

Nonsmoothable involutions on spin 4-manifolds

CHANGTAO XUE and XIMIN LIU*

School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, China

*Department of Mathematics, South China University of Technology, Guangzhou 510641, China

E-mail: xuechangtao123@163.com, ximinliu@scut.edu.cn

MS received 19 March 2010; revised 4 October 2010

Abstract. Let X be a closed, simply-connected, smooth, spin 4-manifold whose intersection form is isomorphic to $n(-E_8) \oplus mH$, where H is the hyperbolic form. In this paper, we prove that for n such that $n \equiv 2 \pmod{4}$, there exists a locally linear pseudofree \mathbb{Z}_2 -action on X which is nonsmoothable with respect to any possible smooth structure on X .

Keywords. Group action; locally linear; involution; nonsmoothable.

1. Introduction

A topological finite group G -action on an n -dimensional manifold X is called locally linear if for any point $x \in X$, there exists a G_x -invariant neighborhood V_x of x such that V_x is homeomorphic to \mathbb{R}^n , and G_x acts on V_x in a linear orthogonal way, where G_x is the isotropy group of x .

It is well-known that every smooth action is locally linear. On the other hand, a locally linear action is not necessarily smooth. In [8], Kwasik and Lawson proved the existence of nonsmoothable actions on certain contractible 4-manifolds mainly by gauge theory, and in some cases of involutions, Rohlin's μ -invariant is used. In recent years, many nonsmoothable group actions on 4-manifolds are constructed by many authors [2, 3, 7, 9, 10, 11]. Almost all of them use gauge theory to prove nonsmoothability except the case of nonsmoothable involution on $K3$ in [11] by Nakamura, where he just uses the G -spin theorem to prove nonsmoothability.

In this paper we restrict our attention to involutions on a class of spin 4-manifolds and prove the following theorem.

Theorem 1.1. *Let X be a closed, simply-connected, smooth, spin 4-manifold whose intersection form is isomorphic to $n(-E_8) \oplus mH$, where H is the hyperbolic form. If $n \equiv 2 \pmod{4}$, then there exists a locally linear pseudofree \mathbb{Z}_2 -action on X which is nonsmoothable with respect to any possible smooth structure on X .*

A group action is said to be pseudofree if each nontrivial group element has a discrete fixed point set.

The proof of Theorem 1.1 is divided into two steps. In the first step, we give a constraint on smooth involutions. In the second step, we construct a locally linear action which would violate the constraint.

To obtain a constraint on smooth involutions, we do not use gauge theory, but only use Rohlin's theorem. On the other hand, to construct a locally linear action, we use the realization theorem due to Edmonds and Ewing [4]. In fact, the method of the proof of Theorem 1.1 is essentially the same as Nakamura's method for the construction of a non-smoothable involution on $K3$ in [11]. While Nakamura uses the G -spin theorem instead of Rohlin's theorem, these two methods can be seen as essentially the same by recalling the proof of Rohlin's theorem by the index theorem. By Freedman's theorem [5], the homeomorphism type of a simply-connected 4-manifold is uniquely determined by its intersection form if the intersection form is even. Thus, our result can be applied to many spin 4-manifolds including certain elliptic surfaces. Let $X = E(n)$ be the relatively minimal elliptic surface with rational base. Then the intersection form of X is isomorphic to $n(-E_8) \oplus (2n - 1)H$. So from Theorem 1.1, we can get the following corollary immediately.

COROLLARY 1.2

Let $X = E(n)$ be the relatively minimal elliptic surface with rational base. If $n \equiv 2 \pmod{4}$, then there exists a locally linear pseudofree \mathbb{Z}_2 -action on X which is nonsmoothable with respect to any possible smooth structure on X .

In [7], Kiyono proved that if X is a closed, simply-connected, spin topological 4-manifold not homeomorphic to either S^4 or $S^2 \times S^2$, then for any sufficiently large prime number p , there exists a homologically trivial, pseudofree, locally linear action of \mathbb{Z}_p on X which is nonsmoothable. So it would be interesting to compare our result with Kiyono's result, since we have constructed nonsmoothable \mathbb{Z}_p -actions of the minimal order $p = 2$ on a large class of spin 4-manifolds.

The paper is organized as follows. In §2, we provide some preliminaries. In particular, a constraint on smooth involutions is given. In §3, we prove our main theorem (Theorem 1.1).

2. Preliminaries

In this section, a constraint on smooth involutions and some method of constructing locally linear \mathbb{Z}_2 -actions are given. Note that this section largely depends on Nakamura's paper [11], so we refer the reader to the Nakamura's excellent exposition [11] for more details.

2.1 Construction of locally linear \mathbb{Z}_2 -actions

To construct locally linear \mathbb{Z}_2 -actions, we use the following special case of the realization theorem by Edmonds and Ewing [4].

Theorem 2.1. [4, 11]. *Suppose we are given a \mathbb{Z}_2 -invariant bilinear unimodular even form $\Psi : V \times V \rightarrow \mathbb{Z}$ which satisfies the following:*

- (1) *As a $\mathbb{Z}[\mathbb{Z}_2]$ -module, $V \cong T \oplus F$, where T is a trivial $\mathbb{Z}[\mathbb{Z}_2]$ -module with rank $_{\mathbb{Z}}T = n$, and F is a free $\mathbb{Z}[\mathbb{Z}_2]$ -module.*
- (2) *For any $v \in V$, $\Psi(v, gv) \equiv 0 \pmod{2}$, where g is the generator of \mathbb{Z}_2 .*
- (3) *The G-signature formula is satisfied, i.e., $\sigma(g, (V, \Psi)) = 0$.*

Then, there exists a locally linear \mathbb{Z}_2 -action on a simply-connected 4-manifold X such that its intersection form is Ψ , and the number of fixed points is $n + 2$.

Since the form Ψ is assumed even, the homeomorphism type of X is unique by Freedman's theorem [5].

For our application, we also need their equivariant handle construction.

Let B_0 be a unit ball in \mathbb{C}^2 , on which \mathbb{Z}_2 acts by multiplication of ± 1 . Take a \mathbb{Z}_2 -invariant knot K in $S_0 = \partial B_0$. Then a framing r of K can be represented by an equivariant embedding $f_r : S^1 \times D^2 \rightarrow S_0$ for some \mathbb{Z}_2 action on $S^1 \times D^2$ given by $g(z, w) = (-z, (-1)^{r-1}w)$. So for a given K and a framing r , a 4-manifold $W = B_0 \cup_{f_r} D^2 \times D^2$ with a \mathbb{Z}_2 -action can be constructed.

Let H_1, \dots, H_n be copies of $D^2 \times D^2$ on which \mathbb{Z}_2 acts by $g(z, w) = (-z, -w)$. If a knot K with an even framing r is given, then we can attach H_i to B_0 equivariantly via f_r .

We will choose a \mathbb{Z}_2 -invariant n -component framed link L in ∂B_0 to represent Ψ as follows. Under the assumption of Theorem 2.1, we may assume that $\Psi|_T$ is represented by a matrix (a_{ij}) such that a_{ii} is even and a_{ij} is odd whenever $i \neq j$. Actually, we can take an n -component link L_T in $S_0 = \partial B_0$ representing the matrix (a_{ij}) such that each component of L_T is \mathbb{Z}_2 -invariant. And it is not difficult to realize the other part of Ψ by a link, and therefore we obtain a framed link L in S_0 which realizes the given \mathbb{Z}_2 -invariant form Ψ . Attach H_1, \dots, H_n and free 2-handles to B_0 along L equivariantly. Thus we obtain a 4-manifold X_0 on which \mathbb{Z}_2 acts smoothly, i.e.

$$X_0 = B_0 \cup H_1 \cup \dots \cup H_n \cup (\text{free handles}).$$

The boundary of X_0 is an integral homology 3-sphere Σ with a free \mathbb{Z}_2 -action. Edmonds and Ewing proved that there exists a contractible 4-manifold Z with a locally linear \mathbb{Z}_2 -action such that its boundary is Σ with the given free \mathbb{Z}_2 -action, and it has exactly one fixed point. Then we obtain the required manifold $X = X_0 \cup Z$ with the required action.

Note that each of the components B_0, H_1, \dots, H_n, Z has one fixed point, denoted by P, Q_1, \dots, Q_n, P' . The action constructed above is smooth on X_0 , and is smooth on X except near the final fixed point.

2.2 Atiyah–Bott's criterion for $\varepsilon(P)$

For a smooth even-type (pseudofree equivalently) \mathbb{Z}_2 -action on a simply-connected smooth spin 4-manifold X preserves the unique spin structure and also the spin^c -structure c_0 which is determined by the spin structure. The sign assignment ε is introduced by Atiyah and Bott in [1]. They defined the spin-number to be the Lefschetz number of the corresponding Dirac complex. The spin-number has the form $\text{ind}_g D = \sum_{P \in X^{\mathbb{Z}_2}} \nu(P)$,

where g is the generator of the \mathbb{Z}_2 -action. The sign of each summand $\nu(P)$ depends on P and g , that is the assignment $\varepsilon(P)$. In other words, the sign assignment determined by the lift of the \mathbb{Z}_2 -action to c_0 is $\varepsilon : X^{\mathbb{Z}_2} \rightarrow \pm 1$. By the G -spin theorem [1], we have

$$\text{ind}_g D = k_+ - k_- = \frac{1}{4} \sum_{P \in X^{\mathbb{Z}_2}} \varepsilon(P),$$

$$\text{ind} D = k_+ + k_- = -\frac{1}{8} \sigma(X),$$

where k_+ and k_- are coefficients of the \mathbb{Z}_2 -index of the Dirac operator. Note that k_+ and k_- are even because of the quaternionic structure of Dirac index. Then, we can see that the sum $\sum_{P \in X^{\mathbb{Z}_2}} \varepsilon(P)$ is a multiple of 8 by solving the above equations. Suppose $g \in \mathbb{Z}_2$ is a nontrivial element. Then we can lift the smooth pseudofree involution $g : X \rightarrow X$ to the frame bundle F as $g_* : F \rightarrow F$ if a g -invariant metric is fixed. A spin structure on X is given by a double cover $\varphi : \hat{F} \rightarrow F$, where \hat{F} is a $\text{Spin}(4)$ -bundle. Suppose that g lifts to \hat{F} . The values $\varepsilon(P)$ and $\varepsilon(Q)$ of distinct fixed points P and Q can be compared by Atiyah–Bott’s criterion to the following proposition. We take a path s in F starting from a point $y \in F_P$ and ending at $y' \in F_Q$. Then the path $-g_*s$ has the same starting point and the end point as s , where ‘ $-$ ’ means the multiplication by -1 on each fiber. Thus by connecting s and $-g_*s$, we obtain a circle C in F .

PROPOSITION 2.2 [1, 11]

The preimage $\varphi^{-1}(C)$ has two components if and only if $\varepsilon(P) = \varepsilon(Q)$. In other words, the preimage $\varphi^{-1}(C)$ is connected if and only if $\varepsilon(P) = -\varepsilon(Q)$.

Recall that each component of B_0 and H_i of X_0 constructed in §§2.1 has a fixed point, denoted by P and Q_i . Suppose H_i is attached to B_0 equivariantly along a knot K with a framing r . Then we have the following proposition.

PROPOSITION 2.3 [11]

Suppose K is a trivial knot in ∂B_0 which bounds a \mathbb{Z}_2 -invariant embedded disk D_0 in B_0 containing P . If $r \equiv 2 \pmod{4}$, then $\varepsilon(P) = \varepsilon(Q)$. If $r \equiv 0 \pmod{4}$, then $\varepsilon(P) = -\varepsilon(Q)$.

Let X be an oriented topological manifold and let $G = \mathbb{Z}_2$. The sign assignment can be defined for locally linear actions by using Atiyah–Bott’s criterion itself on topological spin structure $\varphi : \hat{F} \rightarrow F$, and it depends only on equivalence classes of orientation-preserving locally linear G -actions on X . See [11] for more details. On the fiber of F over each fixed point P , there is a point y_P which is mapped to $-y_P$ by the \mathbb{Z}_2 -action. For distinct fixed points P and Q , by taking a path s connecting such a y_P with y_Q , we can define the sign assignment.

DEFINITION 2.4 [11]

For each pair (P, Q) of fixed points, let s be a path in F as above, and C the circle formed by s and $-g_*s$. Define $\varepsilon'(P, Q)$ by $\varepsilon'(P, Q) = 1$, if $\varphi^{-1}(C)$ has 2 components; $\varepsilon'(P, Q) = -1$, if $\varphi^{-1}(C)$ is connected.

Note that this definition does not depend on smooth structures, and is well-defined if X is simply-connected. Furthermore, if the action is realized by a smooth action, then

$$\varepsilon'(P, Q) = \varepsilon(P)\varepsilon(Q). \quad (2.1)$$

2.3 Constraint on smooth involutions

We give a constraint on smooth involutions by considering Rohlin's theorem. Rohlin's theorem gives a criteria for the question that which simply-connected topological manifolds carry smooth structures.

Theorem 2.5 [5]. *If X is a smooth, closed, spin 4-manifold, then the signature $\sigma(X)$ of X satisfies $\sigma(X) \equiv 0 \pmod{16}$.*

Now let X be a smooth, closed, oriented, simply-connected spin 4-manifold, and suppose that \mathbb{Z}_2 acts on X smoothly and pseudofreely in an orientation-preserving way. By [1], the fixed-point set $X^{\mathbb{Z}_2}$ is discrete if and only if the \mathbb{Z}_2 -action lifts to the spin structure. Therefore X/\mathbb{Z}_2 is a spin V -manifold. The quotient singularities are cones of $\mathbb{R}P^3$. It is well known that the spin structures on $\mathbb{R}P^3$ can be divided into two equivalent classes s_{\pm} which are characterized as follows: let \tilde{s}_{\pm} be the unique spin structure on the disk bundle D_{\pm} over S^2 of degree ± 2 , then $s_{\pm} = \tilde{s}_{\pm}|_{\partial D_{\pm}}$. Define the spin type of a fixed point by the spin structure on $\mathbb{R}P^3$ induced from X/\mathbb{Z}_2 . The number of fixed points corresponding to s_{\pm} is denoted by n_{\pm} . We know that the quotient space X/\mathbb{Z}_2 is not a smooth 4-manifold. In order to use Rohlin's theorem, we need to make X/\mathbb{Z}_2 to be smooth. Since $\mathbb{R}P^3$ has two equivalent classes of spin structures, we need to make the spin structures compatible. Remove cones of $\mathbb{R}P^3$ from X/\mathbb{Z}_2 , and glue disk bundles D_+ and D_- so that spin structures are compatible. We get a smooth spin 4-manifold. Then, applying Rohlin's theorem, we have the following formula:

$$\sigma(X/\mathbb{Z}_2) \equiv n_+ - n_- \pmod{16}.$$

Together with the G -signature theorem [1], we have

$$\frac{1}{2}\sigma(X) \equiv n_+ - n_- \pmod{16}.$$

So for a smooth \mathbb{Z}_2 -action on X as above, we have

$$\begin{cases} \sharp X^{\mathbb{Z}_2} = n_+ + n_-, \\ \frac{1}{2}\sigma(X) \equiv n_+ - n_- \pmod{16}. \end{cases} \quad (2.2)$$

There is a relation between the spin types and the sign assignments of two fixed points.

PROPOSITION 2.6

Let P and Q be distinct fixed points. Then, $\varepsilon(P) = \varepsilon(Q)$ iff P and Q have the same spin type; $\varepsilon(P) = -\varepsilon(Q)$ iff P and Q have different spin types.

Proof. The quantity ε was introduced in order to see the lifting way of the \mathbb{Z}_2 -action around each fixed point. Actually, there are two lifting ways. We will explain it in the following.

Let us consider a local model around a fixed point. Let D be a unit 4-ball in \mathbb{R}^4 , and let \mathbb{Z}_2 act on D by $x \mapsto -x$. The frame bundle of D can be assumed as a trivial bundle $F = X \times \mathrm{SO}(4)$. The induced \mathbb{Z}_2 -action on F is given by $(x, v) \mapsto (-x, -v)$ for $(x, v) \in X \times \mathrm{SO}(4)$. Recall that $\mathrm{Spin}(4) = \mathrm{Sp}(1) \times \mathrm{Sp}(1)$, and there is a canonical homomorphism $\phi_0 : \mathrm{Spin}(4) \rightarrow \mathrm{SO}(4)$. Let $\hat{F} = X \times \mathrm{Spin}(4)$. Then the unique spin structure on F is given by $\phi_0 : \hat{F} \rightarrow F$ defined by $\phi = \mathrm{id}_X \times \phi_0$. We can see that there are two ways of lifting the \mathbb{Z}_2 -action to \hat{F} . They are given by involutions ρ_1 and ρ_2 as follows:

$$\rho_1 : (x, q_+, q_-) \mapsto (-x, -q_+, q_-),$$

$$\rho_2 : (x, q_+, q_-) \mapsto (-x, q_+, -q_-).$$

On $S^3 = \partial D$, the \mathbb{Z}_2 -action is the anti-podal map. Then $\hat{F}_1 \triangleq \hat{F}/\rho_1$ and $\hat{F}_2 \triangleq \hat{F}/\rho_2$ give two spin structures on $S^3/\mathbb{Z}_2 = \mathbb{R}P^3$ since $H^1(\mathbb{R}P^3; \mathbb{Z}_2) = \mathbb{Z}_2$, $\mathbb{R}P^3$ has two distinct isomorphism classes of spin structures. By construction, the difference of \hat{F}_1 and \hat{F}_2 is given by a nontrivial \mathbb{Z}_2 -bundle on $\mathbb{R}P^3$, which gives a nonzero class in $H^1(\mathbb{R}P^3; \mathbb{Z}_2) = \mathbb{Z}_2$. So, the isomorphism classes of \hat{F}_1 and \hat{F}_2 are different.

Recall that the singularities of our \mathbb{Z}_2 -actions are cones of $\mathbb{R}P^3$. Now, consider the definitions of the spin types and the sign assignments of fixed points together with the analysis above. The result follows.

For the fixed points obtained in §2.1, we can compare their spin types by the following proposition. The proposition is obtained by considering Propositions 2.3 and 2.6 simultaneously.

PROPOSITION 2.7

Suppose K_i bounds a \mathbb{Z}_2 -invariant embedded disk in B_0 , and r_i is the framing of K_i . Then, $r_i \equiv 2 \pmod{4}$ if and only if P and Q_i have the same spin types; $r_i \equiv 0 \pmod{4}$ if and only if P and Q_i have different spin types.

We obtained a constraint on smooth involutions as eq. (2.2). By the following proposition, we can see that nonsmoothability of the \mathbb{Z}_2 -action in Theorem 1.1 does not depend on smooth structures.

PROPOSITION 2.8

The numbers n_+ and n_- do not depend on smooth structures, i.e. they are invariants of locally linear pseudofree involutions on topological spin 4-manifolds.

Proof. We know that Definition 2.4 extends the sign assignment ε to the case of locally linear \mathbb{Z}_2 -actions on a simply-connected topological 4-manifold X with isolated fixed points. And this definition does not depend on smooth structures. Now, by the relation (2.1) and Proposition 2.6, we have the result.

3. Proof of Theorem 1.1

Let X be a closed, simply-connected, smooth, spin 4-manifold which has the intersection form isomorphic to $n(-E_8) \oplus mH$, n being a positive integer satisfying $n \equiv 2 \pmod{4}$. Therefore $m \geq n+1$ by Furuta's inequality [6]. Suppose an orientation-preserving smooth pseudofree \mathbb{Z}_2 -action on X is given.

Since $\sigma(X) = -8n$, so by (2.2), $-4n \equiv n_+ - n_- \pmod{16}$. Therefore, if $n_+ = n_-$, then $n \equiv 0 \pmod{4}$.

Now we can construct a locally linear \mathbb{Z}_2 -action on X following the method in §2. By Theorem 2.1, if we fix an appropriate \mathbb{Z}_2 -action on the intersection form, then we have a locally linear \mathbb{Z}_2 -action on X .

Define \mathbb{Z}_2 -action on $\Psi = n(-E_8) \oplus mH$ as follows. Let us select a positive integer r less than m such that $r \equiv m \pmod{2}$. Let \mathbb{Z}_2 act on a $\frac{m-r}{2}H \oplus \frac{m-r}{2}H$ summand by permutation of two $\frac{m-r}{2}H$'s. Similarly, let \mathbb{Z}_2 act on a $n(-E_8) = \frac{n}{2}(-E_8) \oplus \frac{n}{2}(-E_8)$ summand by permutation of two $\frac{n}{2}(-E_8)$'s, and on the rest rH trivially. The trivial part is denoted by T .

Now consider the matrix

$$A = \begin{pmatrix} 0 & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 1 & \cdots & 0 & 1 & \cdots & 1 \\ 1 & \cdots & 1 & 2 & \cdots & 1 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & \cdots & 2 \end{pmatrix},$$

which is a $2r \times 2r$ matrix such that the first r diagonal entries are 0, and the rest r diagonal entries are 2, and all off-diagonal entries are 1.

We claim that the symmetric form represented by A is isomorphic to rH . It is clear that the form is even and indefinite. By diagonalizing the matrix A , we can see that the numbers of $+1$ and -1 on the diagonal are equal. The determinant of A is $(-1)^r$. So the claim follows. Hence, we may assume that $\Psi|_T$ is represented by the matrix A . Furthermore, matrix A can be realized by a link whose every component bounds a \mathbb{Z}_2 -invariant embedded disk as follows. Let $p : S^3 \rightarrow S^2$ be the Hopf fibration. Then the inverse image of distinct $2r$ points in S^2 by p forms a required link. As in §2, by the equivariant handle construction, we can construct a smooth action on a manifold X_0 with boundary, and by Theorem 2.1, this action can be extended to the whole X as a locally linear action.

By now we can construct a pseudofree, locally linear \mathbb{Z}_2 -action on X which satisfies

$$\sharp X^{\mathbb{Z}_2} = 2r + 2.$$

Denote the fixed points by $P, Q_1, \dots, Q_{2r}, P'$ as in §2. Recall that the \mathbb{Z}_2 -action we have constructed is smooth except near P' . By Proposition 2.7, a half number of fixed points in $\bigcup_{1 \leq i \leq 2r} Q_i$ have the same spin type with P , and the rest fixed points in $\bigcup_{1 \leq i \leq 2r} Q_i$ have different spin type with P . If the \mathbb{Z}_2 -action can be smooth on X , by the fact that $\sum_{P \in X^{\mathbb{Z}_2}} \varepsilon(P)$ is a multiple of 8 and Proposition 2.3, we have $\sum_{P \in X^{\mathbb{Z}_2}} \varepsilon(P) = 0$. Thus, P and P' have different spin types. By Proposition 2.6, we can see that

$$(n_+, n_-) = (r + 1, r + 1).$$

But for X whose intersection form is isomorphic to $n(-E_8) \oplus mH$ with $n \equiv 2 \pmod{4}$, we can not have $n_+ = n_-$ by the relation (2.2). Thus Theorem 1.1 is proved.

Remark 3.1. Note that for the positive integer n such that $n \equiv 0 \pmod{4}$, by the consideration in §§2.3, we have

$$n_+ - n_- \equiv 0 \pmod{16}.$$

The pairs (n_+, n_-) with $n_+ = n_-$ satisfy it, so we can not conclude the nonsmoothability as the case of $n \equiv 2 \pmod{4}$. Whether there exists a smooth involution on a closed, simply-connected, smooth, spin 4-manifold which has the intersection form isomorphic to $n(-E_8) \oplus mH$ with $n \equiv 0 \pmod{4}$ and $n_+ = n_-$ would be an interesting question. But we cannot construct such an example now.

Acknowledgements

The authors would like to thank the referee for many valuable suggestions, especially for pointing out that the main theorem of this paper can be extended to a larger class of spin 4-manifolds. It is also a pleasure to thank N Nakamura for his help, in particular, for explaining how to prove Proposition 2.6. This work is supported by NSFC (Nos 10771023 and 10931005).

References

- [1] Atiyah M F and Bott R, A Lefschetz fixed point formula for elliptic complexes II applications, *Ann. Math.* **88** (1968) 451–491
- [2] Bryan J, Seiberg-Witten theory and $\mathbb{Z}/2^p$ actions on spin 4-manifolds, *Math. Res. Lett.* **5** (1998) 165–183
- [3] Chen W and Kwasik S, Symmetries and exotic smooth structures on a $K3$ surface, *J. Topology* **1(4)** (2008) 923–962
- [4] Edmonds A L and Ewing J H, Realizing forms and fixed point data in dimension four, *Am. J. Math.* **114** (1992) 1103–1126
- [5] Freedman M H and Quinn F, The topology of 4-manifolds, Princeton Mathematical Series, 39 (Princeton: Princeton University Press) (1990)
- [6] Furuta M, Monopole equation and $\frac{11}{8}$ -conjecture, *Math. Res. Lett.* **8** (2001) 279–291
- [7] Kiyono K, Nonsmoothable group actions on spin 4-manifolds, arXiv:0809.0119v3
- [8] Kwasik S and Lawson T, Nonsmoothable \mathbb{Z}_p -actions on contractible 4-manifolds, *J. Reine Angew. Math.* **437** (1993) 29–54
- [9] Liu X and Nakamura N, Pseudofree $\mathbb{Z}/3$ -action on $K3$ surfaces, *Proc. Am. Math. Soc.* **135(3)** (2007) 903–910
- [10] Liu X and Nakamura N, Nonsmoothable group actions on elliptic surfaces, *Topol. Appl.* **155** (2008) 946–964
- [11] Nakamura N, Bauer-Furuta invariants under \mathbb{Z}_2 -actions, *Math. Z.* **262** (2009) 219–233