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Effects of non-extensive electrons and positive/negative dust particles on modulational instability of dust-ion-acoustic solitary waves in non-planar geometry

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Abstract. The nonlinear propagation of cylindrical and spherical dust-ion-acoustic (DIA) envelope solitary waves in unmagnetized dusty plasma consisting of dust particles with opposite polarity and non-extensive distribution of electron is investigated. By using the reductive perturbation method, the modified nonlinear Schrödinger (NLS) equation in cylindrical and spherical geometry is obtained. The modulational instability (MI) of DIA waves governed by the NLS equation is also presented. The effects of different ranges of the non-extensive parameter q on the MI are studied. The growth rate of the MI is also given for different values of q. It is found that the basic features of the DIA waves are significantly modified by non-extensive electron distribution, polarity of the net dust-charge number density and non-planar geometry.

Keywords. Nonlinear Schrödinger equation; modulational instability; non-planar; non-extensive; dust-ion-acoustics.

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1. Introduction

The existence of novel dust-ion-acoustic (DIA) waves was first predicted by Shukla and Silin about twenty years ago [1]. Nearly four years later, their prediction was experimentally verified by Barkan *et al* [2]. The linear features of the DIA waves have been rigorously investigated by a number of researchers [3–5]. During the last two decades, the propagation of nonlinear dust-acoustic solitary waves and DIA solitary waves, in dusty plasma with an unbounded planar geometry has been extensively studied theoretically [6–12]. It has been found from both experimental observation [13,14] and theoretical analysis [15,16] that the presence of non-thermal (fast) electrons, which occur in many space plasma situation, particularly in the part of ionosphere or lower part of magnetosphere [17,18], significantly modifies the basic features of DIA waves or introduce new features in them. A few theoretical investigations have been made on DIA waves in dusty plasma containing negative-dust inertial ions and non-thermal electrons. Tribeche and Berbi [15] extended the work of Mamun and Shukla [8] to include the effect of non-thermal electron distribution on one-dimensional (1D) planar DIA solitary wave and shock. Xue [16] considered non-planar cylindrical and spherical geometries and examined the interaction between the compressive and rarefactive DIA waves. It has also been found that in some space environments (viz. upper part of the ionosphere or lower part of the magnetosphere), where dust number density varies from 10 to 100 cm^{-3} and dust size varies from 1 to 10 μm, particles are positively charged [19–25]. In this article, we consider a more general dusty plasma system containing electrons following non-thermal distribution, inertial ions, stationary dust of opposite polarity (positive dust as well as negative dust). The nonlinear DIA waves in cylindrical and spherical geometries are studied.

2. Basic equations

The nonlinear propagation of finite-amplitude non-planar (cylindrical and spherical) DIA waves in the unmagnetized dusty plasma composed of non-extensive distributed electrons, inertial ions and stationary positively as well as negatively charged dust is considered. The usual ion fluid equations, which include the continuity equation, momentum balance equation and Poisson equation, governing the DIA waves in cylindrical or spherical geometry are

$$\frac{\partial n_{\rm i}}{\partial t} + \frac{1}{r^m}\frac{\partial (r^m n_{\rm i} u_{\rm i})}{\partial r} = 0, \tag{1}$$

$$\frac{\partial u_{\rm i}}{\partial t} + u_{\rm i}\frac{\partial u_{\rm i}}{\partial r} = -\frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m \frac{\partial \phi}{\partial r}\right) = (1+s\alpha)n_{\rm e} - n_{\rm i} - s\alpha, \tag{3}$$

where $m = 0, 1, 2$ are for one-dimensional planar, cylindrical and spherical geometries, respectively. The positive (negative) dust number density at equilibrium is $n_{\rm p}$ ($n_{\rm n}$) and $z_{\rm p}$ ($z_{\rm n}$) is the number of excess protons (electrons) residing on the surface of the dust grain. The parameter s represents the polarity of the net dust charge (i.e. $s = 1$ for $z_{\rm p}n_{\rm p} > z_{\rm n}n_{\rm n}$ and $s = -1$ for $z_{\rm n}n_{\rm n} > z_{\rm p}n_{\rm p}$). The ion number density $n_{\rm i}$, the ion fluid velocity $u_{\rm i}$, the electron number density $n_{\rm e}$ and the electrostatic potential ϕ are normalized by the equilibrium density value of ions $n_{\rm i0}$, the ion-acoustic velocity $C_{\rm i} = \sqrt{T_{\rm e}/m_{\rm i}}$ ($m_{\rm i}$ is the ion mass), the equilibrium density value of electrons $n_{\rm e0}$ and by $T_{\rm e}/e$ ($T_{\rm e}$ is the electron temperature in units of Boltzmann constant), respectively. The time and space variables are in units of ion plasma period $\omega_{pe}^{-1} = (m_{\rm i}/4\pi n_{\rm i0}e^2)^{1/2}$ and the Debye length $\lambda_{\rm D} = (T_{\rm e}/4\pi n_{\rm i0}e^2)^{1/2}$ respectively.

To take into account the non-extensive distribution of electrons, we use the following distribution function [26]:

$$F_{\rm e}(v_{\rm e}) = C_q\left[1 + (1-q)\left(\frac{m_{\rm e}v_{\rm e}}{2T_{\rm e}} - \frac{e\phi}{T_{\rm e}}\right)\right]^{1/(q-1)}, \tag{4}$$

where q is the parameter that measures the strength of non-extensivity. It may be noted that the function $F_{\rm e}(v_{\rm e})$ is the particular distribution that maximizes the Tsallis entropy. The normalization constant C_q is given by

$$C_q = n_{\rm e0}\frac{\Gamma(1/(1-q))}{\Gamma(1/(1-q)-(1/2))}\sqrt{\frac{m_{\rm e}(1-q)}{2\pi T_{\rm e}}},\quad \text{for } -1 < q < 1, \tag{5}$$

$$C_q = n_{\rm e0}\frac{(q+1)}{2}\frac{\Gamma(1/(q-1)+(1/2))}{\Gamma(1/(q-1))}\sqrt{\frac{m_{\rm e}(1-q)}{2\pi T_{\rm e}}},\quad \text{for } q > 1, \tag{6}$$

and for $q < -1$, the q-distribution is unnormalizable. The distribution function becomes the well-known Maxwell–Boltzmann velocity distribution in the limiting case $q \to 1$. It is important to note further that $q > 1$ exhibits a thermal cut-off on the maximum value allowed for the electron speed. The latter is given by

$$v_{\rm max} = \sqrt{\frac{2T_{\rm e}}{m_{\rm e}}\left(\frac{e\phi}{T_{\rm e}} + \frac{1}{q-1}\right)}. \tag{7}$$

By integrating the q-distribution over the allowed velocity space, one may obtain the dimensionless electron number density as

$$n_{\rm e} = [1 + (q-1)]^{(q+1)/2(q-1)}. \tag{8}$$

The electron density eq. (8) may be expanded as a power of ϕ

$$n_{\rm e} = [1 + C_1\phi + C_2\phi^2 + C_3\phi^3 + \cdots], \tag{9}$$

where

$$C_1 = (q+1)/2, \quad C_2 = (q+1)(q-3)/8,$$
$$C_3 = (q+1)(q-3)(3q-5)/48.$$

Substituting eq. (9) into eq. (3), and expand up to third order we get

$$\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial \phi}{\partial r}\right) = (1+\alpha s) \times [1 + C_1\phi + C_2\phi^3 + C_3\phi^3] - n_{\rm i} - \alpha s. \tag{10}$$

In order to investigate the modulation of DIA waves in the dusty plasma, we employ the standard reductive perturbation technique to obtain the appropriate nonlinear Schrödinger equation (NLSE). The independent variables are stretched as

$$\xi = \varepsilon(r - v_{\rm g}t), \tag{11}$$

$$\tau = \varepsilon^2 t, \tag{12}$$

where ε is a small parameter and $v_{\rm g}$ is the group velocity of the wave. The dependent variables are expanded as

$$n_{\rm i} = 1 + \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=0}^{\infty}(n_{nl}\exp[il(kr-\omega t)] + \text{c.c.}), \tag{13}$$

$$u_{\rm i} = \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=0}^{\infty}(u_{nl}\exp[il(kr-\omega t)] + \text{c.c.}), \tag{14}$$

$$\phi = \sum_{n=1}^{\infty}\varepsilon^n \sum_{l=0}^{\infty}(\phi_{nl}\exp[il(kr-\omega t)] + \text{c.c.}), \tag{15}$$

where all variables satisfy the reality condition $A_{-l} = A_l^*$, and the asterisk denotes complex conjugate. Substituting expressions (11)–(15) into eqs (1)–(3) and collecting the terms in different powers of ε we obtain each nth-order of reduced equation. For the first power of ε

$$n_{11} = \frac{k}{\omega} u_{11}, \tag{16}$$

$$u_{11} = \frac{k}{\omega} \phi_{11}, \tag{17}$$

$$[k^2 + (1 + \alpha s)C_1]\phi_{11} = n_{11}. \tag{18}$$

Thus, we obtain the following dispersion relation for DIA waves:

$$\omega^2 = \frac{k^2}{k^2 + (1 + \alpha s)C_1}. \tag{19}$$

It is obvious that the phase velocity is significantly increased by the presence of non-thermal electron (the parameter C_1) and that is decreased when the net dust-charge number density is positive ($s = 1$), but it is increased when the net dust-charge number density is negative ($s = -1$). Figures 1 and 2 show the dispersion relations of DIA waves propagated in negative and positive dust particles for different values of q. The normalized frequency ω increases by increasing k, and the frequency is shifted towards lower values as $q \to 1$, i.e., for Maxwell–Boltzmann distribution of the electrons. Thus, deviation from the Maxwellian distribution appears to increase the energy of the wave.

Up to the second power of ε, the group velocity can be obtained, which is compatible with the dispersion relation

$$v_g = [(1 + \alpha s)C_1] \left(\frac{k}{\omega}\right)^3. \tag{20}$$

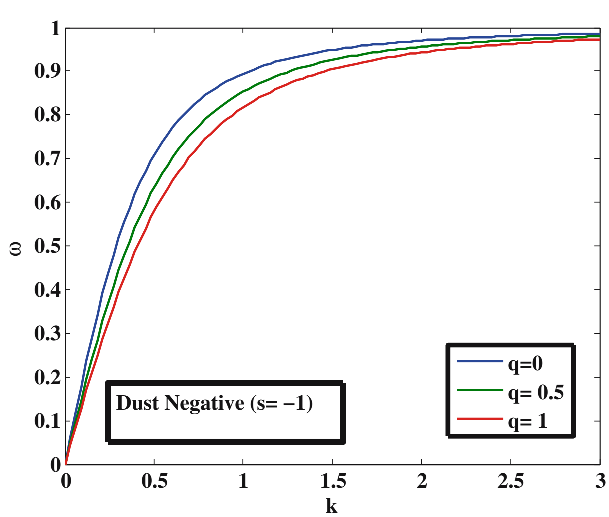

Figure 1. The normalized frequency as a function of the normalized wave number for negative polarity ($s = -1$).

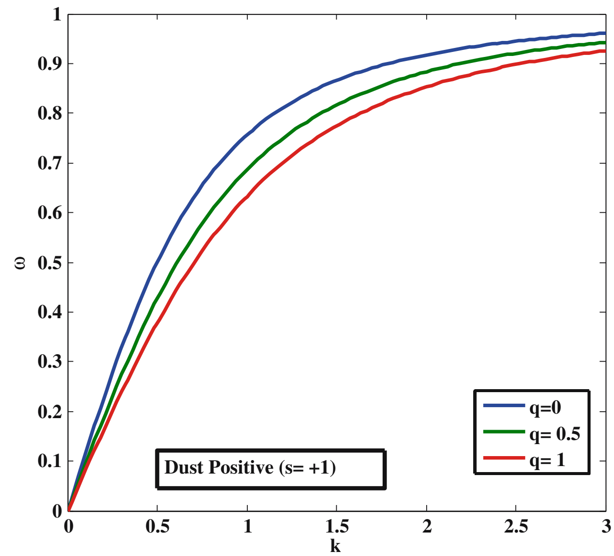

Figure 2. The normalized frequency as a function of the normalized wave number for positive polarity ($s = +1$).

The group velocities as a function of wave number for different values of q are plotted in figures 3 and 4. These figures show that the group velocities for different polarities are very different. Also, the following differential equations come from the second order of ε

$$-i\omega n_{21} + iku_{21} + \frac{\partial u_{11}}{\partial \xi} - v_g \frac{\partial n_{11}}{\partial \xi} = 0, \tag{21}$$

$$-i\omega u_{21} + ik\phi_{21} = v_g \frac{\partial u_{11}}{\partial \xi} - \frac{\partial \phi_{11}}{\partial \xi}, \tag{22}$$

$$n_{21} - [(1 + \delta s)C_1 + k^2]\phi_{21} = -2ik \frac{\partial \phi_{11}}{\partial \xi}. \tag{23}$$

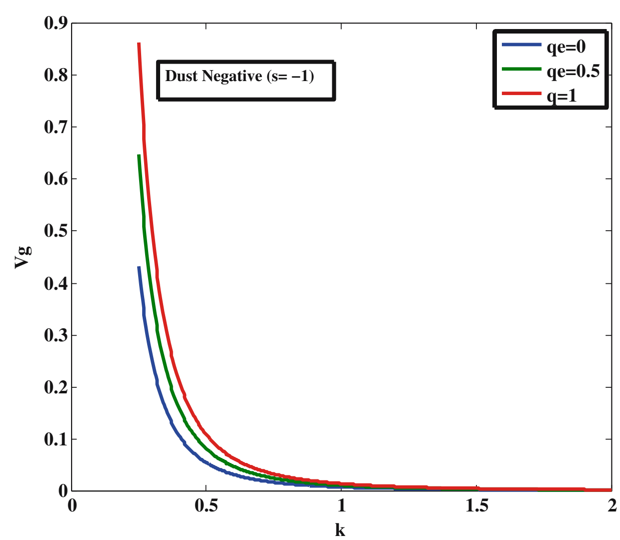

Figure 3. The group velocity as a function of wave number for negative polarity ($s = -1$).

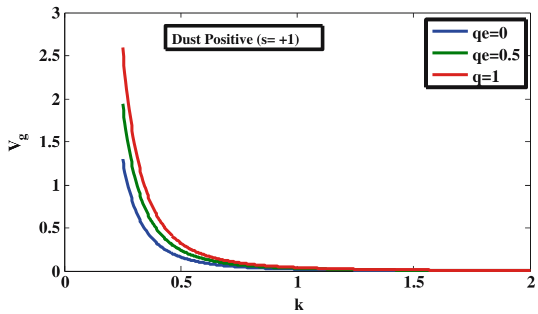

Figure 4. The group velocity as a function of wave number for positive polarity ($s = +1$).

Using the second order with $n = 2, l = 2, n = 2, l = 0$ and the third order with $n = 3$, $l = 1$, we obtain the following nonlinear Schrödinger equation for the first term of the potential:

$$i\frac{\partial\phi_{11}}{\partial\tau} + \frac{m}{2\tau}\phi_{11} + P\frac{\partial^2\phi_{11}}{\partial\xi^2} + Q\phi_{11}\,|\phi_{11}|^2 = 0, \quad (24)$$

where

$$P = -\frac{v_g}{2k}\left[\frac{k}{\omega}\left(1 + \frac{2\omega}{k} + \frac{2\omega^3}{k}\right) - 1\right], \quad (25)$$

$$Q = -\frac{\omega}{2k^2}\left(\begin{array}{l} \frac{k^2}{\omega}(\omega B_{2n} + kB_{2u} + kB_{0u} + \omega B_{0n}) \\ + \frac{k^2}{\omega}(B_{2n} + B_{0u}) - 3\omega^2(1+\alpha s)C_3 \\ -\omega^2(1+\alpha s)C_2(B_{0\phi} + B_{2\phi}) \end{array}\right). \quad (26)$$

The coefficients B are defined in Appendix. The term $m/2\tau$ in eq. (24) represents the cylindrical and spherical geometry effects.

3. Modulational instability

We analyse the MI of DIA waves described by the NLS equation (24), by developing a small modulation $\delta\phi$ according to

$$\phi_{11} = [\bar{\phi}_0 + \delta\phi(\tau,\xi)]\exp\left\{-i\int_{\tau_0}^{\tau}\Delta(\tau')\mathrm{d}\tau' - m/2\ln\tau\right\}. \quad (27)$$

The constant and real ϕ_0 is the amplitude of the pump carrier wave and Δ is a nonlinear frequency shift. Assume the perturbation $\delta\phi$ to be of the form

$$\delta\phi = \delta\phi_0\exp\left[-i\left(K\xi - \int_{\tau_0}^{\tau}\Omega(\tau')\mathrm{d}\tau'\right)\right] + \text{c.c.}\left(K\xi - \int_{\tau_0}^{\tau}\Omega(\tau')\mathrm{d}\tau'\right), \quad (28)$$

where K and Ω in the modulation phase are the wave number and frequency of the modulation, respectively. Substituting expressions (27) and (28) in NLS equation (24), one obtains the nonlinear dispersion relation [9].

$$\Omega^2 = (PK^2)^2\left(1 - \frac{K_c^2}{K^2}\right), \quad (29)$$

where

$$K_c^2 = Q/P(2|\bar{\phi}_0|^2/\tau^m).$$

Further, from eq. (29) in the region $PQ > 0$ and for $K^2 \le K_c^2(\tau)$, the local instability growth rate is given by

$$\Gamma = \mathrm{Im}\,\Omega(\tau) = PK^2\left(\frac{Q}{P}\frac{2|\bar{\phi}_0|^2}{K^2\tau^m} - 1\right)^{1/2}. \quad (30)$$

That is, the instability growth will cease for cylindrical geometry when

$$\tau \ge \tau_{\max} = \left(\frac{2Q|\bar{\phi}_0|^2}{PK^2}\right)^{1/m}, \quad (31)$$

where $m = 1, 2$ are for cylindrical and spherical geometries. There is a new condition of MI in the dusty plasma with non-adiabatic dust charge variations. Equation (31) indicates that the instability period is proportional to the pump carrier wave amplitude ϕ_0 and nonlinearity Q, but inversely proportional to the modulation wave number K and the dispersion P. The total growth of the modulation during the unstable period is defined as

$$\Gamma = \exp(G), \quad (32)$$

$$G = \int_{\tau_0}^{\tau_{\max}}\mathrm{Im}\,\Omega(\tau')\mathrm{d}\tau' = \frac{Q|\bar{\phi}_0|^2}{\tau_0^{m-1}}f(R), \quad (33)$$

where

$$R = (|\bar{\phi}_0|^2/\tau_0^m)(Q/P)1/K^2 \ge 1,$$

for cylindrical geometry and

$$f(R) = \arctan\sqrt{R-1} - \frac{\sqrt{R-1}}{R} \quad (34)$$

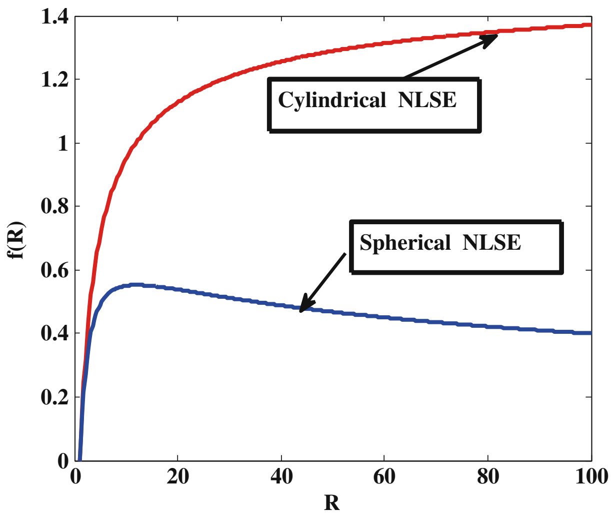

Figure 5. Variation of the function $f(R)$ vs. R.

and for spherical geometry

$$f(R) = \frac{1}{R}\left[\sqrt{R}\ln\frac{\sqrt{R}+\sqrt{R-1}}{\sqrt{R}-\sqrt{R-1}} - 2\sqrt{R-1}\right]. \tag{35}$$

Equation (34) shows that $f(R)$ is an increasing function of R and $f(R)\to\pi/2$ as $R\to\infty$. This means that during the MI period, the total modulation growth $\exp(G)$ increases as R does for cylindrical case, but for spherical geometry the function in eq. (35) indicates that $f(R)$ has a maximum value at $R = R_c$, i.e.

$$\max f(R_c) = \frac{2\sqrt{R_c - 1}}{R_c}, \tag{36}$$

where R_c is determined by the following relation:

$$\sqrt{R_c}\ln\frac{\sqrt{R_c}+\sqrt{R_c-1}}{\sqrt{R_c}-\sqrt{R_c-1}} = 4\sqrt{R_c-1}. \tag{37}$$

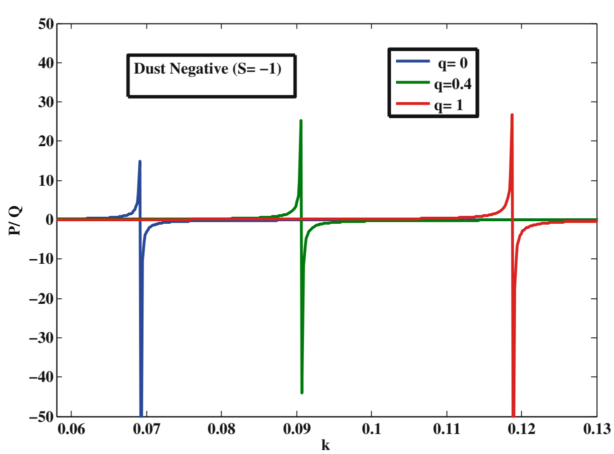

Figure 6. Variation of P/Q with the carrier wave number k for different values of q, the non-extensive parameter, for negative polarity ($s = -1$).

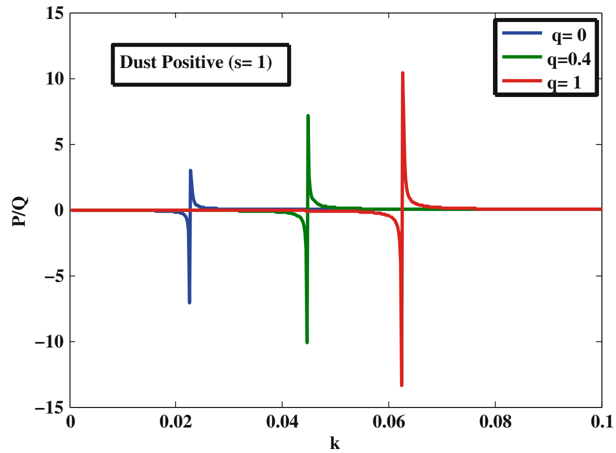

Figure 7. Variation of P/Q with the carrier wave number k for different values of q, the non-extensive parameter, for positive polarity ($s = +1$).

Figure 5 illustrates the curves of $f(R)$ for both eqs (34) and (35). It should be noted that the modulation instability period given by (31) for cylindrical geometry $m = 1$ is longer than that determined for spherical geometry, $m = 2$. During the unstable period, the modulation instability growth rate is always an increasing function of R in cylindrical geometry but not in spherical geometry. This suggests that spherical waves are more structurally stable to perturbations than cylindrical waves, which is also found in Korteweg–de Vries (KdV) solitons. The above discussions also show that the MI becomes a possible limiting factor for coherent transmission in cylindrical and spherical dusty plasmas.

The signs of parameters P and Q are important for understanding the physics of solitary waves and instability. We have modulational instability if $PQ > 0$ and

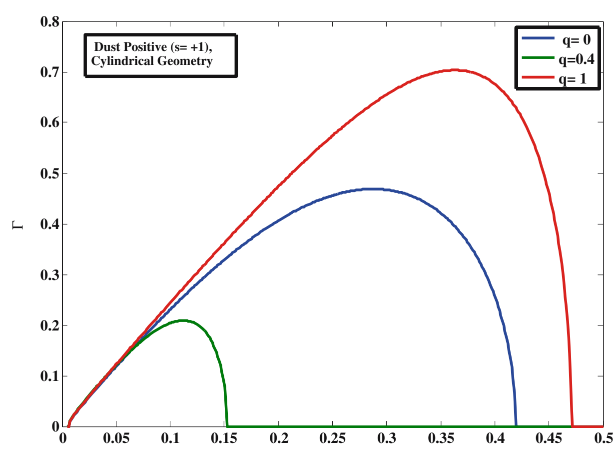

Figure 8. The growth rate Γ as a function of K for different values of q for cylindrical geometry and positive dust ($s = +1$).

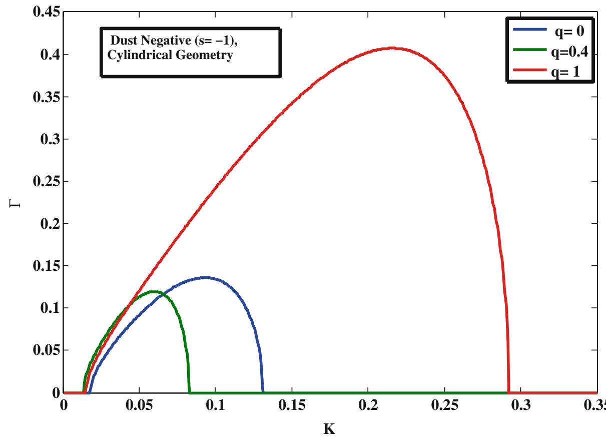

Figure 9. The growth rate Γ as a function of K for different values of q for cylindrical geometry and negative dust ($s = -1$).

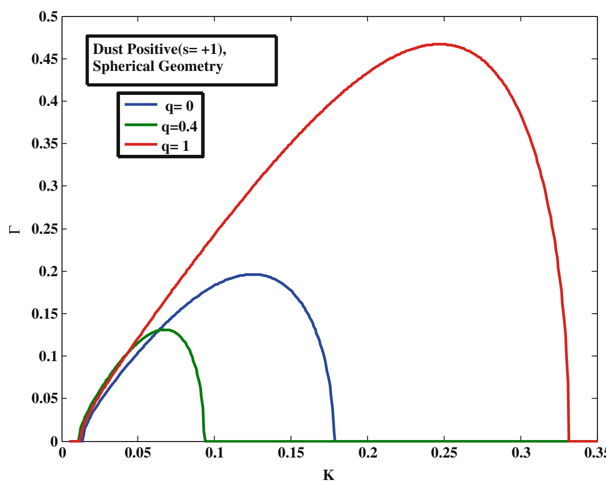

Figure 10. The growth rate Γ as a function of K for different values of q for spherical geometry and positive dust ($s = +1$).

$K \leq K_c$. Our analysis shows that the coefficient P is always negative, but Q can be negative or positive. Apparently, the coefficients of dispersion term P and nonlinear term Q are related to the values of k, q, s. To investigate the effect of parameters in more detail, we plot the ratio of P/Q vs. the carrier wave number k for different parameters in figures 6 and 7. In these figures, $Q = 0$ corresponds to zero dispersion point leading to $P/Q \to \pm\infty$. It is observed from the graphs that the wave remains stable at large wave number $K > K_c$ or when $PQ < 0$.

For more information about the nature of MI, we have plotted growth rate Γ as a function of K for three different values of non-extensive parameter q for cylindrical and spherical geometries of the NLS equation.

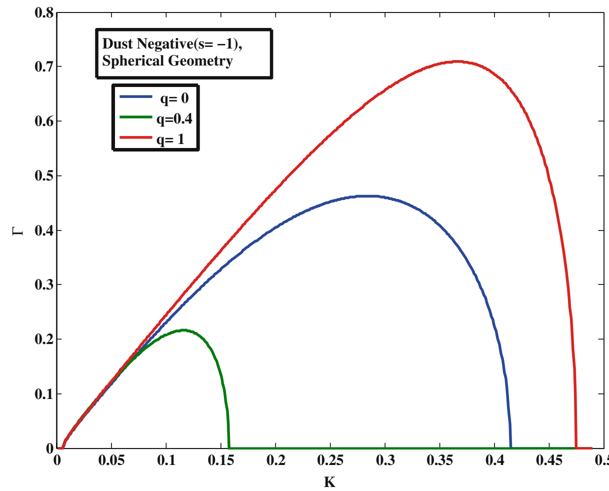

Figure 11. The growth rate Γ as a function of K for different values of q for spherical geometry and negative dust ($s = -1$).

Although the general trend between Maxwellian distribution of electrons and non-extensive distribution is similar, but results in figures 8–11 show difference in magnitude of growth rate between them and clearly, the magnitude of growth is different for cylindrical and spherical geometries. Another aspect worth noting here is that the values of maximum growth rate and the corresponding K not only decrease with increasing q but also shift towards lower K value.

4. Conclusion

In summary, a modified NLS equation describing slow modulation of DIA waves in cylindrical and spherical dusty plasma composed of non-extensive distributed electrons, inertial ions and stationary positively as well as negatively charged dust is derived by the standard reductive perturbation method. The property of the MI in non-planar geometry differs from that of the planar waves. The effects of the non-extensive parameter q, dust polarity s and geometry on the instability and growth rate are investigated. A critical threshold modulation wave number to establish the MI in non-planar case existed. For cylindrical and spherical DIA waves, this critical wave number is related to τ, the MI period. The instability threshold is admittedly rather small for waves in negative dust charge, but seems to be significant for waves in positive dust charge. The growth rate is significantly affected (reduced) by the superthermal electrons and hence MI can be controlled by the existence of a long tail in the plasma species distribution. Our investigation may provide a better understanding of the nonlinear wave phenomena in laboratory experiments and in interstellar and spatial observations, where non-extensive plasma distributions may be present.

Appendix

$$B_{2\phi} = \frac{(-3k^4/\omega^2) + (1+\alpha s)C_2}{k^2 - \omega^2[4k^2 + (1+\alpha s)C_2]},$$

$$B_{2u} = \frac{k}{\omega}B_{2\phi} + \frac{k^3}{2\omega^2},$$

$$B_{2n} = \frac{k}{\omega}B_{2u} + \frac{k^4}{\omega^4},$$

$$B_{0\phi} = \frac{2v_{\mathrm{g}}(k^3/\omega^3) + v_{\mathrm{g}}(k^2/\omega^2) - v_{\mathrm{g}}^2(1+\alpha s)C_2}{v_{\mathrm{g}}^2(1+\alpha s)C_1 - v_{\mathrm{g}}},$$

$$B_{0u} = \frac{1}{v_{\mathrm{g}}}\left[\left(\frac{k}{\omega}\right)^2 + B_{0\phi}\right],$$

$$B_{0n} = (1+\alpha s)C_1 B_{0\phi} + 2(1+\alpha s)C_2.$$

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