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Robust control of a class of chaotic and hyperchaotic driven systems

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Abstract. This paper proposes new conditions which are sufficient for robust control of a class of chaotic and hyperchaotic driven systems. The drive–driven systems are characterized by non-identical uncertain complex dynamics where complexities are mainly introduced by the switching nature of their vector fields. The controller design is achieved using linear matrix inequalities (LMIs) and the so-called S-procedure and then validated using two numerical examples. To illustrate the robustness of the proposed approach, a comparative study is also established with regard to a related approach.

Keywords. Robust control; chaotic drive–driven systems; linear matrix inequalities; norm-bounded uncertainties.

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1. Introduction

Over the last decade, discontinuity of vector fields on dynamical systems had been considered as one of the main causes of the system's instability [1]. A well-known class of systems with discontinuous vector fields is the piecewise linear (PWL) system's class. For such complex dynamics, several research works have been reported in the framework of control, observer and synchronization design methods (see for example [2–5] and references therein). Besides, a few research works are recently devoted to generate chaos and hyperchaos dynamics by proposing new PWL systems [6,7]. However, very few results are published on chaos synchronization for such complex systems [8–10].

Over the past ten years, robust chaos synchronization via state feedback control has been widely studied where some attractive results have been reported using linear matrix inequality (LMI) tools [11,12]. Different types of uncertainties such as parametric uncertainties [13], nonlinear uncertainties [14], randomly occurring uncertainties [15], unknown uncertainties [16], matched and unmatched uncertainties [17] etc., are considered. Nevertheless, to our best knowledge, the problem of robust chaos synchronization of PWL systems with norm-bounded uncertainties is still a pending problem.

Motivated by this, we investigate, in this paper, the robust control of the chaotic PWL drive–driven systems. The synchronization problem between the drive and the driven is formulated as a global stability problem of synchronization error using a Lyapunov approach and solved using LMI tools and the well-known S-procedure. The effectiveness of the proposed solution will be shown by simulation results using two numerical examples.

This paper is structured as follows: The control problem is described in §2. The LMI-sufficient conditions are designed in §3. In §4, the efficiency of the proposed approach is illustrated by simulation results on the well-known Chua's modified model and a new family of hyperchaotic multiscroll attractors. A comparative study is finally organized to show the robustness of the proposed approach compared to a related one.

2. Problem formulation

Consider the particular class of chaotic PWL drive–driven systems with norm-bounded uncertainties described by

$$\begin{cases} \dot{x} = (A_j+\Delta A_j)x+b_j, \;\; x\in\Lambda_j, \;\; j\in\{1,\ldots,N\} \\ \dot{z} = (A_i+\Delta A_i)z+b_i+Bu, \; z\in\Lambda_i, \; i\in\{1,\ldots,N\} \\ u = K(z-x) \end{cases} \tag{1a}$$

where

$$\Delta A_j = D_j V_j E_j, \quad \Delta A_i = D_i V_i E_i \tag{1b}$$

are the norm-bounded uncertainties in the state matrices A_i and A_j and $x \in \Re^n$ and $z \in \Re^n$ are the state vectors of the drive and the driven systems, respectively. $A_i \in \Re^{n\times n}$, $A_j \in \Re^{n\times n}$, $b_i \in \Re^n$ and $b_j \in \Re^n$ are two constant matrices and two constant vectors, respectively. $B \in \Re^{n\times m}$ and $u \in \Re^m$ are the control matrix and the control vector, respectively. $K \in \Re^{m\times n}$ is the state feedback gain matrix. D_j, V_j, E_j and D_i, V_i, E_i are real constant matrices with appropriate dimensions such that $V_j^T V_j \le I$ and $V_i^T V_i \le I$.

Λ_j and Λ_i are partition of the state-space into polyhedral cells defined respectively by the following polytopic description [8]:

$$\Lambda_j = \{x | H_j^T x + h_j \le 0\}, \tag{2a}$$

$$\Lambda_i = \{z | H_i^T z + h_i \le 0\}, \tag{2b}$$

where $H_j \in \Re^{n\times r_j}$, $h_j \in \Re^{r_j \times 1}$, $H_i \in \Re^{n\times r_i}$ and $h_i \in \Re^{r_i \times 1}$.

The objective is to design a control law u and to choose an appropriate constant matrix B such that the synchronization error $e = z - x \to 0$ as the time $t \to \infty$ and the control u is realizable.

3. LMI-sufficient conditions

From (1), the error dynamics between the driven system and the drive system can be written as

$$\begin{aligned}\dot{e} &= (A_i + D_i V_i E_i + BK)e \\ &\quad + (A_{ij} + D_i V_i E_i - D_j V_j E_j)x + b_{ij},\end{aligned} \tag{3}$$

where $e = z - x$, $A_{ij} = A_i - A_j$ and $b_{ij} = b_i - b_j$.

Remark 1. The closed loop system (3) is a continuous PWL system because the drive–driven system described by (1) is a continuous PWL system.

DEFINITION 1

The drive–driven system (1) is said to be of global asymptotical synchronization if the synchronization error system (3) is globally asymptotically stable.

Lemma 1 [18]. *Let D and E be real constant matrices with appropriate dimensions, and matrix V (constant or time-varying) satisfies $V^T V \le I$, then we have: For any scalar $\varepsilon > 0$, the following inequality is valid*:

$$DVE + E^T V^T D^T \le \varepsilon D D^T + \varepsilon^{-1} E^T E. \tag{4}$$

Theorem 1. *If a suitable matrix $B \in \Re^{n\times m}$ is chosen such that the pairs (A_i, B) are controllable, for a given decay $\alpha_1 > 0$ and for all $i, j \in \{1, \ldots, N\}$, if there exist constant symmetric positive definite matrix $S \in \Re^{n\times n}$, constant matrix $R \in \Re^{m\times n}$, diagonal negative definite matrices $E_{ij} \in \Re^{r_i\times r_i}$ and $F_{ij} \in \Re^{r_j\times r_j}$ and strictly negative constants β_{ij} and ξ_{ij}, such that the following LMIs are satisfied*:

$$\begin{bmatrix} \xi_{ij} & \xi_{ij}|h_i|^T & \xi_{ij}|h_j|^T \\ * & \frac{1}{2}E_{ij} & 0 \\ * & * & \frac{1}{2}F_{ij} \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} \Delta_1 & A_{ij} & SH_i & 0 & \Delta_3 & \Delta_6 & SE_i^T \\ * & \Delta_2 & H_i & H_j & \Delta_4 & \Delta_7 & 0 \\ * & * & 2E_{ij} & 0 & 0 & 0 & 0 \\ * & * & * & 2F_{ij} & 0 & 0 & 0 \\ * & * & * & * & \Delta_5 & \Delta_8 & 0 \\ * & * & * & * & * & \Delta_9 & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{6}$$

where

$$\begin{aligned}
\Delta_1 &= A_i S + S A_i^T + BR + R^T B^T + 2 D_i D_i^T \\
&\quad + D_j D_j^T + \alpha_1 S - \xi_{ij} b_{ij} b_{ij}^T, \\
\Delta_2 &= \beta_{ij} I + E_i^T E_i + E_j^T E_j, \\
\Delta_3 &= \xi_{ij} b_{ij} |h_i|^T - \frac{1}{2} S H_i M_i, \\
\Delta_4 &= -\frac{1}{2} H_i M_i, \\
\Delta_5 &= \frac{1}{2} E_{ij} - \xi_{ij} |h_i| |h_i|^T, \\
\Delta_6 &= \xi_{ij} b_{ij} |h_j|^T, \\
\Delta_7 &= -\frac{1}{2} H_j M_j, \\
\Delta_8 &= -\xi_{ij} |h_j| |h_j|^T, \\
\Delta_9 &= \frac{1}{2} F_{ij} - \xi_{ij} |h_j| |h_j|^T.
\end{aligned}$$

Then the drive–driven system (1) *is globally asymptotically stable and the driven control law is given by*

$$u = Ke, \tag{7}$$

where

$$K = RS^{-1}. \tag{8}$$

Proof. Following the methodology borrowed in [8], let us construct a unique Lyapunov function $V(e) = e^T Pe$ for the PWL error synchronization system (3) where

$P \in \Re^{n\times n}$ is a symmetric positive definite matrix. Based on the Lyapunov stability theory [19] and for $e \neq 0$ and a given decay $\alpha_1 > 0$, the synchronization error (3) is globally asymptotically stable if

$$\dot{V}(e) \leq -\alpha_1 V(e). \tag{9}$$

For any small positive constants $0 < \alpha_2 \ll 1, 0 < \alpha_3 \ll 1$ we can also write [7]

$$\dot{V}(e) + \alpha_1 V(e) - \alpha_2 x^T x - \alpha_3 \leq 0. \tag{10}$$

Using the synchronization error dynamics (3) we can write

$$\begin{aligned}\dot{V}(e) + \alpha_1 V(e) = {} & e^T((A_i + BK)^T P \\ & + P(A_i + BK) + \alpha_1 P)e \\ & + e^T((D_i V_i E_i)^T P + P(D_i V_i E_i))e \\ & + b_{ij}^T Pe + e^T P b_{ij} + x^T A_{ij}^T Pe \\ & + e^T P A_{ij} x \\ & + x^T (D_i V_i E_i - D_j V_j E_j)^T Pe \\ & + e^T P(D_i V_i E_i - D_j V_j E_j)x \leq 0.\end{aligned} \tag{11}$$

Using Lemma 1, the following inequalities can be obtained:

$$\begin{aligned}& e^T E_i^T V_i^T D_i^T Pe + e^T P D_i V_i E_i e \\ & \quad \leq e^T P D_i D_i^T Pe + e^T E_i^T E_i e,\end{aligned} \tag{12a}$$

$$\begin{aligned}& x^T E_i^T V_i^T D_i^T Pe + e^T P D_i V_i E_i x \\ & \quad \leq e^T P D_i D_i^T Pe + x^T E_i^T E_i x,\end{aligned} \tag{12b}$$

$$\begin{aligned}& (-1)x^T E_j^T V_j^T D_j^T Pe + (-1)e^T P D_j V_j E_j x \\ & \quad \leq e^T P D_j D_j^T Pe + x^T E_j^T E_j x\end{aligned} \tag{12c}$$

and then the following inequality can be deduced using relations (11) and (12):

$$\begin{aligned}& e^T((A_i + BK)^T P + P(A_i + BK) + \alpha_1 P)e \\ & \quad + e^T P D_i D_i^T Pe + e^T E_i^T E_i e + b_{ij}^T Pe + e^T P b_{ij} \\ & \quad + x^T A_{ij}^T Pe + e^T P A_{ij} x + e^T P D_i D_i^T Pe \\ & \quad + x^T E_i^T E_i x + e^T P D_j D_j^T Pe \\ & \quad + x^T E_j^T E_j x \leq 0.\end{aligned} \tag{13}$$

Using relations (10) and (13), we can write the following inequality:

$$W^T F_0 W \leq 0, \tag{14}$$

where

$$w = [e^T \; x^T \; 1]^T$$

and

$$F_0 = \begin{bmatrix} (A_i+BK)^T P + P(A_i+BK) + \alpha_1 P + 2PD_i D_i^T P + PD_j D_j^T P + E_i^T E_i & * & * \\ A_{ij}^T P & -\alpha_2 I + E_i^T E_i + E_j^T E_j & * \\ b_{ij}^T P & 0 & -\alpha_3 \end{bmatrix} \qquad \square$$

In F_0, $*$ denotes the symmetric bloc and $I \in \Re^{n\times n}$ is the identity matrix.

Relying on polytopic expressions (2) of polyhedral cells and for all column vectors with positive elements $\delta_i \in \Re^{r_i \times 1}$, $\gamma_j \in \Re^{r_j \times 1}$ and all small positive constants satisfying $0 < \zeta_j \ll 1$ and $0 < \sigma_j \ll 1$, we can write

$$\delta_i^T H_i^T z + \delta_i^T h_i \leq 0 \rightarrow \delta_i^T H_i^T e + \delta_i^T H_i^T x + \delta_i^T h_i \leq 0$$

$$\gamma_j^T H_j^T x + \gamma_j^T h_j \leq -\xi_j x^T x - \sigma_j$$

which could be written as

$$W^T F_1 W \leq 0, \tag{15}$$

$$W^T F_2 W \leq 0, \tag{16}$$

where

$$F_1 = \begin{bmatrix} 0 & 0 & * \\ 0 & 0 & * \\ \delta_i^T H_i^T & \delta_i^T H_i^T & 2\delta_i^T h_i \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0 & 0 & * \\ 0 & 2\xi_j I & * \\ 0 & \gamma_j^T H_j^T & 2\gamma_j^T h_j + 2\sigma_j \end{bmatrix}.$$

If $\tau_{1,ij} \geq 0$ and $\tau_{2,ij} \geq 0$, using the S-procedure lemma for nonstrict inequalities [20], we can write using the inequalities (14), (15) and (16) that

$$F_0 - \tau_{1,ij} F_1 - \tau_{2,ij} F_2 < 0 \tag{17}$$

which can be written as

$$\begin{bmatrix} \phi_1 & * & * \\ A_{ij}^T P & \phi_2 & * \\ b_{ij}^T P - \tau_{1,ij}\delta_i^T H_i^T & -\tau_{1,ij}\delta_i^T H_i^T - \tau_{2,ij}\gamma_j^T H_j^T & \phi_3 \end{bmatrix} < 0$$

with

$$\phi_1 = (A_i + BK)^T P + P(A_i + BK) + \alpha_1 P + 2PD_i D_i^T P + PD_j D_j^T P + E_i^T E_i,$$

$$\phi_2 = -\alpha_2 I + E_i^T E_i + E_j^T E_j - 2\tau_{2,ij}\xi_j I,$$

$$\phi_3 = -\alpha_3 - 2\tau_{1,ij}\delta_i^T h_i - 2\tau_{2,ij}(\gamma_j^T h_j + \sigma_j).$$

Let

$$\beta_{1,ij} = -\tau_{1,ij}\delta_i^T, \quad \beta_{2,ij} = -\tau_{2,ij}\gamma_i^T,$$

$$\beta_{3,ij} = -\alpha_2 - 2\tau_{2,ij}\xi_j, \quad \beta_{4,ij} = -\alpha_3 - 2\tau_{2,ij}\sigma_j$$

such that

$$\underset{k=1,\ldots,r_i}{\beta_{1,ij}(k)} \le 0, \quad \underset{k=1,\ldots,r_j}{\beta_{2,ij}(k)} \le 0, \quad \beta_{3,ij} \le 0$$

and

$$\beta_{4,ij} \le 0.$$

The inequality (17) can be written as

$$\begin{bmatrix} \phi_1 & * & * \\ A_{ij}^T P & \beta_{3,ij} I + E_i^T E_i + E_j^T E_j & * \\ b_{ij}^T P + \beta_{1,ij}^T H_i^T & \beta_{1,ij}^T H_i^T + \beta_{2,ij}^T H_j^T & 2\beta_{1,ij}^T h_i + 2\beta_{2,ij}^T h_j + \beta_{4,ij} \end{bmatrix} < 0, \tag{18}$$

where

$$\phi_1 = (A_i + BK)^T P + P(A_i + BK) + \alpha_1 P + 2PD_i D_i^T P + PD_j D_j^T P + E_i^T E_i.$$

Let

$$\beta_{1,ij} = E_{ij}^{-1}|h_i|, \quad \beta_{2,ij} = F_{ij}^{-1}|h_j|,$$

$$h_i = M_i|h_i|, \quad h_j = M_j|h_j|$$

where $|h_i| \in \Re^{r_i \times 1}$ and $|h_j| \in \Re^{r_j \times 1}$ are two column vectors defined such that

$$\underset{k=1,\ldots,r_q}{|h_q|(k)} = \underset{k=1,\ldots,r_q}{|h_q(k)|}, \quad q = i, j$$

$E_{ij} \in \Re^{r_i \times r_i}$ and $F_{ij} \in \Re^{r_j \times r_j}$ are diagonal negative definite matrices and $M_i \in \Re^{r_i \times r_i}$ and $M_j \in \Re^{r_j \times r_j}$ are two diagonal matrices defined as follows:

If $\underset{k=1,\ldots,r_q}{h_q(k)} \ge 0$, then $M_q(k,k) = 1, q = i, j$.

If $\underset{k=1,\ldots,r_q}{h_q(k)} < 0$, then $M_q(k,k) = -1, q = i, j$.

Using the Schur complement [20], the bilinear matrix inequality (18) is satisfied if the following conditions are verified:

$$\beta_{4,ij} + 2|h_i|^T M_i E_{ij}^{-1}|h_i| + 2|h_j|^T M_j F_{ij}^{-1}|h_j| < 0 \tag{19}$$

$$\Lambda_1 - \Lambda_2 \Omega^{-1} \Lambda_2^T < 0, \tag{20}$$

where

$$\Lambda_1 = \begin{bmatrix} (A_i + BK)^T P + P(A_i + BK) + \alpha_1 P + 2PD_i D_i^T P + PD_j D_j^T P + E_i^T E_i & * \\ A_{ij}^T P & \beta_{3ij} I + E_i^T E_i + E_j^T E_j \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} Pb_{ij} + H_i E_{ij}^{-1}|h_i| \\ H_i E_{ij}^{-1}|h_i| + H_j F_{ij}^{-1}|h_j| \end{bmatrix}.$$

Let

$$\Omega = \beta_{4,ij} + 2|h_i|^T M_i E_{ij}^{-1}|h_i| + 2|h_j|^T M_j F_{ij}^{-1}|h_j|$$

$$\beta_{4,ij} = \xi_{ij}^{-1},$$

$$\beta_{3,ij} = \beta_{ij},$$

$$\Psi = \left[\, |h_i| \;\; |h_j| \,\right]^T.$$

We obtain from (20):

$$\Omega = \xi_{ij}^{-1} + \Psi^T \left(\frac{1}{2} \begin{bmatrix} M_i E_{ij} & 0 \\ 0 & M_j F_{ij} \end{bmatrix} \right)^{-1} \Psi < 0. \tag{21}$$

Multiplying (21) by ξ_{ij}^2, assuming that $E_{ij} \le M_i E_{ij} \le -E_{ij}$ and $F_{ij} \le M_j F_{ij} \le -F_{ij}$ and using the Schur complement, the LMI criterion (5) is obtained.

Using the matrix inversion lemma [21], from (21), we get

$$\Omega^{-1} = \xi_{ij} - \xi_{ij}^2 \Psi^T \left(\xi_{ij} \Psi \Psi^T + \frac{1}{2} \begin{bmatrix} M_i E_{ij} & 0 \\ 0 & M_j F_{ij} \end{bmatrix} \right)^{-1} \Psi. \tag{22}$$

Let $S = P^{-1}$. Multiplying left and right of expression (20) by $\begin{bmatrix} S & 0 \\ 0 & I \end{bmatrix}$ we get

$$\Lambda_3 - \Lambda_4 \Omega^{-1} \Lambda_4^T < 0, \tag{23}$$

where

$$\Lambda_3 = \begin{bmatrix} SA_i^T + R^T B^T + A_i S + BR + 2D_i D_i^T + D_j D_j^T + SE_i^T E_i S + \alpha_1 S & * \\ A_{ij}^T & \beta_{ij} I + E_i^T E_i + E_j^T E_j \end{bmatrix},$$

$$\Lambda_4 = \begin{bmatrix} b_{ij} + SH_i E_{ij}^{-1} |h_i| \\ H_i E_{ij}^{-1} |h_i| + H_j F_{ij}^{-1} |h_j| \end{bmatrix}.$$

Substituting (22) in (23), assuming that $E_{ij} \leq M_i E_{ij} \leq -E_{ij}$ and $F_{ij} \leq M_j F_{ij} \leq -F_{ij}$ and using the Schur complement, the following LMI is obtained via some transformations:

$$\begin{bmatrix} \Delta & A_{ij} & SH_j & 0 & \Delta_3 & \Delta_6 \\ * & \Delta_2 & H_i & H_i & \Delta_4 & \Delta_7 \\ * & * & 2E_{ij} & 0 & 0 & 0 \\ * & * & * & 2F_{ij} & 0 & 0 \\ * & * & * & * & \Delta_5 & \Delta_8 \\ * & * & * & * & * & \Delta_9 \end{bmatrix} < 0 \tag{24}$$

with

$$\Delta = SA_i^T + R^T B^T + A_i S + BR + 2D_i D_i^T + D_j D_j^T + SE_i^T E_i S + \alpha_1 S - \xi_{ij} b_{ij} b_{ij}^T.$$

Using Schur complement for (24), the LMI criterion (6) is then obtained.

4. Application

4.1 *The chaotic modified Chua's system*

Let us consider the chaotic modified Chua's system described by [22]

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - g(x_2)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 \end{cases} \tag{25a}$$

where

$$g(x_1) = bx_1 + \frac{1}{2}(a-b)\left(|x_1 + c| - |x_1 - c|\right) \tag{25b}$$

and $\beta_m < \beta < \beta_M$ is the norm-bounded uncertainty. This system can be written as the PWL system (1) with

$$A_1 = A_3 = \begin{pmatrix} -b\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -a\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} -\alpha(a-b)c \\ 0 \\ 0 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$b_3 = \begin{pmatrix} \alpha(a-b)c \\ 0 \\ 0 \end{pmatrix},$$

under the associate polytopic description (2) given by

$$H_1 = H_2 = H_3 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}^T, \quad h_1 = \begin{bmatrix} -d \\ c \end{bmatrix},$$

$$h_2 = \begin{bmatrix} -c \\ -c \end{bmatrix}, \quad h_3 = \begin{bmatrix} c \\ -d \end{bmatrix}$$

and where the norm-bounded uncertainty is described by the following matrices:

$$E_1 = E_2 = E_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D_1 = D_2 = D_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{(\beta_M - \beta_m)}{2} & 0 \end{bmatrix}$$

V_i and V_j are two scalars chosen in $[-1, 1]$ $\forall i,\ j \in \{1, 2, 3\}$.

For system (25) described in form (1) under the polytopic description (2), simulation results are conducted for the initial conditions $x_0 = [-1 \quad -0.5 \quad -0.5]^T$

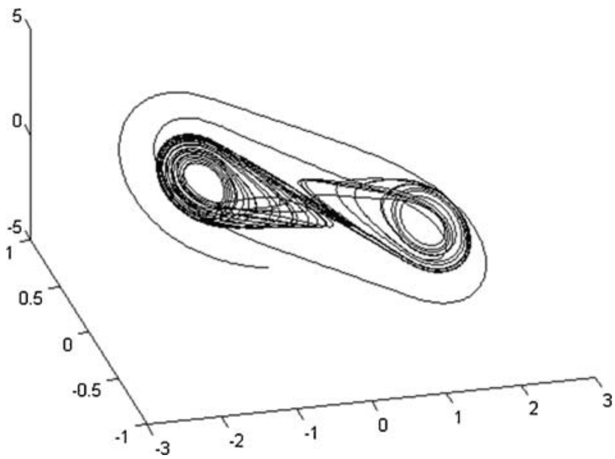

Figure 1. Chaotic dynamics of the Chua's modified system under norm-bounded uncertainty.

and $z_0 = [0 \;\; 0.7 \;\; -0.5]^T$ and, norm-bounded uncertainties designed by $V_j = 0.4$ and $V_i = 0.2$ where the system parameters are given by $\alpha = 9$, $\beta = 100/7$, $c = 1$, $a = -1/7$, $b = 2/7$, $\beta_m = 96/7$, $\beta_M = 106/7$ and $d = 5$. Figure 1 displays the chaotic attractor of the PWL model of the drive system with the norm-bounded uncertainty whereas the uncontrolled error signals of the drive–driven system are shown in figure 2. The LMIs (5) and (6) are solved using the LMI toolbox of MatLab software for the control matrix $B = [5 \times 10^3 \; 0 \; 0]^T$ and the parameter $\alpha_1 = 10^{-4}$. After five iterations, the LMI constraints were found feasible. The feasible solution is given by

$$R = (-0.0002 \quad -0.0030 \quad -0.0030),$$

$$S = \begin{pmatrix} 3.2244 & -0.8455 & 0.0648 \\ * & 1.3885 & 0.3745 \\ * & * & 19.5493 \end{pmatrix}.$$

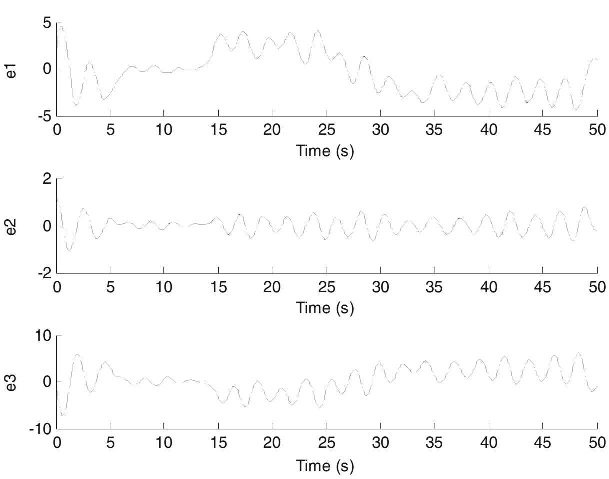

Figure 2. Uncontrolled error signals of the drive–driven modified Chua's system.

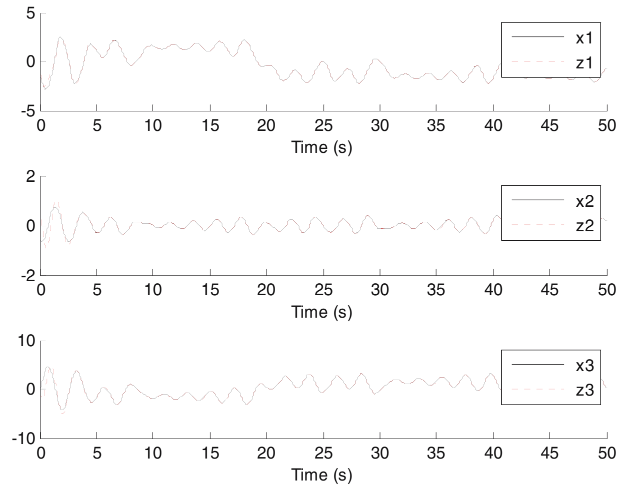

Figure 3. Synchronized state variables of the drive–driven modified Chua's system.

The control gain vector (8) is then deduced as

$$K = \left[\, -0.0007 \quad -0.0026 \quad 0.0001 \right].$$

Figure 3 shows the synchronized state variables of the PWL drive–driven system with norm-bounded uncertainties via the robust state feedback controller displayed in figure 4. Simulation results given in figure 5 prove that the robust chaos synchronization is well achieved. Finally, the switching dynamics of the drive and the driven systems with norm-bounded uncertainties between the polyhedral cells are shown in figures 6 and 7, respectively.

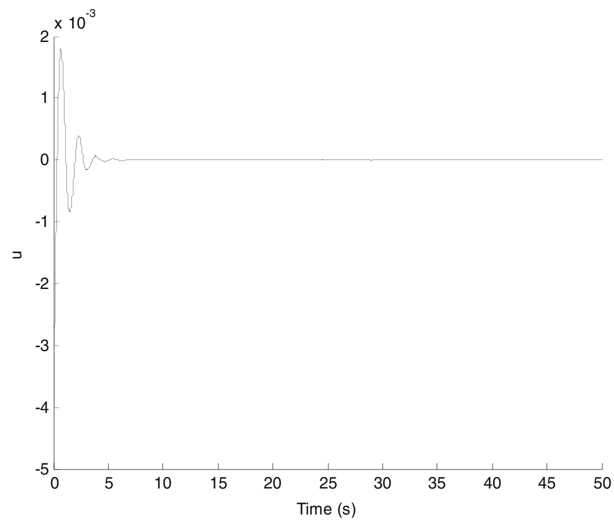

Figure 4. Robust control law of the modified Chua's system with norm-bounded uncertainties.

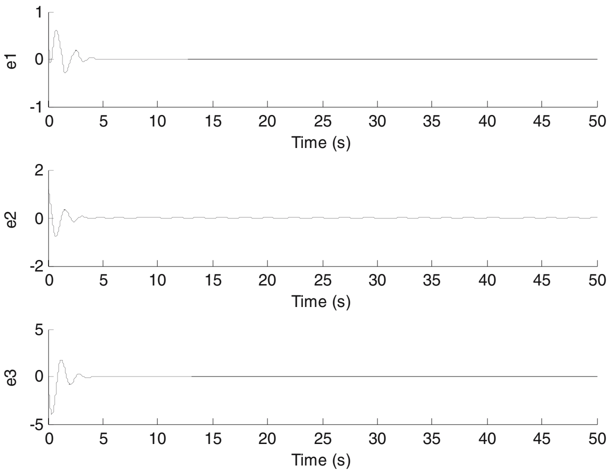

Figure 5. Synchronization errors via robust control of the modified Chua's model.

4.2 *New PWL hyperchaotic family*

Let us consider the new hyperchaotic family recently proposed in [7]. This system can be described as the PWL model (1) with $\forall i, j \in \{1, 2, 3, 4\}$:

$$A_i = A_j = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\alpha_{31} & -\alpha_{32} & -\alpha_{33} & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} 0 \\ 0 \\ 1.8 \\ 1.8 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0.9 \\ 0.9 \end{pmatrix},$$

$$b_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad b_4 = \begin{pmatrix} 0 \\ 0 \\ -0.9 \\ -0.9 \end{pmatrix},$$

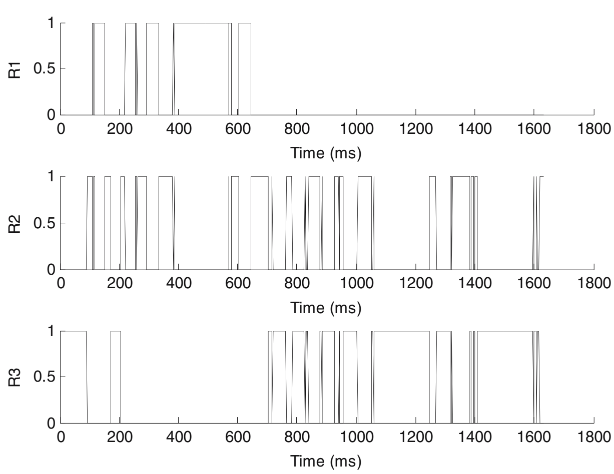

Figure 6. Evolution of the drive's states between polytopic cells of the modified Chua's system.

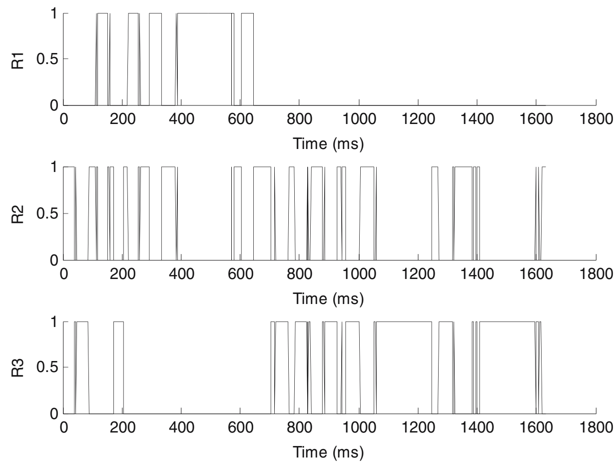

Figure 7. Evolution of the driven's states between polytopic cells of the modified Chua's system.

where $\alpha_{31m} < \alpha_{31} < \alpha_{31M}$ is the norm-bounded uncertainty and the associate polytopic description (2) is given by

$$H_1 = H_2 = H_3 = H_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$h_1 = \begin{bmatrix} -d \\ 0.9 \end{bmatrix}, \quad h_2 = \begin{bmatrix} -0.9 \\ 0.3 \end{bmatrix},$$

$$h_3 = \begin{bmatrix} -0.3 \\ -0.3 \end{bmatrix}, \quad h_4 = \begin{bmatrix} 0.3 \\ -d \end{bmatrix}$$

and where the norm-bounded uncertainty is described by the following matrices:

$$E_i = E_j = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_i = D_j = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\alpha_{31M} - \alpha_{31m}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

V_i and V_j are two scalars chosen in $[-1, 1]$, $\forall i, j \in \{1, 2, 3, 4\}$.

For the last hyperchaotic family described in form (1) under the polytopic description (2), simulation results are conducted for the initial conditions $x_0 = [-1 \ \ -0.5 \ \ -0.5 \ \ 1]^T$ and $z_0 = [0.1 \ \ 0.1 \ \ 0.1 \ \ 0.2]^T$ and scalars $V_j = 0.4$ and $V_i = 0.7$ with the parameters $\alpha_{31} = 1.5, \alpha_{32} = 1$, $\alpha_{33} = 1$, $\alpha_{31m} = 1.11$ and $\alpha_{31M} = 1.81$. Figure 8 shows the hyperchaotic attractor of the PWL drive system under the norm-bounded uncertainty whereas the uncontrolled error signals of the drive–driven system are shown in figure 9. The LMIs (5) and (6) are solved using the LMI

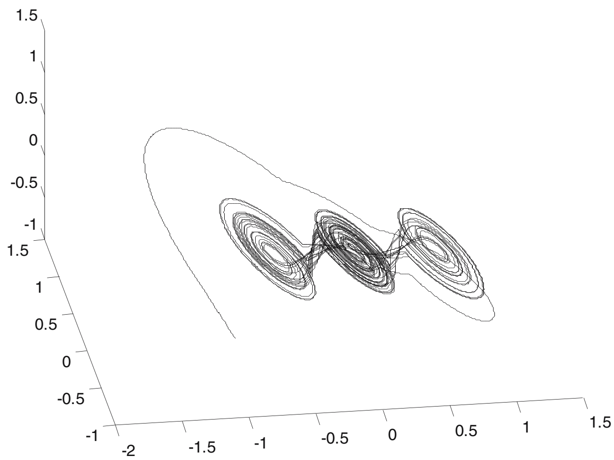

Figure 8. Dynamics of the hyperchaotic new family with norm-bounded uncertainty.

toolbox of MatLab software for the control matrix $B = [\,5\times 10^3 \quad 0 \quad 0\,]^T$ and the parameter $\alpha_1 = 10^{-4}$. The feasible solution is given by

$$R = 10^{-3}(\,-0.4583 \quad -0.7418 \quad 0.5042 \quad 0.3124\,)$$

$$S = \begin{pmatrix} 0.8555 & 0.0734 & 0.0209 & 0.0293 \\ * & 3.5701 & -1.1509 & -1.4825 \\ * & * & 3.0346 & 1.2820 \\ * & * & * & 3.5741 \end{pmatrix}.$$

The control gain vector (7) is then deduced as

$$K = 10^{-3}[-0.5234 \quad -0.1681 \quad 0.1140 \quad -0.0189].$$

Figure 10 shows the state variables of the drive–driven hyperchaotic system under the robust control law given in figure 11. Figure 12 shows the synchronization errors of the drive–driven hyperchaotic system and proves that robust chaos synchronization is well achieved. Finally, figures 13 and 14 respectively show the evolution of the drive and the driven's states between the polytopic cells.

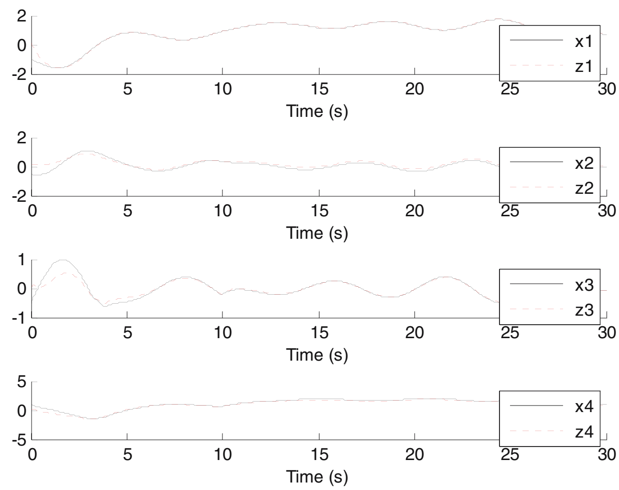

Figure 10. Synchronized state variables of the drive–driven hyperchaotic system.

4.3 *Comparative study*

In order to illustrate the robustness of the synchronization approach proposed in this paper, a comparative study is established with the synchronization approach designed in [8] by using the same example presented in the previous section and described in [7].

Two case studies are then considered for the uncertainties. The first case is performed for $V_j = 0.4$ and $V_i = 0.5$ whereas the second is carried out for the same uncertainties considered in the previous section such as $V_j = 0.4$ and $V_i = 0.7$.

As the uncertainties are not considered for the computation of the controller gain in [8], the feasible

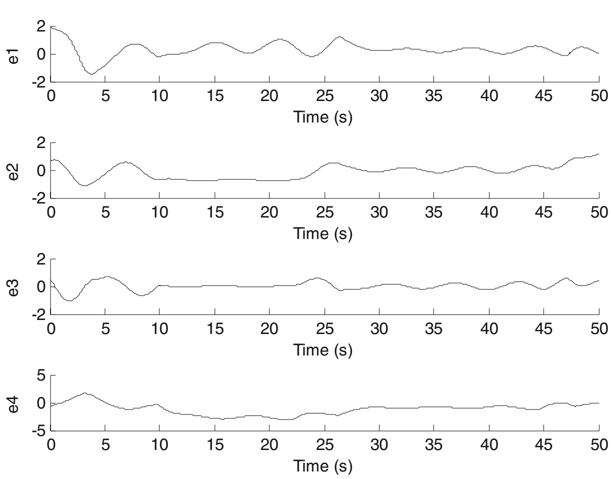

Figure 9. Uncontrolled error signals of the drive–driven hyperchaotic system.

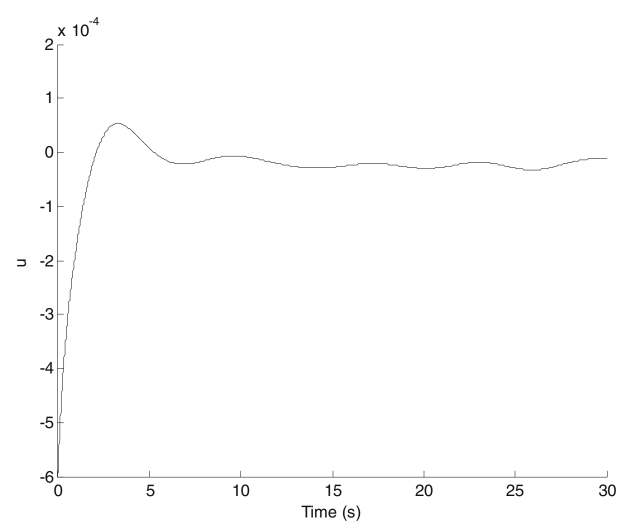

Figure 11. Robust control law of the hyperchaotic system.

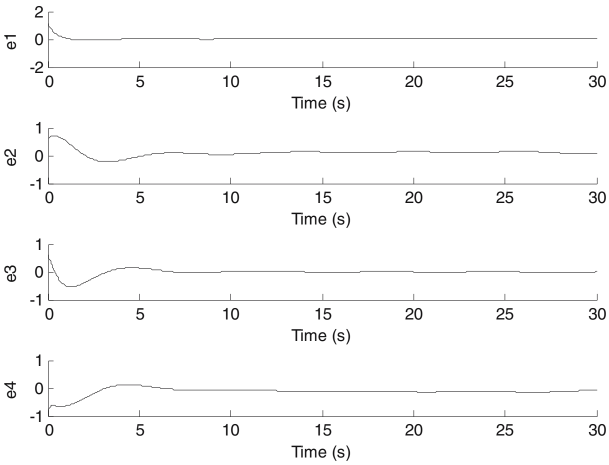

Figure 12. Synchronization errors via robust control for the hyperchaotic system.

solution gives the control gain $K = 10^{-3}[-0.3538\ -0.1241\ \ 0.1537\ \ -0.0199]$ for the two case studies.

For the first case study ($V_j = 0.4$ and $V_i = 0.5$), figure 15 shows synchronization errors using the two approaches. As can be seen, similar dynamics are observed which proves the robustness of the approach [8] for some tolerable uncertainties.

For the second case study ($V_j = 0.4$ and $V_i = 0.7$), stable error dynamics are achieved only when the synchronization approach proposed in this paper is used. Such simulation results are already given in the previous section. Indeed, when the controller designed using the approach in [8] is used, the dynamics of the synchronization errors become unstable. For such a case, the closed loop system does not belong anymore to the class of continuous PWL systems. As the control law no longer has any switching dynamics, simulation results are not delivered here because they are insignificant. This finding proves the nonrobustness of the approach [8] when uncertainties become so significant and confirms the superiority of the approach proposed in this paper compared to the approach given in [8].

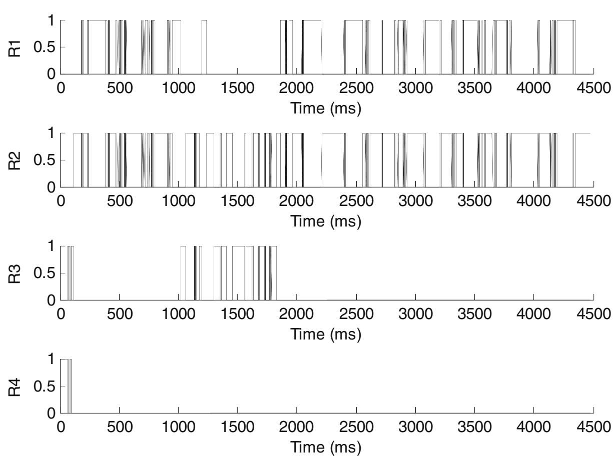

Figure 13. Evolution of the drive's states between polytopic cells of the hyperchaotic system.

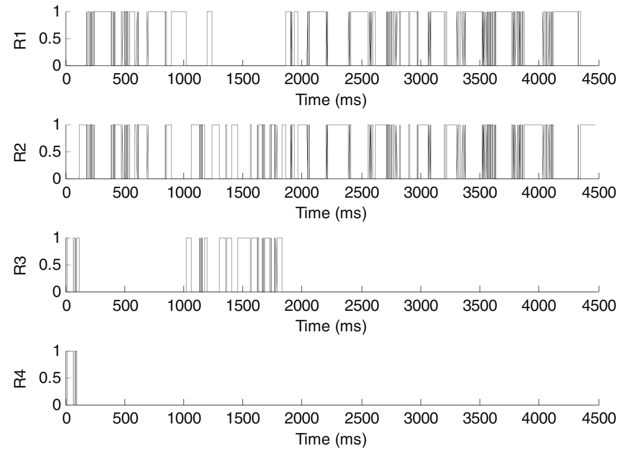

Figure 14. Evolution of the driven's states between polytopic cells of the hyperchaotic system.

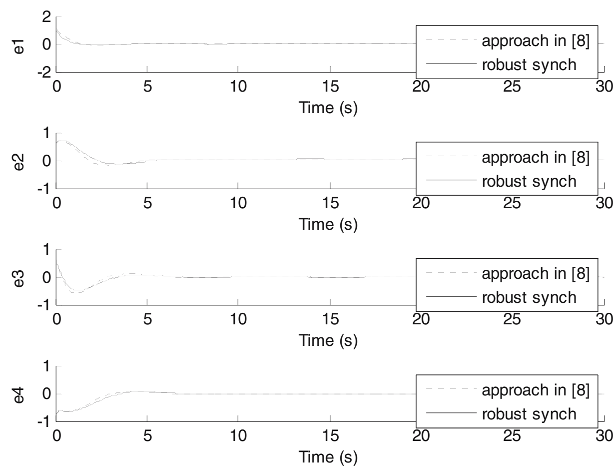

Figure 15. Synchronization errors of the hyperchaotic system for some low and tolerable uncertainties to the approach [8].

5. Conclusion

In this paper, a robust chaos synchronization approach is proposed for PWL chaotic systems with different norm-bounded uncertainties via a robust linear state feedback controller. The suggested synchronization criteria are developed using Lyapunov theory and LMIs tools. The efficiency of the proposed method was demonstrated on the most known chaos generator, the Chua's circuit, and on a multiscroll new hyperchaotic family. Finally, to prove the robustness and the superiority of the proposed approach, a comparative study was done using a related approach.

References

[1] A C J Luo, *Singularity and dynamics on discontinuous vector fields* (Elsevier, Amsterdam, 2006)
[2] M Johansson, *Piecewise linear control systems* (Springer Verlag, New York, 2003)
[3] L Rodrigues and J P How, *Int. J. Control.* **76(5)**, 459 (2003)
[4] H Mkaouar and O Boubaker, *Lecture Notes Comput. Sci.* **6752**, 199 (2011)
[5] M Hovd and S Olaru, *Automatica* **49(2)**, 667 (2013)
[6] J Lü, T Zhou, G Chen and X Yang, *Chaos* **12**, 344 (2002)
[7] L J Ontanon-Garcia, E Jimenez-Lopez, E Campos-Canton and M Basin, *Appl. Math. Comput.* **233**, 522 (2014)
[8] H Mkaouar and O Boubaker, *Commun. Nonlinear Sci. Numer. Simul.* **17(3)**, 1292 (2012)
[9] J M Muñoz-Pacheco, E Zambrano-Serrano, O Félix-Beltrán, L C Gómez-Pavón and A Luis-Ramos, *Nonlinear Dyn.* **70(2)**, 1633 (2012)
[10] T Zhang and G Feng, *IEEE Trans. Circuits Systems I* **54(8)**, 1852 (2007)
[11] F Chen and W Zhang, *Nonlinear Anal.: Theory, Methods Appl.* **67(12)**, 3384 (2007)
[12] Y Liu and S M Lee, *Circuits, Systems, and Signal Processing* **34(12)**, 3725 (2015)
[13] W K Wong, H Li and S Y S Leung, *Commun. Nonlinear Sci. Numer. Simul.* **17(12)**, 4877 (2012)
[14] X Shi and Z Wang, *Nonlinear Dyn.* **60(4)**, 631 (2010)
[15] H Lee, J H Park, S M Lee and O M Kwon, *Int. J. Control* **86(1)**, 107 (2013)
[16] M P Aghababa and M E Akbari, *Appl. Math. Comput.* **218(9)**, 5757 (2012)
[17] C F Huang, K H Cheng and J J Yan, *Commun. Nonlinear Sci. Numer. Simul.* **14(6)**, 2784 (2009)
[18] J C Shen, B S Chen and F C Kung, *IEEE Trans. Automat. Contr.* **AC-36(5)**, 638 (1991)
[19] J J E Slotine and W Li, *Applied nonlinear control* (Prentice Hall, Englewood, 1991)
[20] S P Boyd, L E Ghaoui, E Feron and V Balakrishnan, *Applied mathematics* (SIAM, Philadelphia, 1994)
[21] T Kailath, *Linear systems* (Prentice-Hall, Englewood, 1989)
[22] L O Chua, M Komuro and T Matsumoto, *IEEE Trans. Circuits Systems I* **33(11)**, 1072 (2000)