Pramana – J. Phys. (2017) 88: 30
DOI 10.1007/s12043-016-1338-z

Dynamics of Bianchi type-VI$_0$ holographic dark energy models in general relativity and Lyra's geometry

S D KATORE and D V KAPSE*

Department of Mathematics, Sant Gadge Baba Amravati University, Amravati 444 602, India
*Corresponding author. E-mail: dipti.kapse@gmail.com

MS received 30 January 2016; revised 13 June 2016; accepted 20 July 2016; published online 5 January 2017

Abstract. In this paper, we have studied the anisotropic and homogeneous Bianchi type-VI$_0$ Universe filled with dark matter and holographic dark energy components in the framework of general relativity and Lyra's geometry. The Einstein's field equations have been solved exactly by taking the expansion scalar (θ) in the model is proportional to the shear scalar (σ). Some physical and kinematical properties of the models are also discussed.

Keywords. Bianchi type-VI$_0$ Universe; dark matter; holographic dark energy; general relativity, Lyra's geometry.

PACS Nos **98.80.-k; 95.36.+x; 95.35.+d; 04.50.+h**

1. Introduction

The astronomical observation of SN Ia [1,2], galaxy redshift survey [3] and cosmic microwave background radiation (CMBR) data [4,5] convincingly suggest that the rate of expansion of our Universe is positive, i.e. we live in an accelerating expanding Universe. The most surprising and counterintuitive result coming from these observations is the fact that only ∼4% of the total energy density of the Universe is in the form of baryonic matter, ∼24% is non-baryonic matter and almost ∼72% is of completely unknown component with negative pressure. In literature, the component with negative pressure is named as dark energy (DE) that produces repulsive force which gives rise to the current accelerating expansion of the Universe.

Einstein [6] in 1916 proposed his theory of general relativity (GR) which provides a geometrical description of gravitation. Many physicists attempted to generalize the idea of geometrizing the gravitation to include a geometrical description of electromagnetism. One of the first attempts was made by Weyl [7] who proposed a more general theory by formulating a new kind of gauge theory involving metric tensor to geometrize gravitation and electromagnetism. But Weyl theory was criticized due to non-integrability of the length of the vector under parallel displacement.

Later, Lyra [8] suggested a modification of Riemannian geometry by introducing a gauge function into the structureless manifold which removes the non-integrability condition. This modified geometry is known as Lyra's geometry. Subsequently, Sen [9] formulated a new scalar–tensor theory of gravitation and constructed an analogue of the Einstein's field equations based on Lyra's geometry. He found that the static model with finite density in Lyra's manifold is similar to the static model in Einstein's general relativity. Halford [10] has shown that the constant displacement vector field β in Lyra's geometry plays the role of cosmological constant Λ in general relativity. He has also shown that the scalar–tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as in Einstein's theory [11]. Katore *et al* [12] studied the Einstein–Rosen bulk viscous cosmological model with bulk viscosity and zero-mass scalar field in Lyra's geometry. Ghate [13], Asgar and Ansari [14,15] and Das and Sharma [16] studied the Bianchi type-V string cosmological model in Lyra's geometry with dark energy. SubbaRao [17] studied the Kantowski–Sachs cosmological model in Lyra's geometry in the presence of bulk viscous string cosmological fluid and Sahu *et al* [18] studied the Bianchi type-III cosmological model in Lyra's geometry.

There are many candidates of dark energy, among which holographic dark energy (HDE) has recently been studied by many researchers. Li [19] has obtained an accelerating Universe by considering event horizon as the cosmological scale. The model is consistent with the cosmological observations. Granda and Oliveros [20] proposed a holographic density of the form $\rho_\wedge \approx \alpha H^2 + \gamma \dot{H}$, where H is the Hubble parameter and α, γ are constants which must satisfy the restrictions imposed by the current observational data. They showed that this new model of dark energy represents the accelerated expansion of the Universe and is consistent with the current observational data. Granda and Oliveros [21] have also studied the correspondence between the quintessence, tachyon, k-essence and dilation dark energy models with this holographic dark energy model in the flat FRW Universe. Chattopadhyay [22], Farajollahi *et al* [23], Debnath [24], Malekjani [25], Sarkar [26,27], are some of the researchers who have investigated several aspects of holographic dark energy. Recently, Kiran *et al* [28,29] have studied minimally interacting dark energy models in some scalar–tensor theories. Rao *et al* [30] and Adhav *et al* [31] have discussed interacting dark matter and HDE in Bianchi type-V Universe. Rao *et al* [32] have discussed the five-dimensional FRW holographic dark energy in Brans–Dicke theory.

In this paper, we have studied anisotropic and homogeneous Bianchi type-VI$_0$ Universe filled with dark matter and holographic dark energy components in the framework of general relativity and Lyra's geometry. The Einstein's field equations have been solved exactly by taking the expansion scalar (θ) in the model as proportional to the shear scalar (σ). This paper is outlined as follows: In §2, we consider Bianchi type-VI$_0$ model with HDE and some basic equations. In §2.1, we have obtained the field equations in GR and discussed its solutions and physical properties. Section 2.1.1 is the case when $n = 2$ with its physical properties. In §2.2, we have obtained the field equations in Lyra's geometry and discussed the solutions and physical properties. Section 2.2.1 is the case when $n = 2$ with its physical properties. Finally, the conclusions are summarized in §3.

2. Metric and basic equations

The spatially homogeneous and anisotropic Bianchi type-VI$_0$ space–time is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{e}^{2x}\mathrm{d}y^2 + C^2\mathrm{e}^{-2x}\mathrm{d}z^2, \quad (1)$$

where A, B and C are functions of time t only.

The energy–momentum tensors for dark matter and HDE are respectively given by

$$T_{ij} = \rho_m u_i u_j$$

and

$$\bar{T}_{ij} = (\rho_\wedge + p_\wedge)\, u_i u_j + g_{ij} p_\wedge, \quad (2)$$

where ρ_m, $\rho_\wedge$ are the energy densities of dark matter (cold dark matter) and HDE respectively, $p_\wedge$ is the pressure of HDE.

Some physical parameters are defined as follows:

The generalized mean Hubble parameter (H) is given by

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \quad (3)$$

The spatial volume (V) is given by

$$V = ABC. \quad (4)$$

The scalar expansion (θ), shear scalar (σ), anisotropy parameter (A_m) and deceleration parameter (q) are defined as

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad (5)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right), \quad (6)$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \quad (7)$$

$$q = -1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right). \quad (8)$$

2.1 *General relativity*

The Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(T_{ij} + \bar{T}_{ij}), \quad (9)$$

where g_{ij} is the metric potentials, R is the Ricci scalar, R_{ij} is the Ricci tensor, T_{ij} is the energy–momentum tensor of dark matter and $\bar{T}_{ij}$ is the energy–momentum tensor of HDE.

In a co-moving coordinate system, the Einstein's field equations (9) for Bianchi type-VI$_0$ space–time (1) with the help of eq. (2) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -p_\wedge, \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -p_\wedge, \quad (11)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-p_\wedge, \tag{12}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}=\rho_m+\rho_\wedge, \tag{13}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{14}$$

where the overhead dot denotes derivative with respect to cosmic time t.

Integrating eq. (14), we obtain

$$B=lC, \tag{15}$$

where l is the constant of integration.

Using eq. (15) for $l=1$, the field equations (10)–(14) reduce to

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{A^2}=-p_\wedge, \tag{16}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-p_\wedge, \tag{17}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}=\rho_m+\rho_\wedge. \tag{18}$$

Solutions of field equations

In order to solve field equations (16)–(18) completely, we assume that the expansion scalar (θ) in the model is proportional to shear scalar (σ) as considered by Thorne [33] and Collins *et al* [34], which leads to

$$A=B^n,\quad n>1. \tag{19}$$

Following Granda and Oliveros [21] and Sarkar [26], the HDE density is given by

$$\rho_\wedge=3\big(\alpha H^2+\gamma\dot{H}\big)\ \text{with}\quad M_P^{-2}=8\pi G=1. \tag{20}$$

The equation of state for HDE is

$$p_\wedge=\omega_\wedge\rho_\wedge. \tag{21}$$

Subtracting eq. (16) from (17) and using eq. (19), we obtain

$$\ddot{B}+(n+1)\frac{\dot{B}^2}{B}=\frac{2}{n-1}B^{1-2n}. \tag{22}$$

Putting $\dot{B}=f(B)$, $\ddot{B}=ff'$, $f'=\mathrm{d}f/\mathrm{d}B$ in eq. (22) and then integrating, we obtain

$$\mathrm{d}t=\frac{\mathrm{d}B}{\sqrt{[1/(n-1)]B^{2(1-n)}+l_1B^{-2(n+1)}}}, \tag{23}$$

where l_1 is a constant of integration.

Hence, the model (1) is reduced to

$$\mathrm{d}s^2=-\frac{\mathrm{d}B^2}{[1/(n-1)]B^{2(1-n)}+l_1B^{-2(n+1)}}+B^{2n}\mathrm{d}x^2+B^2[\mathrm{e}^{2x}\mathrm{d}y^2+\mathrm{e}^{-2x}\mathrm{d}z^2]. \tag{24}$$

After using the suitable transformation

$$B=T$$

and

$$\mathrm{d}t=\frac{\mathrm{d}T}{\sqrt{[1/(n-1)]T^{2(1-n)}+l_1T^{-2(n+1)}}}$$

eq. (24) transforms to

$$\mathrm{d}s^2=-\frac{\mathrm{d}T^2}{[1/(n-1)]T^{2(1-n)}+l_1T^{-2(n+1)}}+T^{2n}\mathrm{d}x^2+T^2[\mathrm{e}^{2x}\mathrm{d}y^2+\mathrm{e}^{-2x}\mathrm{d}z^2]. \tag{25}$$

For $n=1$ a singularity arises. Therefore, for realistic model we take $n>1$ and $l_1<0$.

Some physical properties

The physical parameters such as the Hubble parameter (H), the anisotropic parameter (A_m), the shear scalar (σ), the expansion scalar (θ) and the spatial volume (V) of model (25), which are of cosmological importance, are respectively given by

$$H=\frac{n+2}{3}\left[\frac{1}{n-1}T^{-2n}+l_1T^{-2n-4}\right]^{1/2}, \tag{26}$$

$$A_m=\frac{2(n-1)^2}{(n+2)^2}, \tag{27}$$

$$\sigma^2=\frac{(n-1)^2}{3}\left[\frac{1}{n-1}T^{-2n}+l_1T^{-2n-4}\right], \tag{28}$$

$$\theta=(n+2)\left[\frac{1}{n-1}T^{-2n}+l_1T^{-2n-4}\right]^{1/2}, \tag{29}$$

$$V=T^{n+2}. \tag{30}$$

We observe that the spatial volume $V\to 0$ as $T\to 0$. Therefore, the model starts evolving at $T=0$ and expands with cosmic time. At $T=0$ the spatial volume vanishes and other parameters θ,σ,H diverge. Hence the model (25) has a Big Bang type of initial singularity.

The deceleration parameter (q) is found to be

$$q = -1 + \left(\frac{3}{n+2}\right)\frac{([n/(n-1)]T^4 + l_1(n+2))}{([1/(n-1)]T^4 + l_1)}. \tag{31}$$

All the above results resemble the results of Asgar and Ansari [14]. From eq. (31), we observed that $q > 0$ for $T > [-(n+2)l_1]^{1/4}$ and $q < 0$ for $T < [-(n+2)\, l_1]^{1/4}$ (where $l_1 < 0$). Our model is evolving from decelerating to accelerating phase. According to the recent observations of SNe Ia, the present Universe is accelerating and the value of deceleration parameter is in the range $-1 < q < 0$. In our model (25) the deceleration parameter is consistent with the observations of type-Ia supernovae [2,35–37].

Using eq. (26) in eq. (20), the HDE density is given by

$$\rho_\wedge = k_1 T^{-2n} + k_2 T^{-2n-4}, \tag{32}$$

where

$$k_1 = \left(\frac{\alpha(n+2)}{3} - n\gamma\right)\left(\frac{n+2}{n-1}\right)$$

and

$$k_2 = \left(\frac{\alpha}{3} - \gamma\right)(n+2)^2 l_1.$$

From eq. (16), the pressure of HDE is given by

$$p_\wedge = \frac{n-2}{n-1}T^{-2n} + (2n+1)l_1 T^{-2n-4}. \tag{33}$$

Using eq. (32) in eq. (18), the energy density of dark matter is given by

$$\rho_m = \left(\frac{n+2}{n-1} - k_1\right)T^{-2n} + ((2n+1)l_1 - k_2)T^{-2n-4}. \tag{34}$$

Using eq. (21), the EoS parameter of HDE is given by

$$\omega_\wedge = \frac{((n-2)/(n-1))T^4 + (2n+1)l_1}{k_1 T^4 + k_2}. \tag{35}$$

The behaviour of the energy densities depends on the value of constant l_1. From eqs (32) and (34) we observe that the energy densities of dark matter and HDE are decreasing functions of time. From eq. (33) it is also observed that the pressure of HDE is a decreasing function of time. The obtained EoS parameter of HDE is time varying and it is evolving with negative sign which may be attributed to the current acceleration of the expansion [38]. The EoS parameter of the HDE also behaves like quintessence EoS.

The coincidence parameter is

$$\bar{r} = \frac{\rho_\wedge}{\rho_m} = \frac{k_1 T^4 + k_2}{((n+2)/(n-1) - k_1)T^4 + (2n+1)l_1 - k_2}. \tag{36}$$

It is observed that coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem (unlike ΛCDM).

The density parameter of dark matter (Ω_m) and HDE ($\Omega_\wedge$) are as follows:

$$\Omega_m = \frac{\rho_m}{3H^2}, \quad \Omega_\wedge = \frac{\rho_\wedge}{3H^2}. \tag{37}$$

Using eqs (26), (27), (32), (33), (34) and (35) in eq (37) , we get the overall density parameter as

$$\Omega = \Omega_m + \Omega_\wedge = \frac{3}{(n+2)^2}\left[\frac{[(n+2)/(n-1)]T^4 + (2n+1)l_1}{[1/(n-1)]T^4 + l_1}\right]. \tag{38}$$

From the above equation one can observe that the sum of the energy density parameter approaches $3/(n+2)$ as $T \to \infty$. So, at late times the Universe becomes flat. Therefore, for sufficiently large time, this model predicts that the anisotropy of the Universe will damp out and the Universe will become isotropic. This result also shows that in the early Universe, i.e. during the radiation and matter-dominated era the Universe was anisotropic and the Universe approaches isotropy as dark energy starts to dominate the energy density of the Universe.

2.1.1 *A particular case for* $n = 2$. For $n = 2$, eq. (23) reduces to

$$\mathrm{d}t = \frac{B^3 \mathrm{d}B}{\sqrt{B^4 + l_1}}. \tag{39}$$

On integrating eq. (39), we get

$$B^2 = [4(t - l_3)^2 - l_1]^{1/2}, \tag{40}$$

where l_3 is the constant of integration.

Using eq. (40) and taking $l = 1$ in eq. (15), we get

$$C^2 = [4(t - l_3)^2 - l_1]^{1/2}. \tag{41}$$

From eqs (19) and (40), we get

$$A^2 = [4(t - l_3)^2 - l_1]. \tag{42}$$

Therefore, metric (1) reduces to

$$\mathrm{d}s^2 = -\mathrm{d}T^2 + [4T^2 - l_1]\mathrm{d}x^2 + [4T^2 - l_1]^{1/2} \times (\mathrm{e}^{2x}\mathrm{d}y^2 + \mathrm{e}^{-2x}\mathrm{d}z^2), \tag{43}$$

where $t - l_3 = T$.

Some physical properties

The physical parameters such as Hubble parameter (H), anisotropic parameter (A_m), shear scalar (σ), expansion scalar (θ) and spatial volume (V) of eq. (43), are respectively given by

$$H = \frac{8T}{3}[4T^2 - l_1]^{-1}, \tag{44}$$

$$A_m = \frac{1}{8}, \tag{45}$$

$$\sigma^2 = \frac{4T^2}{3}[4T^2 - l_1]^{-2}, \tag{46}$$

$$\theta = 8T[4T^2 - l_1]^{-1}, \tag{47}$$

$$V = [4T^2 - l_1]. \tag{48}$$

It is observed that the spatial volume increases as time increases. There is a point-type singularity in (43) at $T = l_1/4$ [39]. Equation (43) starts with a big bang at $T = l_1/4$ and the expansion in the model decreases as time increases. In general, the model represents an expanding, shearing and non-rotating Universe. As $\sigma/\theta =$ constant, the model does not approach isotropy.

The deceleration parameter (q) is found to be

$$q = -1 + \frac{3}{8T^2}[4T^2 + l_1]. \tag{49}$$

The deceleration parameter $q > 0$ for $T > [-3l_1/4]^{1/2}$ and $q < 0$ for $T < [-3l_1/4]^{1/2}$.

Using eq. (44) in eq. (20), the HDE density is given by

$$\rho_\wedge = \frac{8}{3}[4(2\alpha - 3\gamma)T^2 - 3\gamma l_1]\,[4T^2 - l_1]^{-2}. \tag{50}$$

Using eqs (40) and (42) in eq. (16), the pressure of HDE is given by

$$p_\wedge = 5l_1[4T^2 - l_1]^{-2}. \tag{51}$$

Using eq. (50) in eq. (18), the energy density of dark matter is given by

$$\rho_m = \left[\left(16 - \frac{32}{3}(2\alpha - 3\gamma)\right)T^2 + (8\gamma + 1)l_1\right] \times [4T^2 - l_1]^{-2}. \tag{52}$$

Using eq. (21), the EoS parameter of HDE is given by

$$\omega_\wedge = \frac{15l_1}{8}[4(2\alpha - 3\gamma)T^2 - 3\gamma l_1]^{-1}. \tag{53}$$

We observe that the energy density of dark matter, energy density and pressure of HDE are decreasing functions of time. The EoS parameter of HDE is obtained as time varying and it is evolving with negative sign which may be attributed to the current acceleration of the expansion [38]. The EoS parameter of the HDE also behaves like quintessence EoS.

The coincidence parameter is

$$\bar{r} = \frac{\rho_\wedge}{\rho_m} = \frac{8}{3}\frac{4(2\alpha - 3\gamma)T^2 - 3\gamma l_1}{(16 - (32/3)(2\alpha - 3\gamma))T^2 + (8\gamma + 1)l_1}. \tag{54}$$

It is observed that coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem (like ΛCDM).

Using eqs (50) and (52), we get the overall density parameter as

$$\Omega = \Omega_m + \Omega_\wedge = \frac{3}{4} + \frac{3l_1}{64}T^{-2}. \tag{55}$$

Sum of the energy density parameter approaches 3/4 as $T \to \infty$.

2.2 *Lyra's geometry*

The Einstein's modified field equation in normal gauge for Lyra's manifold obtained by Sen [8] is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -(T_{ij} + \bar{T}_{ij}), \tag{56}$$

where ϕ_i is the displacement vector defined as $\phi_i = (0, 0, 0, \beta(t))$.

In a co-moving coordinate system, the modified Einstein's field equation (56) for Bianchi type-VI$_0$ space–time with the help of eq. (2) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} + \frac{3}{4}\beta^2 = -p_\wedge, \tag{57}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} + \frac{3}{4}\beta^2 = -p_\wedge, \tag{58}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \frac{3}{4}\beta^2 = p_\wedge, \tag{59}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} - \frac{3}{4}\beta^2 = \rho_m + \rho_\wedge, \tag{60}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{61}$$

where the overhead dot denotes derivative with respect to cosmic time t.

The conservation of the right-hand side of eq. (56) leads to

$$\left(R_i^j - \frac{1}{2} R g_i^j\right)_{;j} + \frac{3}{2}(\phi_i \phi^j)_{;j} - \frac{3}{4}(g_i^j \phi_k \phi^k)_{;j} = 0. \tag{62}$$

Equation (62) is reduced to

$$\frac{3}{2}\phi_i \left[\frac{\partial \phi^j}{\partial x^j} + \phi^l \Gamma^j_{lj}\right] + \frac{3}{2}\phi^j \left[\frac{\partial \phi_i}{\partial x^j} - \phi_l \Gamma^l_{ij}\right] - \frac{3}{4} g_i^j \phi_k \left[\frac{\partial \phi^k}{\partial x^j} + \phi^l \Gamma^k_{lj}\right] - \frac{3}{4} g_i^j \phi^k \left[\frac{\partial \phi_k}{\partial x^j} - \phi_l \Gamma^l_{kj}\right] = 0, \tag{63}$$

leading to

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2 \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{64}$$

Integrating eq. (61), we obtain

$$B = lC, \tag{65}$$

where l is a constant of integration.

Solutions of field equations

Using the same conditions (19) and (20) we have solved the field equations (57)–(61).

Using eqs (19) and (65) for $l = 1$, subtracting eq. (57) from (58), we obtain

$$\ddot{B} + (n+1)\frac{\dot{B}^2}{B} = \frac{2}{n-1} B^{1-2n}. \tag{66}$$

Thus, we get the model (25).

Some physical properties

The physical parameters such as Hubble parameter (H), anisotropic parameter (A_m), shear scalar (σ), expansion scalar (θ), spatial volume (V) and deceleration parameter (q) are found to be the same as in the previous general relativity model.

The displacement vector (β), the HDE density $(\rho_\wedge)$, the pressure of HDE $(p_\wedge)$, energy density of dark matter (ρ_m) and the EoS parameter of HDE $(\omega_\wedge)$ are given by

$$\beta = l_2 T^{-(n+2)}, \tag{67}$$

$$\rho_\wedge = k_1 T^{-2n} + k_2 T^{-2n-4}, \tag{68}$$

$$p_\wedge = \frac{n-2}{n-1} T^{-2n} + \left((2n+1)l_1 - \frac{3}{4} l_2^2\right) T^{-2n-4}, \tag{69}$$

$$\rho_m = \left(\frac{n+2}{n-1} - k_1\right) T^{-2n} + \left((2n+1)l_1 - k_2 - \frac{3}{4} l_2^2\right) \times T^{-2n-4}, \tag{70}$$

$$\omega_\wedge = \frac{((n-2)/(n-1))T^4 + \left((2n+1)l_1 - (3/4)l_2^2\right)}{k_1 T^4 + k_2}, \tag{71}$$

where l_2 is a constant of integration,

$$k_1 = \left(\frac{\alpha(n+2)}{3} - n\gamma\right)\left(\frac{n+2}{n-1}\right)$$

and

$$k_2 = \left(\frac{\alpha}{3} - \gamma\right)(n+2)^2 l_1.$$

From eq. (67) it is noted that the displacement vector (β) is a decreasing function of time, which is corroborated with Halford as well as with the recent observations (Perlmutter [2,40,41]; Reiss [1,35]) leading to the conclusion that Λ (cosmological constant) is a decreasing function of time t. From eq. (70) it is observed that the energy densities of dark matter is a decreasing function of time. From eq. (69) it is also observed that the pressure of HDE is a decreasing function of time. The energy density of HDE in Lyra's geometry is similar to the energy density of HDE in GR. The EoS parameter of HDE is obtained as time varying and it is evolved with negative sign and its range is in good agreement with large scale structure data. The EoS parameter of the HDE behaves like quintessence EoS.

The coincidence parameter is

$$\bar{r} = \frac{\rho_\wedge}{\rho_m} = \frac{k_1 T^4 + k_2}{((n+2)/(n-1) - k_1)T^4 + ((2n+1)l_1 - k_2 - (3/4)l_2^2)}. \tag{72}$$

It is observed that coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem (like ΛCDM).

The overall density parameter is obtained as

$$\Omega=\Omega_m+\Omega_\wedge=\frac{3}{(n+2)^2}\times\left[\frac{[(n+2)/(n-1)]T^4+((2n+1)l_1-(3/4)l_2^2)}{[1/(n-1)]T^4+l_1}\right]. \tag{73}$$

The sum of the energy density parameter approaches $3/(n+2)$ as $T\to\infty$.

2.2.1 *A particular case for $n=2$.* For $n=2$, we get the same eqs (40)–(50) as in the case of $n=2$ in general relativity model in §2.1.1.

The displacement vector is given by

$$\beta=l_4[4T^2-l_1]^{-1}, \tag{74}$$

where l_4 is a constant of integration.

From eq. (57), the pressure of HDE is given by

$$p_\wedge=\left(5l_1-\frac{3}{4}l_4^2\right)[4T^2-l_1]^{-2}. \tag{75}$$

Using eq. (50) in eq. (60), the energy density of dark matter is given by

$$\rho_m=\left[\left(16-\frac{32}{3}(2\alpha-3\gamma)\right)T^2+\left((8\gamma+1)l_1-\frac{3}{4}l_4^2\right)\right]\times[4T^2-l_1]^{-2}. \tag{76}$$

Using eq. (21), the EoS parameter of HDE is given by

$$\omega_\wedge=\frac{3(5l_1-(3/4)l_4^2)}{8}[4(2\alpha-3\gamma)T^2-3\gamma l_1]^{-1}. \tag{77}$$

The coincidence parameter

$$\bar{r}=\frac{\rho_\wedge}{\rho_m}=\frac{8}{3}\frac{4(2\alpha-3\gamma)T^2-3\gamma l_1}{(16-(32/3)(2\alpha-3\gamma))T^2+((8\gamma+1)l_1-(3/4)l_4^2)}. \tag{78}$$

It is observed that coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution, thereby avoiding the coincidence problem (like ΛCDM).

The overall density parameter is

$$\Omega=\Omega_m+\Omega_\wedge=\frac{3}{4}+\frac{3}{64}\left(l_1-\frac{3}{4}l_4^2\right)T^{-2}. \tag{79}$$

The sum of the energy density parameter approaches 3/4 as $T\to\infty$. So at late times the Universe becomes flat.

3. Conclusion

In this paper, we have studied the Bianchi type-VI$_0$ cosmological models with HDE in the framework of GR and Lyra's geometry. The exact solutions of field equations have been obtained by assuming that expansion scalar (θ) in the model is proportional to shear scalar (σ). For $n=1$, our models do not survive. The physical parameters such as Hubble parameter, anisotropic parameter, shear scalar, expansion scalar, spatial volume and deceleration parameter are found to be equal in GR and Lyra's geometry. The matter density, the pressure of HDE, the EoS parameter of HDE, the coincidence parameter and overall density parameter in Lyra's geometry slightly differ by the term β from GR. It is found that if the displacement vector $\beta\to 0$, the Lyra's geometry tends to GR in all respects. Also, we have discussed a specific case $n=2$ for both the GR and Lyra's geometry.

References

[1] A G Riess *et al*, *Astron. J.* **116**, 1009 (1998)
[2] S Perlmutter *et al*, *Astrophys. J.* **517**, 565 (1999)
[3] C Fedli, L Moscardini and M Bertelmann, *Astron. Astrophys.* **500**, 667 (2009)
[4] R R Caldwell and M Doran, *Phys. Rev. D* **69**, 103517 (2004)
[5] Z-Yi Huang *et al*, *J. Cosmol. Astropart. Phys.* **05**, 013 (2006)
[6] A Einstein, *Ann. Phys.* **354**, 769 (1916)
[7] H Weyl, *Math. Z.* **2**, 384 (1918)
[8] G Lyra, *Math. Z.* **54**, 52 (1951)
[9] D K Sen, *Z. Phys. C* **149**, 311 (1957)
[10] W D Halford, *Austr. J. Phys.* **23**, 863 (1970)
[11] W D Halford, *J. Math. Phys.* **13**, 1699 (1972)
[12] S D Katore *et al*, *Prespacetime J.* **3**, 83 (2012)
[13] H R Ghate and A S Sontakke, *Prespacetime J.* **5**, 619 (2013)
[14] A Asgar and M Ansari, *The African Rev. Phys.* **9**, 0019 (2014)
[15] A Asgar and M Ansari, *IOSR J. Appl. Phys. (IOSR-JAP)* **5**, 1 (2015)
[16] K Das and G Sharma, *Prespacetime J.* **5**, 494 (2014)
[17] M V SubbaRao, *Astrophys. Space Sci.* **356**, 149 (2015)
[18] S K Sahu *et al*, *Int. J. Theor. Phys.* **54**, 807 (2015)
[19] M Li, *Phys. Lett. B* **603**, 1 (2004)
[20] L N Granda and A Oliveros, *Phys. Lett. B* **669**, 275 (2008)
[21] L N Granda and A Oliveros, *Phys. Lett. B* **671**, 199 (2009)
[22] S Chattopadhyay and U Debnath, *Astrophys. Space Sci.* **319**, 183 (2009)
[23] H Farajollahi, J Sadeghi and M Pourali, *Astrophys. Space Sci.* **341**, 695 (2012)
[24] U Debnath, *Astrophys. Space Sci.* **337**, 503 (2012)
[25] M Malekjani, *Astrophys. Space Sci.* **347**, 405 (2013)
[26] S Sarkar, *Int. J. Theor. Phys.* **52**, 1482 (2013)
[27] S Sarkar, *Astrophys. Space Sci.* **349**, 985 (2014)
[28] M Kiran, D R K Reddy and V M U Rao, *Astrophys. Space Sci.* **354**, 577 (2014)
[29] M Kiran, D R K Reddy and V M U Rao, *Astrophys. Space Sci.* **356**, 407 (2015)

[30] V M U Rao, M Vijaya Santhi and N Sandhya Rani, *Prespacetime J.* **6**, 226 (2014)
[31] K S Adhav, S L Munde, G B Tayade and V D Bokey, *Astrophys. Space Sci.* **359**, 24 (2015)
[32] V U M Rao, M Vijaya Santhi and N Sandhya Rani, *Prespacetime J.* **6**, 961 (2015)
[33] K S Thorne, *Astrophys. J.* **148**, 51 (1967)
[34] C B Collins, E N Glass and D A Wilkinson, *Gen. Relativ. Gravit.* **12**, 805 (1980)
[35] A G Riess *et al*, *Astrophys. J.* **607**, 665 (2004)
[36] J L Torny *et al*, *Astrophys. J.* **594**, 1 (2003)
[37] A Clocchiatti *et al*, *Astrophys. J.* **642**, 1 (2006)
[38] E Komatsu *et al*, *Astrophys. J. Suppl. Ser.* **180**, 330 (2009)
[39] M A H MacCallum, *Comput. Math. Phys.* **20**, 57 (1971)
[40] S Perlmutter *et al*, *Astrophys. J.* **483**, 565 (1997)
[41] S Perlmutter *et al*, *Nature* **391**, 51 (1998)