *or* R *satisfies the standard identity* s_4 *and* d, δ *are inner derivations, induced respectively by the elements* a *and* b *such that* $a + b \in Z(R)$.

In this result, again the assumption of char $(R) \neq 2$ is existing.

In [8], De Filippis studied this situation replacing two derivations d and δ by two generalized derivations H and G respectively. De Filippis [8] proved the following:

Let R *be a prime ring,* U *the Utumi quotient ring of* R, $C = Z(U)$ *the extended centroid of* R, L *a non-central Lie ideal of* R *and* H, G *two non-zero generalized derivations of* R. *Suppose that there exists an integer* $n \geq 1$ *such that* $(H(u)u - uG(u))^n = 0$, *for all* $u \in L$, *then one of the following holds*:

(1) *there exists* $c \in U$ *such that* $H(x) = xc, G(x) = cx$;
(2) R *satisfies the standard identity* s_4 *and* char $(R) = 2$;
(3) R *satisfies* s_4 *and there exist* $a, b, c \in U$, *such that* $H(x) = ax + xc, G(x) = cx + xb$ *and* $(a - b)^n = 0$.

Like previous studies on derivations, there is also a study of generalized derivations with left annihilator conditions. Recently, Carini *et al.* [5] obtained the following result:

Let R *be a prime ring of characteristic different from* 2, U *the Utumi quotient ring of* R, $C = Z(U)$ *the extended centroid of* R, L *a non-central Lie ideal of* R, H *and* G *non-zero generalized derivations of* R. *Suppose that there exists* $0 \neq a \in R$ *such that* $a(H(u)u - uG(u)) = 0$, *for all* $u \in L$, *then one of the following holds*:

(1) *there exist* $b', c' \in U$ *such that* $H(x) = b'x + xc'$, $G(x) = c'x$ *with* $ab' = 0$;
(2) R *satisfies* s_4 *and there exist* $b', c', q' \in U$ *such that* $H(x) = b'x + xc'$, $G(x) = c'x + xq'$, *with* $a(b' - q') = 0$.

In this result, assumption of characteristic $\neq 2$ exists.

In the present article, we generalize the above results by studying the situation $a(H(u)u - uG(u))^n = 0$ for all $u \in L$ without any restriction of characteristic, where H, G are two generalized derivations of R, L is a noncentral Lie ideal of R, $a \in R$ and $n \geq 1$ is a fixed integer.

We need the following remarks:

Remark 1. Let R be a prime ring and L a noncentral Lie ideal of R. If char $(R) \neq 2$, by Lemma 1 of [3] there exists a nonzero ideal I of R such that $0 \neq [I, R] \subseteq L$. If char $(R) = 2$ and $\dim_C RC > 4$ i.e., char $(R) = 2$ and R does not satisfy s_4, then by Theorem 13 of [17] there exists a nonzero ideal I of R such that $0 \neq [I, R] \subseteq L$. Thus if either char $(R) \neq 2$ or R does not satisfy s_4, then we may conclude that there exists a nonzero ideal I of R such that $[I, I] \subseteq L$.

Remark 2. Let R be a prime ring and U be the Utumi quotient ring of R and $C = Z(U)$, the center of U (see [2] for more details). It is well-known that any derivation of R can be uniquely extended to a derivation of U. In Theorem 3 of [18], Lee proved that every generalized derivation g on a dense right ideal of R can be uniquely extended to a generalized derivation of U. Furthermore, the extended generalized derivation g has the form $g(x) = ax + d(x)$ for all $x \in U$, where $a \in U$ and d is a derivation of U.

2. Main results

We begin with the following lemmas:

Lemma 2.1. *Let R be a prime ring with extended centroid C and $a, c, w, p \in R$. If $p \neq 0$ such that $p(a[x_1, x_2]^2 + [x_1, x_2]w[x_1, x_2] + [x_1, x_2]^2c)^n = 0$ for all $x_1, x_2 \in R$, where $n \geq 1$ a fixed integer, then either R satisfies a nontrivial generalized polynomial identity (GPI) or $c, w \in C$ and $p(a + w + c) = 0$.*

Proof. Assume that R does not satisfy any nontrivial GPI. Let $T = U *_C C\{X_1, X_2\}$, the free product of U and $C\{X_1, X_2\}$, the free C-algebra in noncommuting indeterminates X_1 and X_2. If R is commutative, then R satisfies trivially a nontrivial GPI, a contradiction. So, R must be noncommutative.

Then, since $p(a[x_1, x_2]^2 + [x_1, x_2]w[x_1, x_2] + [x_1, x_2]^2c)^n = 0$ is a GPI for R, we see that

$$p(a[X_1, X_2]^2 + [X_1, X_2]w[X_1, X_2] + [X_1, X_2]^2c)^n = 0 \tag{1}$$

in $T = U *_C C\{X_1, X_2\}$. If $c \notin C$, then c and 1 are linearly independent over C. Thus, (1) implies

$$p(a[X_1, X_2]^2 + [X_1, X_2]w[X_1, X_2] + [X_1, X_2]^2c)^{n-1}[X_1, X_2]^2c = 0 \tag{2}$$

in T and then by the same argument, we obtain $p([X_1, X_2]^2c)^n = 0$ in T implying $c = 0$, since $p \neq 0$, a contradiction. Therefore, we conclude that $c \in C$ and hence (1) reduces to

$$p(((a + c)[X_1, X_2] + [X_1, X_2]w)[X_1, X_2])^n = 0 \tag{3}$$

in T. If $w \notin C$, then (3) reduces to

$$p(((a + c)[X_1, X_2] + [X_1, X_2]w)[X_1, X_2])^{n-1}[X_1, X_2]w[X_1, X_2] = 0 \tag{4}$$

in T and then by the same argument again we have that

$$p([X_1, X_2]w[X_1, X_2])^n = 0 \tag{5}$$

in T implying $w = 0$, a contradiction. Therefore, $w \in C$. Thus, (1) becomes

$$p((a + w + c)[X_1, X_2]^2)^n = 0$$

implying $p(a + w + c) = 0$.

Lemma 2.2. *Let R be a noncommutative prime ring with extended centroid C and $a, b, c, p \in R$. Suppose that $p \neq 0$ such that $p(a[x_1, x_2]^2 + [x_1, x_2]b[x_1, x_2] + [x_1, x_2]^2c)^n = 0$ for all $x_1, x_2 \in R$, where $n \geq 1$ is a fixed integer. Then $b, c \in C$ and $p(a + b + c) = 0$, unless R satisfies s_4.*

Proof. Suppose that R does not satisfy s_4. We have that R satisfies a generalized polynomial identity

$$f(x_1, x_2) = p(a[x_1, x_2]^2 + [x_1, x_2]b[x_1, x_2] + [x_1, x_2]^2c)^n. \tag{6}$$

If R does not satisfy any nontrivial GPI, by Lemma 2.1, we obtain $w, c \in C$ and $p(a + b + c) = 0$ which gives the conclusion. So, we assume that R satisfies a nontrivial GPI. Since R and U satisfy the same generalized polynomial identities (see [7]), U satisfies $f(x_1, x_2)$. In case C is infinite, we have $f(x_1, x_2) = 0$ for all $x_1, x_2 \in U \otimes_C \bar{C}$, where $\bar{C}$ is the algebraic closure of C. Moreover, both U and $U \otimes_C \bar{C}$ are prime and centrally closed algebras [11]. Hence, replacing R by U or $U \otimes_C \bar{C}$ according to C finite or infinite, without loss of generality we may assume that $C = Z(R)$ and R is C-algebra centrally closed. By Martindale's theorem [22], R is then a primitive ring having nonzero socle $\mathrm{soc}(R)$ with C as the associated division ring. Hence, by Jacobson's theorem (p. 75 of [14]), R is isomorphic to a dense ring of linear transformations of a vector space V over C.

If $\dim_C V = 2$, then $R \cong M_2(C)$, that is, R satisfies s_4, a contradiction. So, let $\dim_C V \geq 3$.

We show that for any $v \in V$, v and cv are linearly C-dependent. Suppose that v and cv are linearly independent for some $v \in V$. Since $\dim_C V \geq 3$, there exists $u \in V$ such that v, cv, u are linearly C-independent set of vectors. By density, there exist $x_1, x_2 \in R$ such that

$$x_1v = v, \quad x_1cv = -cv, \quad x_1u = 0; \quad x_2v = 0, \quad x_2cv = u, \quad x_2u = v.$$

Then $0 = p(a[x_1, x_2]^2 + [x_1, x_2]b[x_1, x_2] + [x_1, x_2]^2c)^n v = pv$.

This implies that if $pv \neq 0$, then by contradiction we may conclude that v and cv are linearly C-dependent. Now choose $v \in V$ such that v and cv are linearly C-independent. Set $W = \mathrm{Span}_C\{v, cv\}$. Then $pv = 0$. Since $p \neq 0$, there exists $w \in V$ such that $pw \neq 0$ and then $p(v - w) = pw \neq 0$. By the previous argument we have that w, cw are linearly C-dependent and $(v - w), c(v - w)$ too. Thus there exist $\alpha, \beta \in C$ such that $cw = \alpha w$ and $c(v - w) = \beta(v - w)$. Then $cv = \beta(v - w) + cw = \beta(v - w) + \alpha w$ i.e., $(\alpha - \beta)w = cv - \beta v \in W$. Now $\alpha = \beta$ implies that $cv = \beta v$, a contradiction. Hence $\alpha \neq \beta$ and so $w \in W$. Again, if $u \in V$ with $pu = 0$ then $p(w + u) \neq 0$. So, $w + u \in W$ forcing $u \in W$. Thus it is observed that $w \in V$ with $pw \neq 0$ implies $w \in W$ and $u \in V$ with $pu = 0$ implies $u \in W$. This implies that $V = W$ i.e., $\dim_C V = 2$, a contradiction.

Hence, in any case, v and cv are linearly C-dependent for all $v \in V$. Thus for each $v \in V$, $cv = \alpha_v v$ for some $\alpha_v \in C$. It is very easy to prove that α_v is independent of the choice of $v \in V$. Thus we can write $cv = \alpha v$ for all $v \in V$ and $\alpha \in C$ fixed. Now let $r \in R$, $v \in V$. Since $cv = \alpha v$,

$$[c, r]v = (cr)v - (rc)v = c(rv) - r(cv) = \alpha(rv) - r(\alpha v) = 0.$$

Thus $[c, r]v = 0$ for all $v \in V$ i.e., $[c, r]V = 0$. Since $[c, r]$ acts faithfully as a linear transformation on the vector space V, $[c, r] = 0$ for all $r \in R$. Therefore, $c \in Z(R)$.

Therefore, from (6) we have that R satisfies the generalized polynomial identity

$$f(x_1, x_2) = p(a'[x_1, x_2]^2 + [x_1, x_2]b[x_1, x_2])^n, \tag{7}$$

where $a' = a + c$. Now if v and bv are linearly C-independent for some $v \in V$, there exists $w \in V$ such that v, bv, w will be a linearly C-independent set of vectors, since $\dim_C V \geq 3$. Then again by density, there exist $x_1, x_2 \in R$ such that

$$x_1 v = 0,\ x_1 bv = v,\ x_1 w = v + (b - a')v;\ x_2 v = bv,\ x_2 bv = w,\ x_2 w = 0.$$

In this case we get $0 = p(a'[x_1, x_2]^2 + [x_1, x_2]b[x_1, x_2])^n v = pv$. Since $p \neq 0$, by the same argument as stated above, this leads to a contradiction. Hence, by the above argument we conclude that $b \in C$. Therefore, the identity (7) becomes that

$$p(a''[x_1, x_2]^2)^n = 0 \tag{8}$$

for all $x_1, x_2 \in R$, where $a'' = a' + b$. If $pa'' = 0$, then $p(a + b + c) = 0$ and we are done. So, let $pa'' \neq 0$.

Again, if v and $a''v$ are linearly C-independent for some $v \in V$, then since $\dim_C V \geq 3$, there exists $w \in V$ such that $v, a''v, w$ will be a linearly C-independent set of vectors. Again, by density, there exist $x_1, x_2 \in R$ such that

$$x_1 v = 0,\quad x_1 a''v = v,\quad x_1 w = a''v + v;$$
$$x_2 v = a''v,\quad x_2 a''v = w,\quad x_2 w = 0.$$

Then $0 = p(a''[x_1, x_2]^2)^n v = pa''v$. Since $pa'' \neq 0$, by the above argument, this leads to a contradiction for $\dim_C V \geq 3$. Hence, we conclude that v and $a''v$ are linearly C-dependent for all $v \in V$, implying $a'' \in C$. Then the identity (8) reduces to $0 = p(a'')^n[x_1, x_2]^{2n}$ for all $x_1, x_2 \in R$. Since $pa'' \neq 0$, $a'' \neq 0$ and so a'' is invertible in C. Then we have $0 = p[x_1, x_2]^{2n}$ for all $x_1, x_2 \in R$. Let $t = [x_1, x_2]^{2n}$. Then $pt = 0$. We can write $p[x, typ]^{2n} = 0$ for all $x, y \in R$. Since $pt = 0$, it reduces to $p(xtyp)^{2n} = 0$. This can be written as $(typx)^{2n+1} = 0$ for all $x, y \in R$. By Levitzki's lemma (Lemma 1.1 of [13]), $typ = 0$ for all $y \in R$. Since R is prime ring, either $p = 0$ or $t = 0$. Since $pa'' \neq 0$, $p \neq 0$ and hence $t = [x_1, x_2]^{2n} = 0$ for all $x_1, x_2 \in R$. Then by Herstein's result (Theorem 2 of [12]), R is commutative, a contradiction. Hence, $a'' = a+b+c = 0$.

Theorem 2.3 *Let R be a prime ring with its Utumi ring of quotient U, H and G two generalized derivations of R, L a noncentral Lie ideal of R. Suppose that there exists $0 \neq a \in R$ such that $a(H(u)u - uG(u))^n = 0$ for all $u \in L$, where $n \geq 1$ is a fixed integer. Then there exist $b', c' \in U$ such that $H(x) = b'x + xc'$, $G(x) = c'x$ for all $x \in R$ with $ab' = 0$, unless R satisfies s_4, the standard identity in four variables.*

Proof. Suppose that R does not satisfy s_4. Since L is a noncentral Lie ideal of R, by Remark 1, there exists a nonzero ideal I of R such that $[I, I] \subseteq L$. Hence, by our assumption we have

$$a(H([x_1, x_2])[x_1, x_2] - [x_1, x_2]G([x_1, x_2]))^n = 0$$

for all $x_1, x_2 \in I$. Since I, R and U satisfy the same generalized polynomial identities (see [7]) as well as he same differential identities (see [19]), they also satisfy the same generalized differential identities by Remark 2. Hence,

$$a(H([x_1, x_2])[x_1, x_2] - [x_1, x_2]G([x_1, x_2]))^n = 0$$

for all $x \in U$, where $H(x) = bx + d(x)$ and $G(x) = cx + \delta(x)$, for some $b, c \in U$ and derivations d, δ of U. Hence, U satisfies

$$\begin{aligned} &a(b[x_1, x_2]^2 + d([x_1, x_2])[x_1, x_2] - [x_1, x_2]c[x_1, x_2] \\ &\quad - [x_1, x_2]\delta([x_1, x_2]))^n = 0. \end{aligned} \tag{9}$$

Now we divide the proof into two cases:

Case I. Let $d(x) = [p, x]$ for all $x \in U$ and $\delta(x) = [q, x]$ for all $x \in U$ i.e., d and δ are both inner derivations of U. Then from (9), we obtain that U satisfies

$$a((b + p)[x_1, x_2]^2 - [x_1, x_2](c + p + q)[x_1, x_2] + [x_1, x_2]^2 q)^n = 0. \tag{10}$$

By Lemma 2.2, since R does not satisfy s_4, we have $q, c+p+q \in C$ and $a(b+p-(c+p+q)+q) = 0$ which gives $q, c+p \in C$ and $a(b-c) = 0$. Hence, $H(x) = bx+[p, x] = (b+p)x - xp = (b-c+c+p)x - xp = (b-c)x + x(c+p) - xp = (b-c)x + xc$ for all $x \in U$ and $G(x) = cx + [q, x] = cx$ for all $x \in U$. Thus, we get $H(x) = b'x + xc'$, $G(x) = c'x$ for all $x \in R$ with $ab' = 0$, where $b' = b - c$ and $c' = c$.

Case II. Next assume that d and δ are not both inner derivations of U, but they are C-dependent modulo inner derivations of U. Suppose $d = \lambda\delta + ad_p$, that is, $d(x) = \lambda\delta(x) + [p, x]$ for all $x \in U$, where $\lambda \in C$, $p \in U$. Then d can not be the inner derivation of U. From (9), we have that U satisfies

$$\begin{aligned} &a(b[x_1, x_2]^2 + \lambda\delta([x_1, x_2])[x_1, x_2] + [p, [x_1, x_2]][x_1, x_2] \\ &\quad -[x_1, x_2]c[x_1, x_2] - [x_1, x_2]\delta([x_1, x_2]))^n = 0. \end{aligned}$$

This gives

$$\begin{aligned} &a((b + p)[x_1, x_2]^2 - [x_1, x_2](p + c)[x_1, x_2] \\ &\quad +\lambda\delta([x_1, x_2])[x_1, x_2] - [x_1, x_2]\delta([x_1, x_2]))^n = 0. \end{aligned}$$

Since δ is not an inner derivation of U, by Kharchenko's theorem [15], we have that U satisfies

$$\begin{aligned} &a((b + p)[x_1, x_2]^2 - [x_1, x_2](p + c)[x_1, x_2] \\ &\quad +\lambda([u, x_2] + [x_1, v])[x_1, x_2] - [x_1, x_2]([u, x_2] + [x_1, v]))^n = 0. \end{aligned} \tag{11}$$

In particular for $u = v = 0$, we have that U satisfies

$$a((b+p)[x_1, x_2]^2 - [x_1, x_2](p+c)[x_1, x_2])^n = 0.$$

By Lemma 2.2, since R does not satisfy s_4, this implies $p+c \in C$ and $a(b+p-p-c) = 0$, that is, $a(b-c) = 0$. Since L is noncentral, R and U can not be commutative. So, there exists $q \in U$ such that $q \notin C$. Now in (11), we put $u = [q, x_1]$ and $v = [q, x_2]$ for some $q \notin C$, and then U satisfies

$$a((b-c)[x_1, x_2]^2 + \lambda[q, [x_1, x_2]][x_1, x_2] - [x_1, x_2][q, [x_1, x_2]])^n = 0, \quad (12)$$

that is

$$a((b-c+\lambda q)[x_1, x_2]^2 - [x_1, x_2](\lambda q + q)[x_1, x_2] + [x_1, x_2]^2 q)^n = 0. \quad (13)$$

Again by Lemma 2.2, since R does not satisfy s_4, this yields that $q \in C$, a contradiction.

The situation when $\delta = \lambda d + ad_q$ is similar.

Next assume that d and δ are C-independent modulo inner derivations of U. Since neither d nor δ is inner, by Kharchenko's theorem [15], we have from (9) that U satisfies

$$\begin{aligned} a(b[x_1, x_2]^2 &+ ([u_1, x_2] + [x_1, u_2])[x_1, x_2] - [x_1, x_2]c[x_1, x_2] \\ &-[x_1, x_2]([v_1, x_2] + [x_1, v_2]))^n = 0. \end{aligned} \quad (14)$$

Now assuming $u_1 = u_2 = 0$ and replacing v_1 with $[q, x_1]$ and v_2 with $[q, x_2]$ for some $q \notin C$ in (14), we obtain that U satisfies

$$a(b[x_1, x_2]^2 - [x_1, x_2]c[x_1, x_2] - [x_1, x_2][q, [x_1, x_2]])^n = 0 \quad (15)$$

which gives

$$a(b[x_1, x_2]^2 - [x_1, x_2](c+q)[x_1, x_2] + [x_1, x_2]^2 q)^n = 0. \quad (16)$$

By Lemma 2.2, this gives $q \in C$, which is a contradiction. Hence the theorem is proved.

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References

[1] Argac N and De Filippis V, Co-centralizing derivations and nilpotent values on Lie ideals, *Indian J. Pure Appl. Math.* **41**(**3**) (2010) 475–483

[2] Beidar K I, Martindale III W S and Mikhalev A V, Rings with generalized identities, Pure and Applied Math. 196 (New York: Marcel Dekker) (1996)

[3] Bergen J, Herstein I N and Kerr J W, Lie ideals and derivations of prime rings, *J. Algebra* **71** (1981) 259–267

[4] Brešar M, Centralizing mappings and derivations in prime rings, *J. Algebra* **156** (1993) 385–394

[5] Carini L, De Filippis V and Dhara B, Annihilators on co-commutators with generalized derivations on Lie ideals, *Publ. Math. Debrecen* **76/4** (2010) 395–409

[6] Carini L and De Filippis V, Commutators with power central values on a Lie ideal, *Pacific J. Math.* **193**(**2**) (2000) 269–278
[7] Chuang C L, GPI's having coefficients in Utumi quotient rings, *Proc. Amer. Math. Soc.* **103**(**3**) (1988) 723–728
[8] De Filippis V, Power cocentralizing generalized derivations on prime rings, *Proc. Indian Acad. Sci. (Math. Sci.)* **120**(**3**) (2010) 285–297
[9] De Filippis V, Generalized derivations and commutators with nilpotent values on Lie ideals, *Tamsui Oxf. J. Math. Sc.* **22**(**2**) (2006) 167–175
[10] De Filippis V, Lie ideals and annihilator conditions on power values of commutators with derivation, *Indian J. Pure Appl. Math.* **32** (2001) 649–656
[11] Erickson T S, Martindale III W S and Osborn J M, Prime nonassociative algebras, *Pacific J. Math.* **60** (1975) 49–63
[12] Herstein I N, Center-like elements in prime rings, *J. Algebra* **60** (1979) 567–574
[13] Herstein I N, Topics in ring theory, (Chicago, IL: Univ. of Chicago Press) (1969)
[14] Jacobson N, Structure of rings, Amer. Math. Soc. Colloq. Pub., 37, Amer. Math. Soc., Providence, RI (1964)
[15] Kharchenko V K, Differential identity of prime rings, *Algebra and Logic.* **17** (1978) 155–168
[16] Lanski C, Differential identities, Lie ideals, and Posner's theorems, *Pacific J. Math.* **134**(**2**) (1988) 275–297
[17] Lanski C and Montgomery S, Lie structure of prime rings of characteristic 2, *Pacific J. Math.* **42**(**1**) (1972) 117–136
[18] Lee T K, Generalized derivations of left faithful rings, *Comm. Algebra* **27**(**8**) (1999) 4057–4073
[19] Lee T K, Semiprime rings with differential identities, *Bull. Inst. Math. Acad. Sinica* **20**(**1**) (1992) 27–38
[20] Lee P H, Lie ideals of prime rings with derivations, *Bull. Inst. Math. Acad. Sinica* **11** (1983) 75–80
[21] Lee P H and Wong T L, Derivations cocentralizing Lie ideals, *Bull. Inst. Math. Acad. Sinica* **23** (1995) 1–5
[22] Martindale III W S, Prime rings satisfying a generalized polynomial identity, *J. Algebra* **12** (1969) 576–584
[23] Posner E C, Derivations in prime rings, *Proc. Amer. Math. Soc.* **8** (1957) 1093–1100
[24] Wang Y, Annihilator conditions with derivations in prime rings of characteristic 2, *Indian J. Pure Appl. Math.* **39**(**6**) (2008) 459–465
[25] Wang Y and You H, A note on commutators with power central values on Lie ideals, *Acta Math. Sinica, English Series* **22**(**6**) (2006) 1715–1720