Proc. Indian Acad. Sci. (Math. Sci.) Vol. 120, No. 2, April 2010, pp. 209–215.

Equivalence relations of AF-algebra extensions

CHANGGUO WEI

School of Mathematical Sciences, Ocean University of China,
Qingdao 266071, People's Republic of China
E-mail: weicgqd@yahoo.com.cn

MS received 19 March 2009; revised 26 June 2009

Abstract. In this paper, we consider equivalence relations of C^*-algebra extensions and describe the relationship between the isomorphism equivalence and the unitary equivalence. We also show that a certain group homomorphism is the obstruction for these equivalence relations to be the same.

Keywords. AF-algebra; extension; isomorphism.

1. Introduction

Brown, Douglas and Fillmore [3] gave the famous BDF theory to study essentially normal operators on an infinite dimensional Hilbert space and extensions of C^*-algebra $C(X)$ by $\mathcal{K}$ in the 1970's, where X is a compact metric space and $\mathcal{K}$ is the C*-algebra of compact operators on an infinite dimensional separable Hilbert space H. The theory of extensions of C*-algebras has developed rapidly since then, and has become an important tool for classifications of C*-algebras together with K-theory and index theory (see [2]).

The available classification results of C^*-algebra extensions focus on studying the usual unitary, weak and stable unitary Ext groups (i.e. $\mathrm{Ext}_s(A, B)$, $\mathrm{Ext}_w(A, B)$ and $\mathrm{Ext}(A, B)$), see for example [4, 8, 12, 13]. These Ext groups classify C^*-algebra extensions up to unitary, weak and stable unitary equivalence respectively, but they provide very little information about the isomorphisms of these extensions.

On the other hand, the classification of amenable C^*-algebras originated from G A Elliott's work on AF-algebras in 1976. He classified A$\mathbb{T}$-algebras of real rank zero in 1993 [5]. Since then a number of classification results appeared [6, 11], etc.

Extension algebras form an important class of C^*-algebras. There are many classification results of such C^*-algebras ([9, 10, 14], etc). Unlike the classification of C^*-algebra extensions, Ext groups do not classify extension algebras. So one has to study the equivalence relation induced by isomorphisms.

In this paper, we use homological approach to consider equivalence relations of C^*-algebra extensions and describe the relationship between the isomorphism equivalence and the unitary equivalence. Using these results and the classification theorems of AF-algebra extensions (see [4] and [8]), we show a necessary and sufficient condition for the isomorphism equivalence and the unitary equivalence to be the same.

2. Preliminaries

Let A and B be C^*-algebras. Recall that an extension of A by B is a short exact sequence $0 \to B \overset{\alpha}{\to} E \overset{\beta}{\to} A \to 0$. Denote this extension by e or (E, α, β) and the set of all such extensions by $E(A, B)$.

The extension (E, α, β) is called trivial if the above sequence splits, i.e. if there is a homomorphism $\gamma\colon A \to E$ such that $\beta \circ \gamma = \mathrm{id}_A$. We call (E, α, β) essential, if $\alpha(A)$ is an essential ideal in E.

Let $0 \to B \overset{\alpha}{\to} E \overset{\beta}{\to} A \to 0$ be an extension of A by B. Then there is a unique homomorphism $\sigma\colon E \to M(B)$ such that $\sigma \circ \alpha = \iota$, where $M(B)$ is the multiplier algebra of B, and ι is the inclusion map from B to $M(B)$. It is known that σ is injective if and only if the extension is essential.

The Busby invariant of (E, α, β) is a homomorphism τ from A to the corona algebra $\mathcal{Q}(B) = M(B)/B$ defined by $\tau(a) = \pi(\sigma(b))$ for $a \in A$, where $\pi\colon M(B) \to \mathcal{Q}(B)$ is the quotient map, and $b \in E$ such that $\beta(b) = a$. Therefore the Busby invariant of (E, α, β) is the unique homomorphism making the following diagram commute

$$\begin{tikzcd}
0 \arrow[r] & B \arrow[r, "\alpha"] \arrow[d, equal] & E \arrow[r, "\beta"] \arrow[d, "\sigma"] & A \arrow[r] \arrow[d, "\tau"] & 0 \\
0 \arrow[r] & B \arrow[r] & M(B) \arrow[r, "\pi"] & \mathcal{Q}(B) \arrow[r] & 0.
\end{tikzcd}$$

If A is unital and the Busby invariant is unital, then (E, α, β) is called unital.

Let $e_i\colon 0 \to B \overset{\alpha_i}{\to} E_i \overset{\beta_i}{\to} A \to 0$ be two extensions of A by B with Busby invariants τ_i for $i = 1,\ 2$. Then (E_1, α_1, β_1) and (E_2, α_2, β_2) are called congruent (called 'strongly isomorphic' in [2]), denoted by $e_1 \equiv e_2$, if there exists an isomorphism η such that the following diagram is commutative

$$\begin{tikzcd}
0 \arrow[r] & B \arrow[r, "\alpha_1"] \arrow[d, equal] & E_1 \arrow[r, "\beta_1"] \arrow[d, "\eta"] & A \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & B \arrow[r, "\alpha_2"] & E_2 \arrow[r, "\beta_2"] & A \arrow[r] & 0.
\end{tikzcd}$$

Denote by $\mathcal{E}\mathrm{xt}(A, B)$ the set of congruent equivalence classes of extensions of A by B. Hence there is a one-to-one correspondence between congruent equivalence classes of extensions of A by B and homomorphisms from A to $\mathcal{Q}(B)$ [2].

(E_1, α_1, β_1) and (E_2, α_2, β_2) are called isomorphic (called 'weakly isomorphic' in [2]), denoted by $e_1 \cong e_2$, if there exist isomorphisms β, η, α such that the following diagram is commutative:

$$\begin{tikzcd}
0 \arrow[r] & B \arrow[r, "\alpha_1"] \arrow[d, "\beta"] & E_1 \arrow[r, "\beta_1"] \arrow[d, "\eta"] & A \arrow[r] \arrow[d, "\alpha"] & 0 \\
0 \arrow[r] & B \arrow[r, "\alpha_2"] & E_2 \arrow[r, "\beta_2"] & A \arrow[r] & 0.
\end{tikzcd}$$

(E_1, α_1, β_1) and (E_2, α_2, β_2) are called unitarily equivalent, denoted by $e_1 \sim e_2$, if there exists a unitary $u \in M(B)$ such that $\tau_2(a) = \pi(u)\tau_1(a)\pi(u)^*$ for all $a \in A$. Denote by $\mathrm{Ext}(A, B)$ the set of unitary equivalence classes of extensions of A by B.

Let H be a separable infinite dimensional Hilbert space, and $\mathcal{K}$ be the ideal of compact operators in $B(H)$. If B is a stable C^*-algebra (i.e. $B \otimes \mathcal{K} \cong B$), then the sum of two extensions τ_1 and τ_2 is defined to be the homomorphism $\tau_1 \oplus \tau_2$, where $\tau_1 \oplus \tau_2$: $A \to \mathcal{Q}(B) \oplus \mathcal{Q}(B) \subseteq M_2(\mathcal{Q}(B)) \cong \mathcal{Q}(B)$.

Let $\operatorname{Ext}(A, B)$ be the set of stable strong equivalence classes (see [2] for the definition). If A is a separable nuclear C^*-algebra, then $\operatorname{Ext}(A, B)$ is an abelian group by the theorem of Arveson in [1]. But $\operatorname{Ext}(A, B)$ is an abelian semigroup in general.

Let $e \in E(A, B)$, $\alpha \in \operatorname{Aut}(A)$ and $\beta \in \operatorname{Aut}(B)$. Then there are two induced extensions βe and $e\alpha$ (see [15] for details).

Let A be an AF-algebra. Then there is a dimension group $(K_0(A), D(A))$ of A (see [8] for the definition of dimension groups).

3. Main results

Lemma 3.1 [7, 15]. *Let e_1, e_2 be two extensions of A by B, and let $\alpha \in \operatorname{Aut}(A)$ and $\beta \in \operatorname{Aut}(B)$. Then the following are equivalent*:

(1) $e_1 \equiv e_2$;
(2) $e_1\alpha \equiv e_2\alpha$;
(3) $\beta e_1 \equiv \beta e_2$.

The above lemma implies that α^* and β_* are isomorphisms of $\mathcal{E}\mathrm{xt}(A, B)$. We also need the following basic facts about the equivalence relations in $E(A, B)$.

PROPOSITION 3.2

Let $e_1, e_2 \in E(A, B)$. Then $e_1 \sim e_2$ if and only if there is a unitary $u \in M(B)$ such that $e_2 \equiv \operatorname{Ad}(u)e_1$, where $\operatorname{Ad}(u)e_1$ is the extension induced by $\operatorname{Ad}(u)$.

PROPOSITION 3.3

Let $e_1, e_2 \in E(A, B)$, $\alpha \in \operatorname{Aut}(A)$ and $\beta \in \operatorname{Aut}(B)$. Then the following are equivalent:

(1) $e_1 \sim e_2$;
(2) $e_1\alpha \sim e_2\alpha$;
(3) $\beta e_1 \sim \beta e_2$.

PROPOSITION 3.4

Let e_i: $0 \to B \to E_i \xrightarrow{\psi_i} A \to 0$ be essential extensions with Busby invariants τ_i. Let σ_i be the inclusion maps from E_i to $M(B)$. Then the following statements are equivalent:

(1) $e_1 \cong e_2$.
(2) *There is an isomorphism ϕ: $M(B) \to M(B)$ such that $\phi(B) = B$ and $\psi \circ \tau_1(A) = \tau_2(A)$, where ψ: $\mathcal{Q} \to \mathcal{Q}$ is the isomorphism induced by ϕ.*

Proof.

(1) $\Longrightarrow$ (2). Let (α, β, γ): $e_1 \to e_2$ be an extension isomorphism and let e_0 denote the extension $0 \to B \to M(B) \to \mathcal{Q} \to 0$. Then we have the following commutative diagram:

$$\begin{array}{rcccccccc}
e_1\colon 0 & \longrightarrow & B & \longrightarrow & E_1 & \longrightarrow & A & \longrightarrow & 0\\
 & & \big\downarrow{\scriptstyle \alpha} & & \big\downarrow{\scriptstyle \beta} & & \big\downarrow{\scriptstyle \gamma} & & \\
e_2\colon 0 & \longrightarrow & B & \longrightarrow & E_2 & \longrightarrow & A & \longrightarrow & 0\\
 & & \big\downarrow{\scriptstyle \mathrm{id}} & & \big\downarrow{\scriptstyle \sigma_2} & & \big\downarrow{\scriptstyle \tau_2} & & \\
e_0\colon 0 & \longrightarrow & B & \longrightarrow & M(B) & \longrightarrow & Q & \longrightarrow & 0\\
 & & \big\downarrow{\scriptstyle \alpha^{-1}} & & \big\downarrow{\scriptstyle \phi^{-1}} & & \big\downarrow{\scriptstyle \psi^{-1}} & & \\
e_0\colon 0 & \longrightarrow & B & \longrightarrow & M(B) & \longrightarrow & Q & \longrightarrow & 0,
\end{array}$$

where $\phi\colon M(B) \to M(B)$ is the unique extension of $\alpha\colon B \to B$. So it follows that there is a homomorphism $(\mathrm{id}, \phi^{-1} \circ \sigma_2 \circ \beta, \psi^{-1} \circ \tau_2 \circ \gamma)$ from e_1 to e_0. Hence $\phi^{-1} \circ \sigma_2 \circ \beta = \sigma_1$ and $\psi^{-1} \circ \tau_2 \circ \gamma = \tau_1$. Then we have $\phi \circ \sigma_1 = \sigma_2 \circ \beta$ and $\tau_2 \circ \gamma = \psi \circ \tau_1$. Therefore, $\phi(B) = B$ and $\psi \circ \tau_1(A) = \tau_2(A)$.
(2) $\Longrightarrow$ (1). Since $\psi \circ \pi = \pi \circ \phi$, we have

$$\psi \circ \tau_1(A) = \psi \circ \pi(E_1) = \pi \circ \phi(E_1).$$

Since $B \subset E_i$, $\pi^{-1}(\psi \circ \tau_1(A)) = \phi(E_1)$. By $\psi \circ \tau_1(A) = \tau_2(A)$, we have $\pi^{-1}(\tau_2(A)) = \phi(E_1)$, that is, $\phi(E_1) = E_2$. Let $\alpha = \phi|_B$, $\beta = \sigma_2^{-1} \circ \phi \circ \sigma_1$ and $\gamma = \tau_2^{-1} \circ \psi \circ \tau_1$. Then $(\alpha, \beta, \gamma)\colon e_1 \to e_2$ is an extension isomorphism. □

PROPOSITION 3.5

Suppose A and B are two AF*-algebras. Let $e \in E(A, B)$, $\alpha_1, \alpha_2 \in$* Aut(A) *and $\beta_1, \beta_2 \in$* Aut(B)*. Then*

(1) $e\alpha_1 \sim e\alpha_2 \iff K_0(e)_* K_0(\alpha_1) \equiv K_0(e)_* K_0(\alpha_2)$, *where $K_0(e)_*$:* $\mathrm{Hom}(K_0(A), K_0(A)) \to \mathrm{Ext}(K_0(A), K_0(B))$ *is the homomorphism induced by $K_0(e)$.*
(2) $\beta_1 e \sim \beta_2 e \iff K_0(\beta_1)K_0(e) \equiv K_0(\beta_2)K_0(e)$, *where $K_0(\beta_i)K_0(e)$ is the extension induced by $K_0(\beta_i)$.*

Proof. We only need to show (1).

(1) If $e\alpha_1 \sim e\alpha_2$, then we have $e \sim e\alpha_2{\alpha_1}^{-1}$ by Proposition 3.3. So there is an isomorphism $\phi\colon E_2 \to E_1$ such that the following diagram is commutative:

$$\begin{array}{rcccccccc}
e\alpha_2{\alpha_1}^{-1}\colon 0 & \longrightarrow & B & \longrightarrow & E_2 & \longrightarrow & A & \longrightarrow & 0\\
 & & \big\downarrow{\scriptstyle \mathrm{id}} & & \big\downarrow{\scriptstyle \phi} & & \big\downarrow{\scriptstyle \alpha_2{\alpha_1}^{-1}} & & \\
e\colon 0 & \longrightarrow & B & \longrightarrow & E_1 & \longrightarrow & A & \longrightarrow & 0.
\end{array}$$

Applying the functor K_0 to the above diagram, we have a commutative diagram at K-theory level in the following:

$$\begin{array}{rcccccccc}
K_0(e\alpha_2\alpha_1^{-1})\colon 0 & \longrightarrow & K_0(B) & \longrightarrow & K_0(E_2) & \longrightarrow & K_0(A) & \longrightarrow & 0\\
 & & \big\downarrow{\scriptstyle \mathrm{id}} & & \big\downarrow{\scriptstyle K_0(\phi)} & & \big\downarrow{\scriptstyle K_0(\alpha_2{\alpha_1}^{-1})} & & \\
K_0(e)\colon 0 & \longrightarrow & K_0(B) & \longrightarrow & K_0(E_1) & \longrightarrow & K_0(A) & \longrightarrow & 0.
\end{array}$$

Hence

$$K_0(\alpha_2\alpha_1{}^{-1})^*(K_0(e)) = K_0(e\alpha_2\alpha_1^{-1}).$$

It follows from [8] or [4] that $K_0(e) \equiv K_0(e\alpha_2\alpha_1^{-1})$. Therefore $K_0(e)K_0(\alpha_2\alpha_1^{-1}) \equiv K_0(e)$, and then $K_0(e)K_0(\alpha_1) \equiv K_0(e)K_0(\alpha_2)$. Hence

$$(K_0(e))_*K_0(\alpha_1) \equiv (K_0(e))_*K_0(\alpha_2).$$

Conversely, if $K_0(e)_*K_0(\alpha_1) \equiv K_0(e)_*K_0(\alpha_2)$, then $K_0(e) \equiv K_0(e\alpha_2\alpha_1^{-1})$. So we have $e \sim e\alpha_2\alpha_1^{-1}$ by Lemma I.5 of [8]. Then by Proposition 3.3, we have $e\alpha_1 \sim e\alpha_2$. □

Theorem 3.6. *Let* $e_1, e_2 \in E(A, B)$, $\alpha \in \mathrm{Aut}(A)$, $\beta \in \mathrm{Aut}(B)$ *and* $\eta \in \mathrm{Hom}(E_1, E_2)$. *Then*

(1) $e_1 \overset{(\beta,\eta,\alpha)}{\cong} e_2 \iff e_1 \equiv \beta^{-1}e_2\alpha$.

(2) *If* β *is inner, then* $e_1 \overset{(\beta,\eta,\alpha)}{\cong} e_2 \iff e_1 \sim e_2\alpha$.

Proof.

(1) Suppose $(\beta, \eta, \alpha)\colon e_1 \to e_2$ is an isomorphism of extensions, i.e. there is a commutative diagram:

$$\begin{array}{ccccccccc}
0 & \longrightarrow & B & \xrightarrow{i_1} & E_1 & \xrightarrow{\psi_1} & A & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle \beta} & & \downarrow{\scriptstyle \eta} & & \downarrow{\scriptstyle \alpha} & & \\
0 & \longrightarrow & B & \xrightarrow{i_2} & E_2 & \xrightarrow{\psi_2} & A & \longrightarrow & 0.
\end{array}$$

Let e' be the extension $0 \to B \xrightarrow{j} E_2 \xrightarrow{\psi} A \to 0$, where $j = i_2 \circ \beta$ and $\psi = \alpha_1^{-1} \circ \psi_2$. Then there is a commutative diagram:

$$\begin{array}{ccccccccc}
0 & \longrightarrow & B & \xrightarrow{i_1} & E_1 & \xrightarrow{\psi_1} & A & \longrightarrow & 0 \\
 & & \downarrow{\scriptstyle \mathrm{id}} & & \downarrow{\scriptstyle \eta} & & \downarrow{\scriptstyle \mathrm{id}} & & \\
0 & \longrightarrow & B & \xrightarrow{j} & E_2 & \xrightarrow{\psi} & A & \longrightarrow & 0.
\end{array}$$

Hence $e_1 \equiv e'$. Since $e' \equiv \beta^{-1}e_2\alpha$, $e_1 \equiv \beta^{-1}e_2\alpha$.

Conversely, let $\alpha \in \mathrm{Aut}(A)$, $\beta \in \mathrm{Aut}(B)$, and $e_1 \equiv \beta^{-1}e_2\alpha$. By Lemma 3.1, we have $\beta e_1 \equiv e_2\alpha$. It follows that there is an isomorphism $\eta\colon E_1 \to E_2$ such that the following diagram is commutative:

$$\begin{array}{cccccccccc}
\beta e_1\colon & 0 & \longrightarrow & B & \xrightarrow{i_1\circ\beta^{-1}} & E_1 & \xrightarrow{\psi_1} & A & \longrightarrow & 0 \\
 & & & \downarrow{\scriptstyle \mathrm{id}} & & \downarrow{\scriptstyle \eta} & & \downarrow{\scriptstyle \mathrm{id}} & & \\
e_2\alpha\colon & 0 & \longrightarrow & B & \xrightarrow{i_2} & E_2 & \xrightarrow{\alpha^{-1}\circ\psi_2} & A & \longrightarrow & 0.
\end{array}$$

Then we have a commutative diagram:

$$
\begin{array}{ccccccccc}
e_1\colon 0 & \longrightarrow & B & \xrightarrow{i_1} & E_1 & \xrightarrow{\psi_1} & A & \longrightarrow & 0 \\
 & & \big\downarrow{\scriptstyle\beta} & & \big\downarrow{\scriptstyle\eta} & & \big\downarrow{\scriptstyle\alpha} & & \\
e_2\colon 0 & \longrightarrow & B & \xrightarrow{i_2} & E_2 & \xrightarrow{\psi_2} & A & \longrightarrow & 0,
\end{array}
$$

i.e. $(\beta, \eta, \alpha)\colon e_1 \to e_2$ is an extension isomorphism.

(2) Let $\beta = \mathrm{Ad}(u)$ and $(\beta, \eta, \alpha)\colon e_1 \to e_2$ be an isomorphism, where u is a unitary in $M(B)$. It follows from (1) that

$$e_1 \equiv \beta^{-1} e_2 \alpha \equiv \mathrm{Ad}(u^*) e_2 \alpha.$$

By Proposition 3.2, we have $e_1 \sim e_2\alpha$.

Conversely, if there is a unitary $w \in M(B)$ such that $e_1 = \mathrm{Ad}(w)e_2\alpha$, then by (1) there is an isomorphism $\eta\colon E_1 \to E_2$ such that $(\mathrm{Ad}(w^*), \eta, \alpha)\colon e_1 \to e_2$ is an extension isomorphism. □

Theorem 3.7. *Let $B = \mathcal{K}$. Then the isomorphism equivalence and the unitary equivalence coincide in $E(A, B)$ if and only if $e \sim e\gamma$ for any $e \in E(A, B)$ and $\gamma \in \mathrm{Aut}(A)$.*

Proof.
($\Longrightarrow$). For every $e \in E(A, B)$ and $\gamma \in \mathrm{Aut}(A)$ we have $e \cong e\gamma$. It follows that $e \sim e\gamma$ from the assumption immediately.
($\Longleftarrow$). Let $e_1,\ e_2 \in E(A, B)$ and $e_1 \sim e_2$. Then it is easy to see that $e_1 \cong e_2$. Conversely, suppose there is an isomorphism $(\alpha, \beta, \gamma)\colon e_1 \to e_2$. Since $\alpha \in \mathrm{Aut}(\mathcal{K})$, α is inner. By Theorem 3.6 we have $e_1 \sim e_2\gamma$. The assumption implies $e_2\gamma \sim e_2$, so we have $e_1 \sim e_2$. Hence, the two equivalence relations are equivalent in $E(A, B)$. □

Theorem 3.8. *Let $B = \mathcal{K}$ and A be an* AF*-algebra. Then the isomorphism equivalence and the unitary equivalence coincide in $E(A, B)$ if and only if the group homomorphism $\gamma \longmapsto K_0(\gamma)^*$ from* $\mathrm{Aut}(A)$ *to* $\mathrm{Aut}(\mathrm{Ext}(K_0(A), D(A); K_0(B), D(B)))$ *is trivial.*

Proof.
($\Longleftarrow$). For any $e \in E(A, B)$ and $\gamma \in \mathrm{Aut}(A)$, let $(\mathrm{id}, \eta, \gamma)\colon e\gamma \to e$ be an extension isomorphism. Then there is a commutative diagram in K-theory:

$$
\begin{array}{ccccccccc}
K_0(e\gamma)\colon 0 & \longrightarrow & K_0(B) & \longrightarrow & K_0(E_2) & \longrightarrow & K_0(A) & \longrightarrow & 0 \\
 & & \big\downarrow{\scriptstyle \mathrm{id}} & & \big\downarrow{\scriptstyle K_0(\eta)} & & \big\downarrow{\scriptstyle K_0(\gamma)} & & \\
K_0(e)\colon 0 & \longrightarrow & K_0(B) & \longrightarrow & K_0(E_1) & \longrightarrow & K_0(A) & \longrightarrow & 0.
\end{array}
$$

Hence, we have

$$K_0(e\gamma) = K_0(e)K_0(\gamma) = K_0(\gamma)^*(K_0(e)).$$

From the assumption it follows that $K_0(\gamma)^* = \mathrm{id}$. Hence $K_0(e\gamma) \equiv K_0(e)$. This implies that the two extensions $K_0(e\gamma)$ and $K_0(e)$ are congruent as dimension groups with interval, and then $e\gamma \sim e$ by [8]. It follows that the two equivalence relations coincide in $E(A, B)$ by Theorem 3.7.

($\Longrightarrow$). Let $\gamma \in \mathrm{Aut}(A)$ and e an extension of A by B. Then $e \sim e\gamma$ since $e\gamma \overset{(\mathrm{id},\eta,\gamma)}{\cong} e$. It follows that there is an isomorphism of dimension groups $(\mathrm{id}, K_0(\eta), K_0(\gamma))$ from $K_0(e\gamma)$ to $K_0(e)$. Therefore, $K_0(e\gamma) = K_0(\gamma)^*(K_0(e))$. Since $e \sim e\gamma$, $K_0(e\gamma) \equiv K_0(e)$. Hence $K_0(\gamma)^* = \mathrm{id}$. So the group homomorphism $\gamma \longmapsto K_0(\gamma)^*$ is trivial. $\square$

COROLLARY 3.9

Let A be an AF-*algebra. If any automorphism of A is inner*, *then the isomorphism equivalence and the unitary equivalence coincide in* $E(A, \mathcal{K})$.

Proof. Let $\gamma \in \mathrm{Aut}(A)$. Since γ is inner, it follows that $K_0(\gamma)$ is the unit of $\mathrm{Aut}(K_0(A), D(A))$. Hence $K_0(\gamma)^*$ is the unit of $\mathrm{Aut}(\mathrm{Ext}(K_0(A), D(A); K_0(B), D(B)))$. The corollary now follows from Theorem 3.8. $\square$

Acknowledgements

The author would like to thank the referee for his/her helpful suggestions.

References

[1] Arveson W, Notes on extensions of C^*-algebras, *Duke Math. J.* **44(2)** (1977) 329–355
[2] Blackadar B, K-theory for operator algebras, Second edition, Mathematical Sciences Research Institute Publications, 5 (Cambridge: Cambridge University Press) (1998)
[3] Brown L G, Douglas R G and Fillmore P A, Extensions of C^*-algebras and K-homology, *Ann. Math. (2)* **105(2)** (1977) 265–324
[4] Brown L G and Elliott G A, Extensions of AF-algebras are determined by K_0, *C. R. Math. Rep. Acad. Sci. Canada.* **4** (1982) 15–19
[5] Elliott G A, On the classification of C^*-algebras of real rank zero, *J. Reine Angew. Math.* **443** (1993) 179–219
[6] Elliott G A and Gong Guihua, On the classification of C^*-algebras of real rank zero, II, *Ann. Math. (2)* **144(3)** (1996) 497–610
[7] Fuchs L, Infinite abelian groups, vol. I (New York and London: Academic Press) (1970)
[8] Handelman D, Extensions for AF C^*-algebras and dimension groups, *Trans. AMS* **27(2)** (1982) 537–573
[9] Lin Huaxin, On the classification of C^*-algebras of real rank zero with zero K_1, *J. Operator Theory* **35(1)** (1996) 147–178
[10] Lin Huaxin, A classification theorem for infinite Toeplitz algebras, Operator algebras and operator theory (Shanghai, 1997) pp. 219–275, Contemp. Math. 228, Amer. Math. Soc., Providence, RI (1998)
[11] Lin Huaxin, Classification of simple C^*-algebras of tracial topological rank zero, *Duke Math. J.* **125(1)** (2004) 91–119
[12] Lin Huaxin, Unitary equivalence for essential extensions of C^*-algebras, arXiv: math. OA/0403236 v1 15 Mar. 2004
[13] Lin Huaxin, Extensions by C^*-algebras of real rank zero III, *Proc. London Math. Soc. (3)* **76(3)** (1998) 634–666
[14] Lin Huaxin and Su Hongbing, Classification of direct limits of generalized Toeplitz algebras, *Pacific J. Math.* **181(1)** (1997) 89–140
[15] MacLane S, Homology (Berlin. FRG: Springer-Verlag) (1963)