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The general class of Bianchi cosmological models with dark energy and variable Λ and *G* in viscous cosmology

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Abstract. The general class of Bianchi cosmological models with dark energy in the form of modified Chaplygin gas with variable Λ and G and bulk viscosity have been considered. We discuss three types of average scale factor by using a special law for deceleration parameter which is linear in time with negative slope. The exact solutions to the corresponding field equations are obtained. We obtain the solution of bulk viscosity (ξ), cosmological constant (Λ), gravitational parameter (G) and deceleration parameter (q) for different equations of state. The model describes an accelerating Universe for large value of time t, wherein the effective negative pressure induced by Chaplygin gas and bulk viscous pressure are driving the acceleration.

Keywords. Dark energy; bulk viscosity; cosmological constant; cosmological parameter.

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1. Introduction

Recent observations of the luminosity of type-Ia supernovae [1,2] indicate an accelerated expansion of the Universe and lead to the search for a new type of matter which violates the strong energy condition, i.e. $\rho + 3p < 0$ is satisfied. The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the Universe is referred to as dark energy. There are a number of candidates for dark energy [3–5]. The type of dark energy represented by a scalar field is often called quintessence [6,7]. The simplest candidate of dark energy is the cosmological constant. [8–10]. There are other candidates such as phantom field (a scalar field with a negative sign of the kinetic term) [11–13], a quintom (a combination of quintessence and phantom) [14–20] etc. Cosmological models including Chaplygin gas are usually used for unification of dark matter and dark energy. As we know, Chaplygin gas behaves as dark matter at the early Universe while it behaves as a cosmological constant at the late time. Chaplygin gas [20,21] is one of the candidates of the dark energy models to explain the accelerated expansion of the Universe. The Chaplygin gas obeys an equation of state $p = -A_1/\rho$ [21,22], where p and ρ are respectively the pressure and energy density and A_1 is a positive constant. Subsequently, the above equation of state was modified to the form $p = -A_1/\rho^\alpha$ with $0 < \alpha \leq 1$. This model gives cosmological evolution from initial dust-like matter to an asymptotic cosmological constant and a fluid obeying an equation of state $p = \gamma\rho$. This generalized model has been studied by several researchers [23–25]. The simplest form of Chaplygin gas model, called the standard Chaplygin gas (SCG), was used to explain the accelerated expansion of the Universe [26]. The SCG has been extended to the generalized Chaplygin gas (GCG) [27–29]. Subsequently, the GCG is also extended to the modified Chaplygin gas (MCG) [30–34], which can show a radiation era in the early Universe. Also, the dissipative effects in GCG model using the framework of the non-causal Eckart theory [35] have been studied. Zhai *et al* [36] have investigated the viscous GCG model by assuming that there is bulk viscosity in the linear barotropic fluid and GCG.

Among all the possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored in the vacuum state of all the existing fields in the Universe, i.e.

$$\rho_v = \frac{\Lambda}{8\pi G},$$

where Λ is the cosmological constant. However, the constant Λ cannot explain the huge difference between the cosmological constant inferred from observation and the vacuum energy density resulting from quantum field theories. In an attempt to solve this problem, variable Λ was induced such that Λ was large in the early Universe and then decayed with evolution. Variation of Newton's gravitational parameter G was originally suggested by Dirac [37] on the basis of his large numbers hypothesis (LNH). It seems reasonable to consider $G = G(t)$ in an evolving Universe when one considers $\Lambda = \Lambda(t)$. Many extensions of general relativity with $G = G(t)$ have been made ever since Dirac first considered the possibility of a variable G. Sattar and Vishwakarma [38] have suggested the conservation of energy–momentum tensor which consequently renders G and Λ as coupled fields. This leaves Einstein's field equations formally unchanged. Bonanno and Reuter [39] have considered the scaling of $G(t)$ and $\Lambda(t)$ arising from an underlying renormalization group flow near an infrared attractive fixed point. The resulting cosmology [40] explains the high redshift SNe Ia and radiosource observations successfully. Gravitational theories with variable G have been discussed by Zee [41], Smolin [42] and Alder [43] in the context of induced gravity model where G is generated by means of a non-vanishing vacuum expectation value of a scalar field. Recently, a constraint on the variation of G has been obtained by using WMAP and the big-bang nucleosynthesis observations by Copi *et al* [44], which comes out to be $-3 \times 10^{-13}\ \mathrm{yr}^{-1} < (\dot{G}/G)_{\mathrm{today}} < 4 \times 10^{-13}\ \mathrm{yr}^{-1}$.

The present day Universe is homogeneous and isotropic on large scales, which is defined by FRW models. However, the latest observational data of the CMB by WMAP satellite show hints of anomalies that the isotropy seems broken in cosmological data [45]. Large-angle anomalies in the CMB can provide a very important role to understand the very early Universe and the effects of the early Universe on the present day large-scale structure. According to the theories proposed by Misner [46] and Gibbons and Hawking [47], anisotropy at the early stage of the Universe turns into an isotropic present Universe and initial anisotropies die away.

Several researchers [48–50] have suggested that anisotropic Bianchi Universes can play important roles in observational cosmology (see also [46,51–54]). The WMAP data [55–57] seem to require, in addition to the standard cosmological model, a positive cosmological constant that bears a resemblance to the Bianchi morphology [58–60]. According to this, the Universe should have a slightly anisotropic spatial geometry in spite of the inflation, contrary to generic inflationary models [61–65].

Singh and Chaubey [66] have studied the evolution of a homogeneous anisotropic Universe filled with viscous fluid, in the presence of cosmological constant Λ. Pradhan *et al* [67–69] have discussed various viable cosmologies for homogeneous and anisotropic cosmological model. Singh and Chaubey [70] also studied the evolution of a homogeneous anisotropic Universe with varying Λ, G and shear (σ^2) simultaneously. Recently, Chaubey [71,72] has studied the modified Chaplygin gas and generalized gas in the background of Bianchi type-I space–time. Fayaz *et al* [73] have studied the dark energy and viscous fluid cosmology with variable G and Λ in an anisotropic space–time by considering constant deceleration parameter (Berman law). Khurshundyan *et al* [74] have studied three models of $f(R)$ modified gravity including higher-order terms based on different equation of state parameters in the presence of variable G and Λ. In order to obtain a comprehensive model, we also add two modifications to the ordinary model. First, we consider a fluid which obeys the varying equation of state (EoS) and second, we consider time-varying Λ and G. The variation of G and Λ leads to the modification of Einstein's field equations and the conservation laws [74–76]. This is because, if we allow G and Λ to be variables in Einstein's equations, the energy conservation law is violated. Therefore, the study of varying G and Λ can be done through modified field equations and modified conservation law [75,76]. In this paper, we have considered the dark energy and viscous fluid cosmology with variable G and Λ in Bianchi type-III, V, VI_0 and VI_h space–times with a variable deceleration parameter, which is the generalization of Berman law.

The present paper is organized as follows. In §1, a brief introduction is given. Section 2 deals with the basic equations of cosmological model. Cosmological parameters are also defined in this section. In §3, we have obtained cosmological solutions of our model for modified Chaplygin gas in three different subcases. In each case, we have obtained the cosmological parameters, density and pressure for $t \to 0$ and $t \to \infty$. The paper ends with a conclusion given in §4.

2. Model and basic equations

A gravitational action with Ricci scalar curvature R containing a variable gravitational constant $G(t)$ and cosmological constant $\Lambda(t)$ is given by

$$I = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi G(t)}(R - 2\Lambda(t)) + L_m\right], \tag{1}$$

where g is the determinant of the four-dimensional tensor metric g^{ij} and L_m represents the matter Lagrangian.

The simplest models for a uniformly expanding Universe are the FRW models. The main justification of these models was their mathematical simplicity and tractability [77]. Theoretical arguments and possible indications from recent experimental data support the existence of an isotropic phase. The WMAP data have indicated that the Universe was not isotropic at early times. It has been also demonstrated [78,79] that the Universe is not isotropic for all time. The Bianchi models must be considered which are models with less symmetry than standard FRW model. Such models should be examined to include the effects of shear and anisotropy in the early Universe.

The diagonal form of the metric of general class of Bianchi cosmological model is given by

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\mathrm{d}x^2 - a_2^2\mathrm{e}^{-2x}\mathrm{d}y^2 - a_3^2\mathrm{e}^{-2mx}\mathrm{d}z^2. \tag{2}$$

We have the additional classes of Bianchi models as follows: type-III corresponds to $m = 0$, type-V corresponds to $m = 1$, type-VI$_0$ corresponds to $m = -1$, and all other m give VI$_h$, where $m = h - 1$.

Scale factors a_1, a_2 and a_3 are the functions of cosmic time t. These scale factors are in three anisotropic directions.

A very interesting generalization of Einstein's theory of gravitation was proposed by Lau [80] with time-dependent cosmological and gravitational parameters which is consistent with Dirac's large number hypothesis (LNH) [81]. The field equations of this theory are

$$R_{ij} - \frac{1}{2}Rg_{ij} - \Lambda(t)g_{ij} = -8\pi G(t)T_{ij}, \tag{3}$$

where the cosmological parameter Λ and the gravitational parameter G are functions of time. Other symbols have their usual meaning. By appealing to Dirac cosmology, Lau found specific forms for Λ and G. Other generalized theories with variable Λ and constant G include those proposed by Ozer and Taha [82–85]. Der Sarkissian [86] has presented some new cosmological models based on the field eq. (3) with variables Λ and G as functions of time. He has claimed that energy conservation cannot occur unless both Λ and G are constants. But this claim is not entirely correct [87].

The divergence of eq. (3) leads to

$$\Lambda_{,j}\, g^{ij} = G_{,j}\, T^{ij} + GT^{ij}{}_{;j}\,. \tag{4}$$

It can be seen from eq. (3) that energy conservation is possible when

$$\Lambda_{,j}\, g^{ij} = G_{,j}\, T^{ij}. \tag{5}$$

For an example of this type of cosmological model, please refer Dirac [81]. Thus, contrary to the claim of Beesham [87], it is possible to have energy conservation even though both Λ and G vary with time.

It is assumed that the matter is a perfect fluid with bulk viscosity and dark energy. Then the energy–momentum tensor is

$$T_{ij} = \rho u_i u_j + \tilde{p}h_{ij}. \tag{6}$$

Here ρ is the total energy density of dark energy and a perfect fluid and $\tilde{p}$ is the corresponding total pressure. The projection tensor is defined as $h_{ij} = g_{ij} + u_i u_j$ and u^i is the flow vector satisfying $u_i u^j = 1$.

We first define the expressions for the average scale factor and volume scale factor. We define the generalized Hubble parameter H in analogy with a flat FRW model.

The average scale factor a and spatial volume V of the general class of Bianchi cosmological model eq. (2) are defined by

$$V = a^3 = a_1 a_2 a_3. \tag{7}$$

We define the generalized Hubbles parameter H in terms of spatial volume and scale factor as

$$\begin{aligned} H = \frac{1}{3}\frac{\dot{V}}{V} &= \frac{1}{3}(H_1 + H_2 + H_3) \\ &= \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right), \end{aligned} \tag{8}$$

where $H_1 = \dot{a}_1/a_1$, $H_2 = \dot{a}_2/a_2$ and $H_3 = \dot{a}_3/a_3$ are the directional Hubble's parameters. The overhead dot denotes differentiation with respect to cosmic time t.

From eqs (2) and (5), the field eq. (3) leads to

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{m^2+m+1}{a_1^2} = 8\pi G\rho + \Lambda, \tag{9}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{m}{a_1^2} = -8\pi G\tilde{p} + \Lambda, \tag{10}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \frac{m^2}{a_1^2} = -8\pi G\tilde{p} + \Lambda, \tag{11}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{1}{a_1^2}=-8\pi G\tilde{p}+\Lambda, \tag{12}$$

$$(m+1)\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}-m\frac{\dot{a}_3}{a_3}=0. \tag{13}$$

Let us introduce the dynamical scalars such as expansion parameter (θ), shear scalar (σ^2) and the mean anisotropy parameter (A) as

$$\theta = u^i{}_{;i} = 3H, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{15}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{16}$$

where $\Delta H_i = H_i - H,\ i = 1, 2, 3.$

From eqs (9)–(13), we obtain

$$3H^2 = 8\pi G\rho + \sigma^2 + \Lambda + \frac{m^2+m+1}{a_1^2} \tag{17}$$

$$H^2(2q-1)=8\pi G\tilde{p}-\Lambda+\sigma^2-\frac{m^2+m+1}{3a_1^2} \tag{18}$$

$$\dot{\sigma}+3H\sigma-\frac{(m^2+m+1)H}{\sigma a_1^2}=0. \tag{19}$$

The total pressure $\tilde{p}$ is related to equilibrium pressure p by

$$\tilde{p} = p - \xi\theta, \tag{20}$$

where ξ is the coefficient of viscosity. In most of the investigations involving bulk viscosity, it is assumed that bulk viscosity is a power function of the energy density [88–90] given by

$$\xi = \xi_0\rho^r, \quad \xi_0 \geq 0, \tag{21}$$

where ξ_0 and r are constants. The covariant conservation equation is given by

$$\dot{\rho} + \theta(\rho + \tilde{p}) = -\rho\frac{\dot{G}}{G} - \frac{\dot{\Lambda}}{8\pi G}. \tag{22}$$

We assume conservation of matter, viz.

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{23}$$

From eqs (5), (20), (22), (23), we have

$$8\pi\rho\dot{G} + \dot{\Lambda} = 8\pi G\xi\theta^2. \tag{24}$$

From eqs (17) and (18), we obtain the deceleration parameter q as

$$q=\frac{1}{2}+\frac{3\left(8\pi G\tilde{p}+\sigma^2-\Lambda-\frac{m^2+m+1}{a_1^2}\right)}{2\left(8\pi G\rho+\sigma^2+\Lambda+\frac{m^2+m+1}{a_1^2}\right)}. \tag{25}$$

From eq. (25), it is observed that the deceleration parameter q is a function of cosmic time t. Here we take the deceleration parameter q as linear in time with a negative slope, proposed by Akarsu and Dereli [91]. This law covers the law of Berman (where the deceleration parameter is constant) used for obtaining exact cosmological models, in the context of dark energy, to account for the current acceleration of the Universe. This new law gives an opportunity to generalize many of these dark energy models having better consistency with the cosmological observations. The linearly varying deceleration parameter q is defined as

$$\begin{aligned} q &= -\frac{a\ddot{a}}{\dot{a}^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) - 1 \\ &= -\frac{\dot{H}}{H^2} - 1 = -kt + n - 1, \end{aligned} \tag{26}$$

where k and n are positive constants. The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign indicates accelerated expansion. For $n > 1 + kt$, $q > 0$. Therefore, the model represents a decelerating Universe whereas for $kt < n \leq 1 + kt$, we get $-1 \leq q < 0$ which describes an accelerating model of the Universe.

Solving eq. (26) for the scale factor, we obtain the law of variation for average scale factor a as

$$a = (nlt + c_1)^{1/n}, \quad k = 0, \quad n > 0, \tag{27}$$

$$a = c_2\mathrm{e}^{lt}, \quad k = 0, \quad n = 0, \tag{28}$$

$$a = c_3\mathrm{e}^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}, \quad k > 0, \quad n > 1, \tag{29}$$

where c_1, c_2 and c_3 are constants of integration. Equation (25) implies that the condition for the expanding Universe is $n = (q + 1 + kt) > 0$.

3. Modified Chaplygin gas model

In this section, we consider the case where the dark energy is represented by a modified Chaplygin gas (MCG). The EoS of the MCG model [92–94] is,

$$p = \gamma\rho - \frac{A_1}{\rho^\alpha}, \quad \text{where } 0 < \alpha < 1. \tag{30}$$

The modified Chaplygin gas EoS corresponds to a mixture of ordinary matter and dark energy. For $\rho = (A_1/\gamma)^{1/(1+\alpha)}$ the content of the matter is dust, i.e. $p = 0$.

It has already been suggested that for $\alpha = 1$, MCG reduces to standard Chaplygin gas (SCG) [95,96].

In SCG model, when the co-moving volume of the Universe is small ($\rho \to \infty$) and $\gamma = 1/3$, this equation of state corresponds to a radiation-dominated era. When density tends to zero, the equation of state corresponds to a cosmological fluid with negative pressure (the dark energy). Chaplygin gas plays a dual role at different epochs of the history of the Universe: at early time it behaves like a dust (i.e., for small scale factor a) and at late times it behaves as a cosmological constant (i.e., for large values of a).

Case 1. When $k = 0, n > 0$ and $a_3 = V^b$, where b is any constant, then from eqs (7), (13) and (27), we get

$$a_1(t) = (nlt + c_1)^{(3+3mb-3b)/n(m+2)} \tag{31}$$

$$a_2(t) = (nlt + c_1)^{(3+3m-3b-6mb)/n(m+2)} \tag{32}$$

$$a_3(t) = (nlt + c_1)^{3b/n}. \tag{33}$$

The directional Hubble parameters H_1, H_2 and H_3 have values

$$H_1 = \left(\frac{3+3mb-3b}{m+2}\right)\frac{l}{nlt+c_1} \tag{34}$$

$$H_2 = \left(\frac{3+3m-3b-6mb}{m+2}\right)\frac{l}{nlt+c_1} \tag{35}$$

$$H_3 = \frac{3bl}{nlt+c_1}. \tag{36}$$

From eq. (8), the average generalized Hubble parameter H has the value

$$H = \frac{l}{nlt+c_1}. \tag{37}$$

From eqs (14), (15) and (16), the dynamical scalars are given by

$$\theta = \frac{3l}{nlt+c_1} \tag{38}$$

$$\sigma^2 = [3+3m-18b-18mb+27mb^2 - 18m^2b + 27m^2b^2 + 27b^2 + 3m^2] \times \frac{l^2}{(m+2)^2(nlt+c_1)^2} \tag{39}$$

$$A = [2+2m-12b-12mb+18mb^2 - 12m^2b + 18m^2b^2 + 18b^2 + 2m^2] \times \frac{1}{(m+2)^2}. \tag{40}$$

Using eqs (27) and (30) into energy conservation eq. (23), we obtain the energy density

$$\rho = \left(\frac{A_1 - (nlt+c_1)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}}{1+\gamma}\right)^{\frac{1}{1+\alpha}}. \tag{41}$$

From eq. (30), the pressure is given by

$$p = \gamma\left(\frac{A_1 - (nlt+c_1)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}}{1+\gamma}\right)^{\frac{1}{1+\alpha}} - \frac{A_1(1+\gamma)^{\frac{1}{1+\alpha}}}{\left(A_1 - (nlt+c_1)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}\right)^{\frac{1}{1+\alpha}}}. \tag{42}$$

From eq. (21), the coefficient of viscosity is

$$\xi = \xi_0\left(\frac{A_1 - (nlt+c_1)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}}{1+\gamma}\right)^{\frac{r}{1+\alpha}}. \tag{43}$$

Using eqs (42), (43) and (38) in eq. (20), the total pressure $\tilde{p}$ is given by

$$\tilde{p} = \frac{(nlt+c_1)\left\{\gamma(1+\gamma)^{\frac{n-1}{1+\alpha}}\left[A_1 - (nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{2}{1+\alpha}} - A_1(1+\gamma)^{\frac{n+1}{1+\alpha}}\right\}}{(nlt+c_1)(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1 - (nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{1}{1+\alpha}}} - \frac{3\xi_0 l\left[A_1 - (nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{n+1}{1+\alpha}}}{(nlt+c_1)(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1 - (nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{1}{1+\alpha}}}. \tag{44}$$

Using eqs (41) and (43) in eqs (17) and (24), we can obtain $G(t)$ and $\Lambda(t)$ respectively as follows:

$$G = \frac{(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1-(nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}\left\{\frac{Xnl^3-3nl^3(m+2)^2}{(m+2)^2(nlt+c_1)}+\frac{3l(m^2+m+1)(1+mb-b)}{(m+2)(nlt+c_1)^{\frac{6(1+mb-b)}{n(m+2)}-1}}\right\}}{12\pi l\left\{(1+\gamma)^{\frac{n+\alpha}{1+\alpha}}(nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)+1}+3\xi_0 l\left[A_1-(nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{n+\alpha}{1+\alpha}}\right\}} \tag{45}$$

and

$$\Lambda = -\frac{2(1+\gamma)^{\frac{n-1}{1+\alpha}}\left[A_1-(nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]\left\{\frac{Xnl^3-3nl^3(m+2)^2}{(m+2)^2(nlt+c_1)}+\frac{3l(m^2+m+1)(1+mb-b)}{(m+2)(nlt+c_1)^{\frac{6(1+mb-b)}{n(m+2)}-1}}\right\}}{3l\left\{(1+\gamma)^{\frac{n+\alpha}{1+\alpha}}(nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)+1}+3\xi_0 l\left[A_1-(nlt+c_1)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{n+\alpha}{1+\alpha}}\right\}}$$
$$+\left\{\frac{l^2(3(m+2)^2-X)}{(m+2)^2(nlt+c_1)^2}-\frac{(m^2+m+1)}{(nlt+c_1)^{\frac{6(1+mb-b)}{n(m+2)}}}\right\}. \tag{46}$$

where

$$X = (3+3m-18b-18mb+27mb^2-18m^2b$$
$$+27m^2b^2+27b^2+3m^2).$$

Here we observe that, the spatial volume V is zero at $t = t_0 = -c_1/nl$. The energy density and pressure are infinite at this epoch. The rate of expansion and the mean anisotropy parameter are infinite as $t \to t_0$. From eqs (36)–(38), the directional Hubble's parameters H_1, H_2, H_3 are infinite at the initial time $t = t_0$. Thus, the Universe starts evolving with zero volume at $t = t_0$ and expands with cosmic time t. From eqs (40) and (41), $\lim_{t\to\infty}(\sigma^2/\theta) = 0$. So the model approaches isotropy for large cosmic time t. From eq. (40), it is observed that when $b = 1/3$ the anisotropy of the present model becomes zero for all values of m and the model becomes free from anisotropy, which is acceptable as a dark energy model. The conditions of homogeneity and isotropization, formulated by Collins and Hawking [77], are satisfied in the present model.

For $A_1 = (nlt + c_1)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}$ we have $|p| \to \infty$. From eqs (41) and (42), one can see that for large value of time,

$$\rho = \left[\frac{A_1}{1+\gamma}\right]^{\frac{1}{1+\alpha}} \quad \text{and} \quad p = -\left[\frac{A_1}{1+\gamma}\right]^{\frac{1}{1+\alpha}},$$

which shows an accelerating Universe. From eq. (45), it is observed that the cosmological term $\Lambda(t)$ tends zero for large cosmic time t. We also note that G tends to be a constant for large value of cosmic time t. Thus, the cosmological constant term is very small today. It is also observed that the energy conditions are satisfied for this model when

$$t \geq t_\star = \frac{1}{nl}\left[\left(\frac{1}{A_1}\right)^{\frac{n}{3(1+\gamma)(1+\alpha)}} - c_1\right].$$

Here $t_\star \sim 10^{-24}$ for suitable choices of constraints.

For suitable choices of constraints ($\alpha = 0.5, \gamma = 1/3$) in figure 1, red curve (with dots) represents the variation of $\dot{G}/G$ with evolution of Universe for $n = 1$, while green curve (with circles) and blue curve (with stars) represent the variation of $\dot{G}/G$ with evolution of the Universe for $n = 2$ and 3 respectively. From figure 1, it is interesting to note that, the value of $\dot{G}/G$ satisfies Viking Landers on Mars data [97] (i.e. $(\dot{G}/G) \leq 6$) and PSR B1913+16 and PSR B1855+09 data [98] (i.e. $(\dot{G}/G) \leq 9$) for large cosmic time t.

Case 2. When $k = 0, n = 0$ and $a_3 = V^b$, where b is any constant, we have

$$a_1(t) = c_2^{\frac{3+3mb-3b}{m+2}}\mathrm{e}^{\frac{(3+3mb-3b)lt}{m+2}} \tag{47}$$

$$a_2(t) = c_2^{\frac{3+3m-3b-6mb}{m+2}}\mathrm{e}^{\frac{(3+3m-3b-6mb)lt}{m+2}} \tag{48}$$

$$a_3(t) = c_2^{3b}\mathrm{e}^{3blt}. \tag{49}$$

The directional Hubble's parameters H_1, H_2 and H_3 have values

$$H_1 = \frac{(3+3mb-3b)l}{m+2} \tag{50}$$

$$H_2 = \frac{(3+3m-3b-6mb)l}{m+2} \tag{51}$$

$$H_3 = 3bl. \tag{52}$$

From eq. (8), the average generalized Hubble's parameter H has the value given by

$$H = l. \tag{53}$$

From eqs (14), (15) and (16), the dynamical scalars are given by

$$\theta = 3l \tag{54}$$

$$\sigma^2 = \frac{3+3m-18b-18mb+27mb^2-18m^2b+27m^2b^2+27b^2+3m^2}{(m+2)^2} l^2 \tag{55}$$

$$A = \frac{2+2m-12b-12mb+18mb^2-12m^2b+18m^2b^2+18b^2+2m^2}{(m+2)^2}. \tag{56}$$

Using eqs (28) and (30) in energy conservation eq. (23), we obtain the energy density ρ as

$$\rho = \left(\frac{A_1 - \exp(-3lt(1+\gamma)(1+\alpha))}{1+\gamma}\right)^{\frac{1}{1+\alpha}}. \tag{57}$$

From eq. (30), the pressure p is given by

$$p = \gamma \left(\frac{A_1 - \exp(-3lt(1+\gamma)(1+\alpha))}{1+\gamma}\right)^{\frac{1}{1+\alpha}} - \frac{A_1(1+\gamma)^{\frac{1}{1+\alpha}}}{(A_1 - \exp(-3lt(1+\gamma)(1+\alpha)))^{\frac{1}{1+\alpha}}}. \tag{58}$$

From eq. (21), the coefficient of viscosity ξ is given by

$$\xi = \xi_0 \left(\frac{A_1 - \exp(-3lt(1+\gamma)(1+\alpha))}{1+\gamma}\right)^{\frac{r}{1+\alpha}}. \tag{59}$$

Using eqs (58), (59) and (54) in eq. (20), we obtain

$$\tilde{p} = \frac{\gamma(1+\gamma)^{\frac{n-1}{1+\alpha}}\left[A_1 - \exp(-3lt(1+\gamma)(1+\alpha))\right]^{\frac{2}{1+\alpha}} - A_1(1+\gamma)^{\frac{n+1}{1+\alpha}}}{(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1 - \exp(-3lt(1+\gamma)(1+\alpha))\right]^{\frac{1}{1+\alpha}}} - \frac{3\xi_0 l\left[A_1 - \exp(-3lt(1+\gamma)(1+\alpha))\right]^{\frac{n+1}{1+\alpha}}}{(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1 - \exp(-3lt(1+\gamma)(1+\alpha))\right]^{\frac{1}{1+\alpha}}}. \tag{60}$$

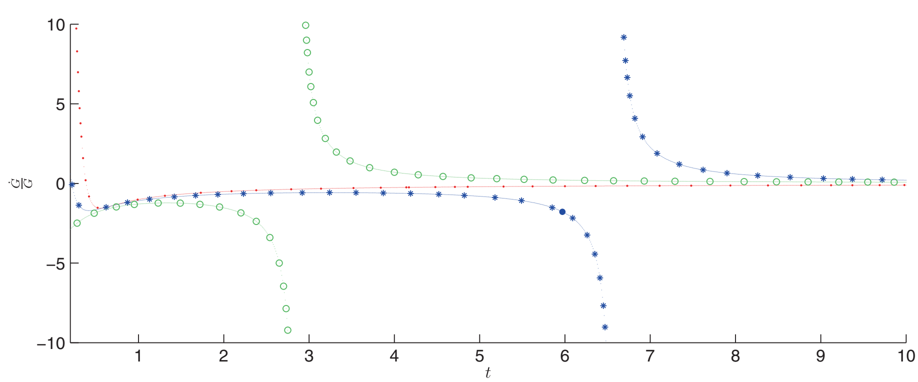

Figure 1. $\dot{G}/G$ vs. t for $n = 1, 2, 3$.

Using eqs (57) and (59) in eqs (17) and (24), we can obtain $G(t)$ and $\Lambda(t)$ respectively as follows:

$$G = \left\{(m^2+m+1)(1+mb-b)(1+\gamma)^{\frac{n}{1+\alpha}}\,(A_1-\exp(-3lt(1+\gamma)(1+\alpha)))^{\frac{\alpha}{1+\alpha}}\right\} \Big/ \left\{4\pi(m+2)c_2^{\frac{6(1+mb-b)}{(m+2)}}\exp\left(\frac{6(1+mb-b)lt}{(m+2)}\right)\left[3\xi_0 l\,\{A_1-\exp(-3lt(1+\gamma)(1+\alpha))\}^{\frac{n+\alpha}{1+\alpha}} + (1+\gamma)^{\frac{n+\alpha}{1+\alpha}}\exp(-3lt(1+\gamma)(1+\alpha))\right]\right\} \tag{61}$$

and

$$\Lambda = \left\{-2(m^2+m+1)(1+mb-b)(1+\gamma)^{\frac{n-1}{1+\alpha}}\,(A_1-\exp(-3lt(1+\gamma)(1+\alpha)))\right\} \Big/ \left\{3(m+2)c_2^{\frac{6(1+mb-b)}{(m+2)}}\exp\left(\frac{6(1+mb-b)lt}{(m+2)}\right)\left[3\xi_0 l\left\{A_1-\exp(-3lt(1+\gamma)(1+\alpha))\right\}^{\frac{n+\alpha}{1+\alpha}} + (1+\gamma)^{\frac{n+\alpha}{1+\alpha}}\exp(-3lt(1+\gamma)(1+\alpha))\right]\right\} + \frac{3(m+2)^2-X}{(m+2)^2}l^2 - \frac{m^2+m+1}{c_2^{\frac{6(1+mb-b)}{(m+2)}}\exp\left(\frac{6(1+mb-b)lt}{(m+2)}\right)}. \tag{62}$$

Here we observe that, the spatial volume V is finite at $t = 0$. The energy density and pressure are infinite at this epoch. The rate of expansion and the mean anisotropy parameter are infinite at $t = 0$. Thus, the Universe starts evolving with finite volume at $t = 0$ and expands with cosmic time t. Collins *et al* [99] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous space satisfies the relation $(\sigma/\theta) =$ constant. From eq. (56), it is observed that when $b = 1/3$ the anisotropy of the present model becomes zero for all values of m and the model becomes free from anisotropy, which is acceptable as a dark energy model. From eqs (52) and (53), it is observed that, Collins condition is satisfied. As $t \to \infty$, the scale factor becomes infinitely large, whereas the shear scalar tends to zero.

For $A_1 = \exp(-3lt(1+\gamma)(1+\alpha))$ we have $|p| \to \infty$. From eqs (57) and (58), we see that for large value of time,

$$\rho = \left[\frac{A_1}{1+\gamma}\right]^{\frac{1}{1+\alpha}} \quad \text{and} \quad p = -\left[\frac{A_1}{1+\gamma}\right]^{\frac{1}{1+\alpha}},$$

which shows an accelerating Universe. From eq. (62), it is observed that the cosmological term $\Lambda(t)$ tends to zero for large cosmic time t. We also note that G tends to a constant for large value of cosmic time t. Thus, the cosmological constant term is very small today. It is also observed that the energy conditions are satisfied for this model when

$$t \geq t_\star = \frac{1}{3l(1+\gamma)(1+\alpha)}\ln\left(\frac{1}{A_1 c_2^{3(1+\gamma)(1+\alpha)}}\right).$$

Here $t_\star \sim 10^{-24}$ for suitable choices of constraints.

For suitable choices of constraints ($n = 1, \gamma = 1/3$) in figure 2, red curve (with dots) represents the variation of $\dot{G}/G$ with evolution of Universe for $\alpha = 0.1$, while blue curve (with circles) and green curve (with stars) represent the variation of $\dot{G}/G$ with evolution of Universe for $\alpha = 0.5$ and 0.9 respectively. From figure 2, it is interesting to note that, the value of $\dot{G}/G$ satisfies Viking Landers on Mars data [97] (i.e. $(\dot{G}/G) \leq 6$) and PSR B1913+16 and PSR B1855+09 data [98] (i.e. $(\dot{G}/G) \leq 9$) for large cosmic time t.

Case 3. When $k > 0, n > 1$ and $a_3 = V^b$, where b is a constant, we have

$$a_1(t) = c_3^{\frac{3+3mb-3b}{m+2}} \mathrm{e}^{\frac{2(3+3mb-3b)}{n(m+2)} \tanh^{-1}(\frac{kt}{n}-1)} \tag{63}$$

$$a_2(t) = c_3^{\frac{3+3m-3b-6mb}{m+2}} \mathrm{e}^{\frac{2(3+3m-3b-6mb)}{n(m+2)} \tanh^{-1}(\frac{kt}{n}-1)} \tag{64}$$

$$a_3(t) = c_3^{3b} \mathrm{e}^{\frac{6b}{n} \tanh^{-1}(\frac{kt}{n}-1)}. \tag{65}$$

The directional Hubble's parameters H_1, H_2 and H_3 have values

$$H_1 = \frac{2(3+3mb-3b)}{t(2n-kt)(m+2)} \tag{66}$$

$$H_2 = \frac{2(3+3m-3b-6mb)}{t(2n-kt)(m+2)} \tag{67}$$

$$H_3 = \frac{6b}{t(2n-kt)}. \tag{68}$$

From eq. (8), the average Hubble's parameter H has the value

$$H = \frac{2}{t(2n-kt)}. \tag{69}$$

From eqs (14), (15) and (16), the dynamical scalars are given by

$$\theta = \frac{6}{t(2n-kt)} \tag{70}$$

$$\sigma^2 = \frac{12(1+m-6b-6mb+9mb^2-6m^2b+9m^2b^2+9b^2+m^2)}{t^2(2n-kt)^2(m+2)^2} \tag{71}$$

$$A = \frac{2(1+m-6b-6mb+9mb^2-6m^2b+9m^2b^2+9b^2+m^2)}{(m+2)^2}. \tag{72}$$

Using eqs (29) and (30) in energy conservation eq. (23), we obtain the energy density ρ as

$$\rho = \left[\frac{A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}}{1+\gamma}\right]^{\frac{1}{1+\alpha}}. \tag{73}$$

From eq. (30), the pressure p is given by

$$p = \gamma\left[\frac{A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}}{1+\gamma}\right]^{\frac{1}{1+\alpha}} - \frac{A_1(1+\gamma)^{\frac{1}{1+\alpha}}}{\left(A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}\right)^{\frac{1}{1+\alpha}}}. \tag{74}$$

From eq. (21), the coefficient of viscosity ξ is given by

$$\xi = \xi_0\left[\frac{A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}}{1+\gamma}\right]^{\frac{r}{1+\alpha}}. \tag{75}$$

Using eqs (74), (75) and (70) in eq. (20), we obtain

$$\tilde{p} = \frac{t(2n-kt)\left\{\gamma(1+\gamma)^{\frac{n-1}{1+\alpha}}\left[A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{2}{1+\alpha}} - A_1(1+\gamma)^{\frac{n+1}{1+\alpha}}\right\}}{t(2n-kt)(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{1}{1+\alpha}}} - \frac{6\xi_0\left[A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{n+1}{1+\alpha}}}{t(2n-kt)(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1 - \left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{1}{1+\alpha}}}. \tag{76}$$

Using eqs (73) and (75) in eqs (17) and (24), we can obtain $G(t)$ and $\Lambda(t)$ respectively as follows:

$$G=\frac{(1+\gamma)^{\frac{n}{1+\alpha}}\left[A_1-\left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{\alpha}{1+\alpha}}\left\{\frac{(12+B)(kt-n)}{t(2n-kt)}+\frac{3(m^2+m+1)(1+mb-b)t(2n-kt)}{(m+2)c_3^{\frac{6(1+mb-b)}{(m+2)}}\mathrm{e}^{\frac{12(1+mb-b)}{n(m+2)}\tanh^{-1}\left(\frac{kt}{n}-1\right)}}\right\}}{12\pi\left\{kt^2(1+\gamma)^{\frac{n+\alpha}{1+\alpha}}\left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)-1}+6\xi_0\left[A_1-\left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{n+\alpha}{1+\alpha}}\right\}} \tag{77}$$

and

$$\Lambda=-\frac{4(1+\gamma)^{\frac{n-1}{1+\alpha}}\left[A_1-\left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]\left\{\frac{(12+B)(kt-n)}{t(2n-kt)}+\frac{3(m^2+m+1)(1+mb-b)t(2n-kt)}{(m+2)c_3^{\frac{6(1+mb-b)}{(m+2)}}\mathrm{e}^{\frac{12(1+mb-b)}{n(m+2)}\tanh^{-1}\left(\frac{kt}{n}-1\right)}}\right\}}{\left\{6kt^2(1+\gamma)^{\frac{n+\alpha}{1+\alpha}}\left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)-1}+36\xi_0\left[A_1-\left(\frac{kt}{2n-kt}\right)^{\frac{-3}{n}(1+\gamma)(1+\alpha)}\right]^{\frac{n+\alpha}{1+\alpha}}\right\}}$$
$$+\left\{\frac{12-B}{t^2(2n-kt)^2}-\frac{(m^2+m+1)}{c_3^{\frac{6(1+mb-b)}{(m+2)}}\mathrm{e}^{\frac{12(1+mb-b)}{n(m+2)}\tanh^{-1}\left(\frac{kt}{n}-1\right)}}\right\}, \tag{78}$$

where

$$B=\frac{12(1+m-6b-6mb+9mb^2-6m^2b+9m^2b^2+9b^2+m^2)}{(m+2)^2}.$$

Here we observe that the spatial volume V is finite at $t=t_0=0$. The energy density and pressure are infinite at this epoch. The rate of expansion and the mean anisotropy parameter are infinite at $t=t_0$. Thus, the Universe starts evolving with finite volume at $t=t_0$ and expands with cosmic time t. From eqs (70) and (71) $\lim_{t\to\infty}\sigma^2/\theta=0$. Thus, the model approaches isotropy for large cosmic time t. From eq. (72), it is observed that when $b=1/3$ the anisotropy of the present model becomes zero for all values of m and the model becomes free from anisotropy, which is acceptable as a dark energy model. The conditions of homogeneity and isotropization, formulated by Collins and Hawking [77], are satisfied in the present model. As $t\to\infty$, the scale factor becomes infinitely large, whereas the shear scalar tends to zero. For

$$A_1=\left(\frac{kt}{2n-kt}\right)^{\frac{-3(1+\gamma)(1+\alpha)}{n}}$$

we have $|p|\to\infty$. From eqs (73) and (74), it is seen that for large value of time,

$$\rho=\left[\frac{A_1}{1+\gamma}\right]^{\frac{1}{1+\alpha}}\quad\text{and}\quad p=-\left[\frac{A_1}{1+\gamma}\right]^{\frac{1}{1+\alpha}},$$

which shows an accelerating Universe. From eq. (78), it is observed that the cosmological term $\Lambda(t)$ tends to

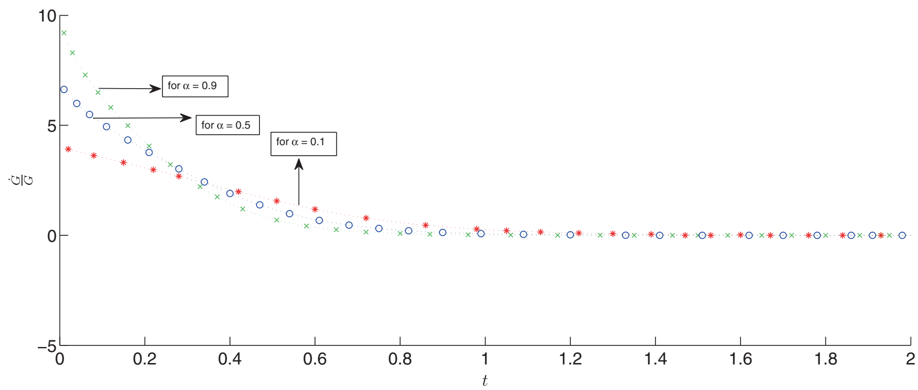

Figure 2. $\dot{G}/G$ vs. t for $n = 1$, $\gamma = 1$ and $\alpha = 0.1, 0.5, 0.9$.

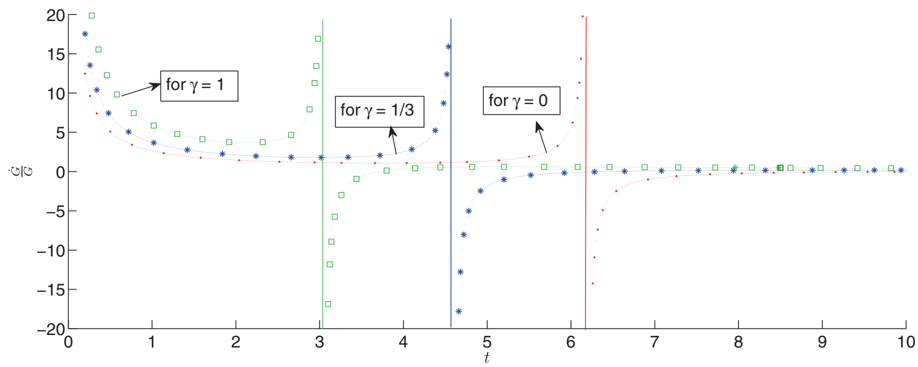

Figure 3. $\dot{G}/G$ vs. t for $n = 1$, $\alpha = 0.5$ and $\gamma = 0, 1/3, 1$.

zero for large cosmic time t. We also note that G tends to be a constant for large values of cosmic time t. Thus, the cosmological constant term is very small today. It is also observed that the energy conditions are satisfied for this model when

$$t \geq t_\star = \frac{n}{k}\left[\tanh\left(\frac{2}{n}\ln(\frac{1}{A_1 c_3^{3(1+\gamma)(1+\alpha)}})\right) + 1\right].$$

Here $t_\star \sim 10^{-24}$ for suitable choices of constraints.

For suitable choices of constraints ($n = 1, \alpha = 0.5$) in figure 3, red curve (with dots) represents the variation of $\dot{G}/G$ with evolution of Universe for $\gamma = 1$, while blue curve (with stars) and green curve (with circles) represent the variation of $\dot{G}/G$ with evolution of Universe for $\gamma = 1/3$ and 1 respectively. From figure 3, it is interesting to note here that, the value of $\dot{G}/G$ satisfies Viking Landers on Mars data [97] (i.e. $(\dot{G}/G) \leq 6$) and PSR B1913+16 and PSR B1855+09 data [98] (i.e. $(\dot{G}/G) \leq 9$) for large cosmic time t.

4. Result and discussion

The evolution of homogeneous and anisotropic cosmological models is studied in the presence of dark energy and bulk viscosity. We have considered dark energy models with bulk viscosity and variable Λ and G for general class of Bianchi cosmological models. The dark energy is represented by modified Chaplygin gas (MCG). To find the solutions, we have taken the deceleration parameter as linear in time with a negative slope. The exact solutions to the corresponding field equations are obtained for all three cases of scale factors in both scenarios of Chaplygin gas. In §3, we have taken an alternative model of dark energy with an exotic equation of state in three different cases of scale factors. It has been shown in all three subcases that the models have good agreement with current features of the Universe. It is also shown that, the model represents a shearing, non-rotating and expanding Universe with a very small finite volume and approaches asymptotically to isotropic model at late time. It is also noted that the cosmological constant is a decreasing function of cosmic time t and it tends to zero for large time t. Thus, the cosmological constant term is very small today. Also during inflation the cosmological term and energy density decrease with time.

5. Conclusions

This paper has dealt with a general class of Bianchi cosmological models with dark energy and bulk viscosity and variable Λ and G, where dark energy is taken in the form of modified Chaplygin gas (MCG). We have used the general class of cosmological models for different values of m as follows: Bianchi type-III, V, VI_0 models correspond to $m = 0$, 1 and -1 and all other values of m give Bianchi type-VI_h. The exact solutions to the corresponding field equations are obtained in quadrature form. Three different cases have been discussed, depending on the nature of relation between the scale factor and the cosmic time t. Here, we observed that viscosity plays the role of an agent driving the present acceleration of the Universe. It is also observed that the cosmological term becomes very small at late time. In all cases, the Universe starts from a non-singular initial state. In each case, the spatial volume, expansion parameter, shear scalar and mean anisotropic parameter tend to zero for large cosmic time t. All the physical parameters have been calculated and discussed for each model. In each case, the cosmological model approaches isotropy for large value of cosmic time t. These models represent a shearing, non-rotating and expanding Universe, which approaches isotropy for large value of t. We have also shown that MCG model corresponds to an accelerated Universe.

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