Pramana – J. Phys. (2016) 87: 47
DOI 10.1007/s12043-016-1235-5

Cylindrically symmetric cosmological model in the presence of bulk stress with varying Λ

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MS received 28 November 2014; revised 30 July 2015; accepted 16 December 2015; published online 6 September 2016

Abstract. Cylindrically symmetric non-static space–time is investigated in the presence of bulk stress given by Landau and Lifshitz. To get a solution, a supplementary condition between metric potentials is used. The viscosity coefficient of the bulk viscous fluid is assumed to be a power function of mass density whereas the coefficient of shear viscosity is considered as proportional to the scale of expansion in the model. Also some physical and geometrical properties of the model are discussed.

Keywords. Cylindrically symmetric space–time; viscous fluid; variable cosmological constant.

PACS Nos 98.80.Es; 04.20.jb; 04.20.−q

1. Introduction

In Einstein's theory of gravity, the Newtonian gravitational constant G and the cosmological term Λ are considered as fundamental constants. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry of space and matter in Einstein's field equations. In an evolving Universe, it is natural to take this constant as a function of time. The generalized Einstein's theory of gravitation with time-dependent G and Λ has been proposed by Lau [1]. The possibility of variables G and Λ in Einstein's theory has also been studied by Dersarkissian [2]. The cosmological model with variables G and Λ have been studied recently by several researchers. Some of the recent discussions on the cosmological constant and consequence on cosmology with a time-varying cosmological constant have been discussed in refs [3–11]. Also, the role of viscosity is important in cosmology for a number of reasons. Weinberger [12], Heller and Klimek [13], Misner [14,15] have studied the effect of viscosity on the evolution of cosmological models. Collins and Stewart [16] have studied the effect of viscosity on the formation of galaxies. Xing-Xiang Wang [17] discussed Kantowski–Sachs string cosmological model with bulk viscosity in general relativity. Also, several aspects of viscous fluid cosmological model in early Universe have been extensively investigated by many researchers. Raj Bali and Dave [18] have studied Bianchi Type-III string cosmological models with time-dependent bulk viscosity. Bianchi Type-V string cosmological model with bulk viscous fluid in general relativity has been presented by Adhav *et al* [19].

Recently, Verma [20] studied spatially homogeneous bulk viscous fluid models with time-dependent gravitational constant and cosmological term. Kantowski–Sachs cosmological model in the presence of massless scalar field with flat potential has been investigated in [21,22]. Mete *et al* [23] have studied Kantowski–Sachs bulk viscous fluid cosmological model with time-dependent Λ-term.

In this paper, we discuss cylindrically symmetric non-static cosmological model with varying Λ, in the presence of bulk stress given by Landau and Lifshitz. This paper is organized as follows:

In §2 we have derived the field equations, while §3 deals with the solution of field equations in the presence of viscous fluid. Some particular cases are discussed in §4. Section 5 is mainly concerned with the physical and kinematical properties of the model and the last section contains concluding remarks.

2. The metric and field equations

We consider the cylindrically symmetric space–time given by Bhattacharya and Karade [24] in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(t)\left[\mathrm{d}\chi^2 + f^2(\chi)\mathrm{d}\phi^2\right] - B^2(t)\mathrm{d}z^2, \quad (1)$$

with the convention $x^1 = \chi, x^2 = \phi, x^3 = z, x^4 = t$, A and B are functions of the proper time t alone, while f is a function of coordinate χ alone.

The Einstein's field equations (in gravitational units $C = 1, G = 1$) read as

$$R_i^j - \frac{1}{2}R\delta_i^j + \Lambda g_i^j = -8\pi T_i^j, \quad (2)$$

where R_i^j is the Ricci tensor $R = g^{ij}R_{ij}$ is the Ricci scalar and T_i^j is the stress energy-tensor in the presence of bulk stress given by Landau and Lifshitz.

$$\begin{aligned} T_i^j &= (\rho + p)\, v_i v^j - p g_i^j \\ &\quad -\eta\left(v_{i;}^j + v_{;i}^j + v^j v^l v_{i;l} + v_i v^l v_{;l}^j\right) \\ &\quad -\left(\xi - \frac{2}{3}\eta\right)\theta\left(g_i^j + v_i v^j\right). \end{aligned} \quad (3)$$

Here ρ, p, η and ξ are the energy density, isotropic pressure, coefficient of shear velocity and bulk viscous coefficient, respectively and v_i is the flow vector satisfying the relations

$$g_{ij}v^i v^j = 1. \quad (4)$$

The semicolon (;) indicates covariant differentiation. We choose the coordinates to be comoving, so that

$$v^1 = 0 = v^2 = v^3, \qquad v^4 = 1. \quad (5)$$

The Einstein's field equations (2) for the line element (1) has been set up as

$$\begin{aligned} &\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \Lambda \\ &\quad = 8\pi\left[-p - 2\eta\frac{A_4}{A} - \left(\xi - \frac{2}{3}\eta\right)\theta\right], \end{aligned} \quad (6)$$

$$\begin{aligned} &2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{f_{11}}{A^2 f} - \Lambda \\ &\quad = 8\pi\left[-p - 2\eta\frac{B_4}{B} - \left(\xi - \frac{2}{3}\eta\right)\theta\right], \end{aligned} \quad (7)$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 B_4}{AB} - \frac{f_{11}}{A^2 f} - \Lambda = 8\pi\rho, \quad (8)$$

where suffix 4 at the symbols A and B denotes ordinary differentiation with respect to t and θ is the shear expansion given by

$$\theta = v_{;i}^j. \quad (9)$$

The functional dependence of the metric together with eqs (7) and (8) imply

$$\frac{f_{11}}{f} = k^2, \qquad k^2 = \text{constant}. \quad (10)$$

If $k = 0$, then $f(\chi) = \text{constant}\ \chi, 0 < \chi < \infty$.

This constant can be made equal to 1 by suitably choosing units for ϕ. Thus we shall have

$$f(\chi) = \chi$$

resulting in the flat model of the Universe as shown by Hawking and Ellis [25].

3. Solution of the field equations

Using eq. (10), the set of eqs (6) to (8) reduces to

$$\begin{aligned} &\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \Lambda \\ &\quad = 8\pi\left[-p - 2\eta\frac{A_4}{A} - \left(\xi - \frac{2}{3}\eta\right)\theta\right], \end{aligned} \quad (11)$$

$$\begin{aligned} &2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \Lambda \\ &\quad = 8\pi\left[-p - 2\eta\frac{B_4}{B} - \left(\xi - \frac{2}{3}\eta\right)\theta\right], \end{aligned} \quad (12)$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 B_4}{AB} - \Lambda = 8\pi\rho. \quad (13)$$

Equations (11)–(13) are three independent equations in seven unknowns A, B, ρ, p, η, ξ and Λ. For the complete determinancy of the system, we need four extra conditions. Firstly, we assume a relation in metric potential as

$$A = B^m \quad (14)$$

and secondly, we assume that the coefficient of shear viscosity is proportional to the scale expansion, i.e.,

$$\eta \propto \theta, \quad (15)$$

where m is a real number.

Equations (11) and (12) lead to

$$\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{B_{44}}{B} - \frac{A_4 B_4}{AB} = 16\pi\eta\left(\frac{A_4}{A} - \frac{B_4}{B}\right). \quad (16)$$

Condition (15) leads to

$$\eta = l\left(2\frac{A_4}{A} + \frac{B_4}{B}\right), \quad (17)$$

where l is the proportionality constant.

Equation (16) together with eqs (14) and (17) lead to

$$BB_{44} + \alpha B_4^2 = 0 \tag{18}$$

which can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}B}(f^2) + \frac{2\alpha}{B}(f^2) = 0, \tag{19}$$

where

$$\alpha = 2m - 16\pi l(2m+1) \tag{20}$$

and

$$B_4 = f(B). \tag{21}$$

From (19), we obtain

$$\left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)^2 = \frac{\beta}{B^{2\alpha}}. \tag{22}$$

Integrating eq. (22), we can easily obtain the metric functions

$$B = [(\alpha+1)(k_1 t + k_2)]^{1/(\alpha+1)}$$

and

$$A = [(\alpha+1)(k_1 t + k_2)]^{m/(\alpha+1)}, \tag{23}$$

where k_1 is the expression term of the integrating constant β and k_2 is a constant.

The metric (1) takes the form

$$\begin{aligned} \mathrm{d}s^2 = {} & \mathrm{d}t^2 - [(\alpha+1)(k_1 t + k_2)]^{2m/(\alpha+1)} \\ & \times\left[\mathrm{d}\chi^2 + f^2(\chi)\mathrm{d}\phi^2\right] \\ & - [(\alpha+1)(k_1 t + k_2)]^{2/(\alpha+1)}\,\mathrm{d}z^2. \end{aligned}$$

After a suitable transformation of coordinates, the above metric reduces to the form

$$\mathrm{d}s^2 = \left[\frac{\beta}{T^{2\alpha}}\right]^{-1}\mathrm{d}T^2 - T^{2m}\left[\mathrm{d}\chi^2 + f^2(\chi)\mathrm{d}\phi^2\right] - T^2\mathrm{d}z^2, \tag{24}$$

where $B = T$.

The pressure and density for the model (24) are given by

$$\begin{aligned} 8\pi p = {} & \frac{[-m^2(32\pi l + 3) + (m+1)(16\pi l + 3\alpha)]\beta}{3T^{2(\alpha+1)}} \\ & - \frac{8\pi\xi(2m+1)\sqrt{\beta}}{T^{(\alpha+1)}} + \Lambda \end{aligned} \tag{25}$$

and

$$8\pi\rho = \frac{m(m+2)\beta}{T^{2(\alpha+1)}} - \Lambda. \tag{26}$$

For the specification of ξ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{27}$$

where $\gamma(0 \le \gamma \le 1)$ is a constant.

Thus, given $\xi(t)$, we can solve the cosmological parameters. In most of the investigations involving bulk viscosity, it is assumed to be a simple power function of the energy density [26–28]

$$\xi(t) = \xi_0\rho^n, \tag{28}$$

where ξ_0 and n are constants. If $n = 1$ eq. (28) may correspond to a relative fluid [29]. However, more realistic models [30] are based on n lying in the regime $0 \le n \le \frac{1}{2}$. Using (28) in (25), we obtain

$$\begin{aligned} 8\pi p = {} & \frac{[-m^2(32\pi l + 3) + (m+1)(16\pi l + 3\alpha)]\beta}{3T^{2(\alpha+1)}} \\ & - \frac{8\pi\xi_0\rho^n(2m+1)\sqrt{\beta}}{T^{(\alpha+1)}} + \Lambda. \end{aligned} \tag{29}$$

4. Particular cases

Case I. Solution for $\xi(t) = \xi_0$

When $n = 0$, eq. (28) reduces to $\xi(t) = \xi_0 =$ constant. Hence, in this case, eq. (29), with the use of (26) and (27), leads to

$$\begin{aligned} & 8\pi\rho(1+\gamma) \\ & = \frac{[-m^2(32\pi l + 3) + (m+1)(16\pi l + 3\alpha) + 3m(m+2)]\beta}{3T^{2(\alpha+1)}} \\ & \quad - \frac{8\pi\xi_0\,(2m+1)\,\sqrt{\beta}}{T^{(\alpha+1)}}. \end{aligned} \tag{30}$$

Eliminating $\rho(t)$ between (26) and (30), we have

$$\begin{aligned} & (1+\gamma)\Lambda \\ & = \frac{[m^2(32\pi l + 3) - (m+1)(16\pi l + 3\alpha) + 3\gamma m(m+2)]\beta}{3T^{2(\alpha+1)}} \\ & \quad + \frac{8\pi\xi_0\,(2m+1)\,\sqrt{\beta}}{T^{(\alpha+1)}}. \end{aligned} \tag{31}$$

Case II. Solution for $\xi(t) = \xi_0\rho$

When $n = 1$, eq. (27) reduces to $\xi(t) = \xi_0\rho$. Hence, in this case, eq. (29), with the use of (26) and (27), leads to

$$8\pi p = \left[\frac{1}{1+\gamma+\frac{\xi_0(2m+1)\sqrt{\beta}}{T^{(\alpha+1)}}}\right] \times \frac{[-m^2(32\pi l+3)+(m+1)(16\pi l+3\alpha)+3m(m+2)]\beta}{3T^{2(\alpha+1)}}. \tag{32}$$

Eliminating $\rho(t)$ between eqs (26) and (32), we have

$$\Lambda = \left[\frac{1}{1+\gamma+\frac{\xi_0(2m+1)\sqrt{\beta}}{T^{(\alpha+1)}}}\right] \times \frac{[m^2(32\pi l+3)-(m+1)(16\pi l+3\alpha)-3m(m+2)]\beta}{3T^{2(\alpha+1)}} + \frac{m(m+2)\beta}{T^{2(\alpha+1)}}. \tag{33}$$

From eq. (33), we observe that if $\alpha > 0, m > 0$ and $\alpha < 0, m < 0$, positive cosmological constant is a decreasing function of time and approaches a small value in the present epoch but when $\alpha < 0, m < 0$ and $\alpha < 0, m > 0$, the cosmological term $-\Lambda$ remains constant.

5. Some physical and kinematical properties

In this section, we discuss some physical and kinematical properties of the velocity vector v^i of the metric (24). The spatial volume (V), the scalar expansion (θ), the shear scalar (σ) and deceleration parameter (q) of the fluid are given by

$$V = \sqrt{-g} = T^{2m+1}, \tag{34}$$

$$\theta = \frac{(2m+1)\sqrt{\beta}}{T^{(\alpha+1)}}, \tag{35}$$

$$\sigma = \sqrt{\frac{7}{18}}\frac{(2m+1)\sqrt{\beta}}{T^{(\alpha+1)}} \tag{36}$$

and

$$q = \frac{2+3\alpha-2m}{1+2m}. \tag{37}$$

For $\alpha > 0$, the expansion factor θ is a decreasing function of T and approaches, asymptotically to zero with ρ and also approaching zero as $T \to \infty$.

6. Conclusions

We have considered a cylindrically symmetric non-static cosmological model in the presence of bulk stress given by Landau and Lifshitz. To solve the field equations, we have assumed the relation between metric coefficient and the viscosity coefficient of bulk viscous fluid. Here the viscosity coefficient of the bulk viscous fluid is assumed to be the power function of mass density, whereas coefficient of the shear viscosity is considered to be proportional to the scale of expansion. The model is expanding, shearing and non-rotating in the standard way.

Acknowledgements

The authors are thankful to the referee for the valuable suggestion for improving this paper to its present form. One of the author (V D Elkar) is grateful to the University Grants Commission, New Delhi, India for providing fellowship under F.I.P.

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