PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 86, No. 5
May 2016
pp. 1031–1042

A nonlinear approach to analyse the development of tropical disturbances

A JEEVAREKHA and P PHILOMINATHAN*

PG and Research Department of Physics, A.V.V.M. Sri Pushpam College, Poondi, Thanjavur 613 503, India.

*Corresponding author. E-mail: philominathan@gmail.com

MS received 6 January 2015; accepted 11 May 2015

DOI: 10.1007/s12043-015-1131-4; ***e*Publication:** 25 November 2015

Abstract. The development of atmospheric disturbances in the tropical region is explained using vibrational resonance, a nonlinear phenomenon. As the Lorenz system is the most plausible model to describe the convective process in a tropical region, the influence of vertical wind shear and tropical waves on the system leading to tropical cyclone has been investigated. The response of the convective region towards vertical wind shear and tropical waves is numerically studied. It was found that the response of the convective system decreases with the absence of any of these environmental factors. The dynamics of the system including resonance phenomenon is studied using phase portraits and Lyapunov dimension. Further, Lyapunov dimension is employed here to characterize the occurrence of resonant peaks.

Keywords. Vibrational resonance; Lorenz equation; tropical disturbances; convection; tropical waves; vertical wind shear.

PACS Nos 05.45.−a; 47.52.+j

1. Introduction

Two complexities involved in weather forecasting are: (i) the necessity of precise measurement of the present state of the atmosphere in order to anticipate the future and (ii) the nonlinearity endowed by the atmospheric and oceanographic systems due to the aperiodic flow, a specific nature of geophysical fluids (GPFs). Since Lorenz [1] found the relevance of sensitive dependence of hydrodynamical systems on initial conditions (SDIC), plenty of investigations have been extended to study chaos, due to its possible applicability in different areas including engineering [2,3], economics [4,5], biology [6,7], atmospheric sciences [1,8] and so on.

GPFs play vital roles in preserving energy balance of the planet via thermohaline circulation and cyclone [9]. The irregularity of GPFs can be seen in anomalies like cloud formation, cyclone development, swirling eddies in oceans and apparently these abnormal activities form an integral part while examining the kinetics of atmospheric and oceanographic systems. Visualizing these irregularities using dynamical equations is a bit difficult as it requires precise measurements of innumerable variables like pressure, viscosity, temperature and so on. With the advent of several numerical and simulation techniques, the complexity in explaining processes like cloud formation and cyclone development has been reduced considerably. In recent years, many researchers have projected successful climate models [10,11], cloud formation theories [12,13] and cyclone development models [14,15].

It is necessary to understand the development of atmospheric disturbances, as it may eventually lead to the formation of storm or cyclones. Though the ways and means of occurrence or the development of atmospheric disturbances are intertwined with a number of parameters, an overall trend leading to the development of tropical disturbances has been considered in this investigation and elucidated: The formation of convective layers with different thicknesses is quite natural due to the differential heating of the Earth's surface and the horizontal temperature gradient of these layers creates pressure difference, which in turn, results in the formation of vertical wind shear. The sheared convective region, on interaction with the tropical waves, initiates abrupt disturbances such as formation of thunderstorms, cumulus cloud development, hurricanes, tropical cyclones etc. In the present study, we wish to implement the concept of resonance in a convective system to perceive the sequence of abrupt changes or consequences like a hike or lowering of temperature, energy, pressure and so on. The interaction of convective system with various components constituting atmospheric circulation like vertical wind shear and tropical waves is studied with the help of the phenomenon called vibrational resonance. Also, we discuss how the enhancement in the response of the convective system causes various environmental factors leading to the development of tropical disturbances.

There are a few interesting modes in which resonance is found to occur in nonlinear systems and some of them are stochastic resonance (SR) [16], chaotic resonance (CR) [17] and vibrational resonance (VR) [18]. Among them, VR has received a great deal of attention, in the recent past, due to its applicability in various fields like engineering [19], medicine [20], atmospheric science [21] etc. The nonmonotonic dependence of the response of the system (Q) (which is under the influence of weak periodic forcing) on the amplitude and frequency of the high-frequency external signal (vibration) can be viewed as peak and the phenomenon is termed as vibrational resonance [18]. This phenomenon has been observed in a wide class of systems with monostable [22], bistable [18] potentials, excitable systems [23], discrete systems [24] and it was analysed experimentally, theoretically and numerically [25]. In the present investigation, vertical wind shear and tropical waves are considered as low-frequency ($f \cos \omega t$) and high-frequency ($g \cos \Omega t$) periodic components respectively. We discuss the role of tropical waves in enhancing the response of convective system using numerical studies.

The outline of the article is as follows: Section 2 focusses on the modelling aspects of the convective region. The role of vertical wind shear and tropical waves in the convective process is analysed and the numerical results are presented. Section 3 is concerned with the role of VR phenomenon in understanding the dynamical behaviour of the convective system, in the presence and absence of tropical waves. The enhancement in the response of the convective system is explained using amplitude–response and frequency–response studies. In §4, the effect of resonance in the dynamics of the chosen system is characterized with the help of Lyapunov exponents, Lyapunov dimension and phase portraits. Section 5 gives some concluding remarks.

2. Lorenz 63 model with biharmonic components

2.1 *System description*

In the process of identifying the most suited and probable models that can describe various activities in atmospheric scenario leading to tropical disturbances, including tropical cyclone, the Lorenz 63 system is found to be appropriate to study finite amplitude convection. Some of the properties of Lorenz system such as boundednesss, SDIC and chaos make it a convenient choice to describe real atmosphere. The nonlinear dissipative Lorenz equations are

$$\begin{aligned} k_1 &= \dot{x} = \sigma(y(t) - x(t)), \\ k_2 &= \dot{y} = rx(t) - y(t) - x(t)z(t), \\ k_3 &= \dot{z} = x(t)y(t) - bz(t). \end{aligned} \tag{1}$$

The description of variables used in eq. (1) is similar to the equation given in [1]. As Prandtl number (σ) and the geometric factor (b) play crucial roles in describing the atmospheric convection system, they are chosen to be 10 and 8/3. Among the three equilibrium points of eq. (1), $(0, 0, 0)^T$ represents stable nonconvective state of the system when $r < 1$, while the system oscillates around two stable equilibrium points $(\sqrt{b(r-1)}, \sqrt{b(r-1)}, r-1)^T$ and $(-\sqrt{b(r-1)}, -\sqrt{b(r-1)}, r-1)^T$ when $r < r_c$, where r_c is the critical value given by

$$r_c = \sigma \frac{(\sigma + b + 3)}{(\sigma - b - 1)}.$$

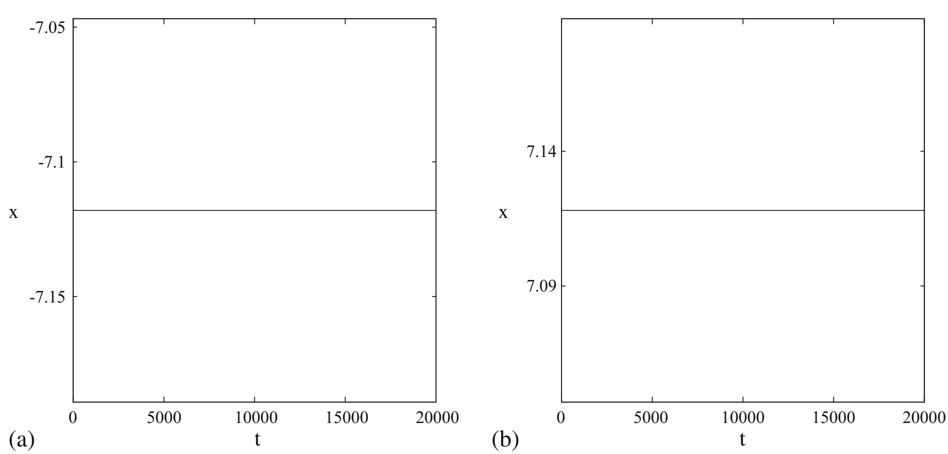

Figure 1. Bistability of Lorenz equation for two sets of initial conditions with the parameters $\sigma = 10$, $b = 8/3$ and $r = 20$. The equilibrium point settles down to (**a**) $x = -7.118$ for $(-10.0, -0.1, 0.0)^T$, (**b**) $x = 7.118$ for $(10.0, 0.1, 0.0)^T$.

When $r > r_c$, all the equilibrium points become unstable due to the nonperiodic evolution of convective system. Hence, the bistable nature of the Lorenz system (figure 1) when $r < r_c$, is considered in the present investigation and the Rayleigh number is fixed at 20.0. Now eq. (1) mimics an organized convection scenario which is a prerequisite to ascribe the development of tropical disturbances.

2.2 *Effect of vertical wind shear on convective region*

With $r = 20$, eq. (1) describes the convective process where fluid layers of different thicknesses are formed due to the temperature variations in ascending and descending fluid currents. Thus, the horizontal temperature gradient causes pressure variation in vertical direction and as a result, the magnitude and direction of the wind changes with height; such changes are the reasons for the formation of vertical wind shear. Chen *et al* [26] considered vertical wind shear as the difference between the 200–800 hPa winds in an annular region between 200 and 800 km radius from cyclone centre at 12 h time intervals for their study. In the present work, we symbolize the change in magnitude as well as direction of the wind by a periodic function $f\cos(\omega t)$, where f is the magnitude of the mean wind speed and ω is the frequency at which either the magnitude or the direction of the wind changes with height in time t. Here time is expressed as the number of hours (or days) over which it is measured. Let us consider ω to be low in order to study the effect of low vertical wind shear.

Thus, the vertical wind shear, which is the consequence of convective instability, plays a crucial role in changing the dynamics of the convective system and is considered as weak external periodic forcing. The Lorenz system (eq. (1)), after the inclusion of vertical wind shear component becomes

$$\begin{aligned} k_1' &= k_1, \\ k_2' &= k_2 + f\cos(\omega t), \\ k_3' &= k_3. \end{aligned} \tag{2}$$

Equation (2) represents the convective region with low vertical wind shear. As the horizontal temperature gradient of the convective fluid layers is responsible for the development of low vertical wind shear, the periodic term representing vertical wind shear is added with k_2. The effect of vertical wind shear on the convective system can be realized with the help of phase portraits (figure 2). Figure 2 depicts the phase portrait of the system (eq. (2)) for $f = 0.25, 0.5, 0.75$. From the figure, it can be observed that by increasing f from 0.25 to 0.75, the amplitude with which the system oscillates also increases. This supports the fact that strongly sheared environment affects the temperature variation between ascending and descending fluid currents and also the intensity of convection [27]. However, the complexity of the system remains unchanged with increase in f and this can be verified by calculating Lyapunov exponents [28]. For all f, the Lyapunov exponents are $\lambda_1 = -0.2247$, $\lambda_1 = -0.230$ and $\lambda_1 = -0.1943$. Negative Lyapunov exponent values for all f is also an indication of nonoccurrence of nonlinear resonant phenomenon. Thus, it can be inferred that with low vertical wind shear alone, no atmospheric disturbances can be observed.

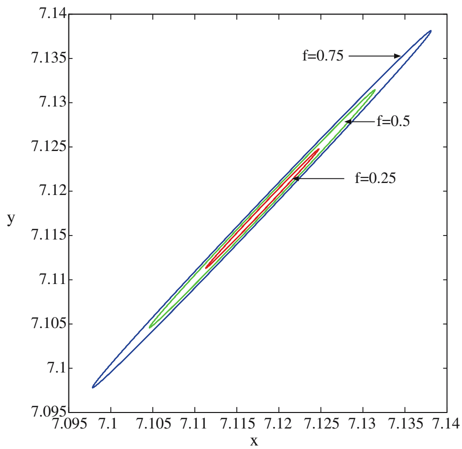

Figure 2. Phase portrait of eq. (2) for $f = 0.25, 0.5, 0.75$ with $\omega = 0.5$. The rest of the parameters are fixed at $\sigma = 10$, $b = 8/3$ and $r = 20$.

2.3 *Effect of tropical waves on low sheared convective region*

It is quite common and well known from the studies of tropical climate that frequent flow of tropical easterly waves in the sheared convective region enhances the possibility of the development of tropical disturbances and thunderstorms. Many investigators [29–32] framed tropical waves as high-frequency periodic forcing and hence we wish to conceptualize tropical waves as a rapidly varying periodic component. By including second periodic forcing term in (2), one gets

$$\begin{aligned} k_1' &= k_1, \\ k_2' &= k_2 + f\cos(\omega t) + g\cos(\Omega t), \\ k_3' &= k_3. \end{aligned} \tag{3}$$

Here, $f\cos(\omega t)$ and $g\cos(\Omega t)$ are considered as general representations of vertical wind shear and tropical waves, where $g\cos(\Omega t)$ symbolizes tropical waves with Ω as the frequency of occurrence of these waves in the convective region and g indicates the wind speed. The value of Ω should be very much greater than that of ω to explain VR. The equilibrium points and stability of the system (eq. (3)) remain unchanged even with the inclusion of biharmonic components. The effect of inclusion of a high-frequency component in eq. (3) and its consequences in the form of VR is studied in the following sections.

3. Vibrational resonance

To characterize VR, the response amplitude of system (3) to the low-frequency component has to be calculated. The response of the convective system under the action of these periodic components is computed using the following formula given by Landa and McClintock [18]:

$$Q = \frac{\sqrt{(Q_s^2 + Q_c^2)}}{f}, \tag{4}$$

where Q_s and Q_c are the sine and cosine components extracted from the numerical solution of (3) (that is by solving (3) using RK4 method) and they are given by

$$Q_s = \frac{2}{NT}\int_0^{NT} x(t)\sin(\omega t)\mathrm{d}t, \tag{5}$$

$$Q_c = \frac{2}{NT}\int_0^{NT} x(t)\cos(\omega t)\mathrm{d}t, \tag{6}$$

where $T = 2\pi/\omega$ is the period of response and N is an integer taken as 200.

3.1 *Interaction of the convective region with vertical wind shear alone*

In order to understand the role of vertical wind shear, we numerically computed the response of eq. (2), in the absence of the tropical wave component. The Q vs. f response (as in figure 3) shows that there exists a steady fall in the response amplitude (Q) with no significant peak. Similar response is found in Q vs. ω plot also. Thus, one could conclude that the susceptibility of the system to external periodic forcing decreases for increasing shear. Further, it is quite clear that with vertical wind shear alone, the chance for the formation of the TC is minimal, despite a few changes in the convective system.

3.2 *Resonant interaction under the action of biharmonic components*

The effect of tropical waves in enhancing the response of the convective system towards vertical wind shear is illustrated in figure 4. The nonmonotonic variation of response amplitude (Q) with respect to Ω is shown in figure 4 and this phenomenon is VR. The amplitude (f) and frequency (ω) of the vertical wind shear component is considered to be small (0.4 and 0.5 respectively), in order to study the nature of low vertical wind

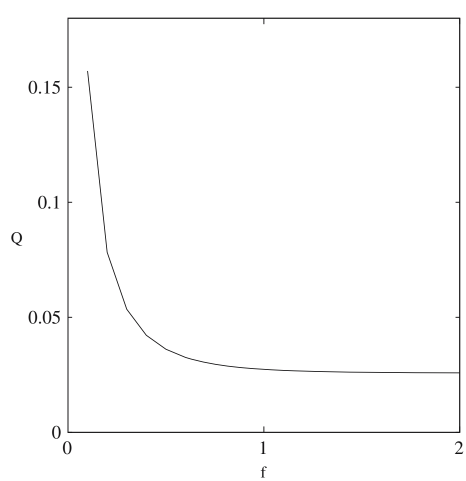

Figure 3. Q vs. f in the absence of $g\cos(\Omega t)$ with $\omega = 0.5$. The rest of the parameters in eq. (2) are kept the same as in figure 2.

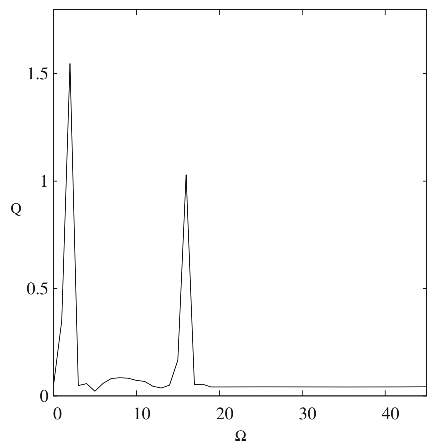

Figure 4. Variation of Q with respect to Ω for $f = 0.4$, $g = 45.0$, $\omega = 0.5$.

shear whereas the magnitude of tropical waves ($g = 45$) is considered to be very high in comparison with the magnitude of vertical wind shear. The temperature variation between the ascending and descending convective fluid currents or strength of convection shows nonlinear dependence on the frequency of tropical waves. Hence VR can be accounted for atmospheric disturbances like cloud development, thunderstorm formation or atmospheric disturbances. Variation of Q for different values of Ω and f is depicted in figure 5. From the figure, one can infer that Q reaches a greater value (red region) for low values for f. Also, while varying Ω from 1 to 50 and f from 0 to 1, resonant peaks are observed only in a certain range of Ω and f. Thus, the influence of the frequency of tropical oscillatory waves for developing atmospheric disturbance can be identified. While achieving this, the values of g and ω are fixed at 45.0 and 0.5 respectively. Figure 6 shows the variation of Q for a wide range of values of g and f. Significant enhancement is obtained, when the

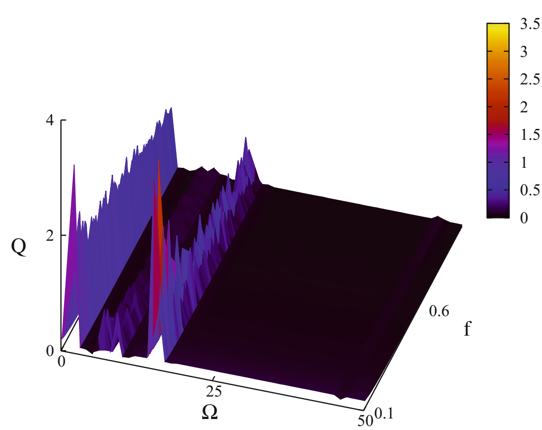

Figure 5. Three-dimensional plot showing the variation of Q, Ω and f for $g = 45$ and $\omega = 0.5$.

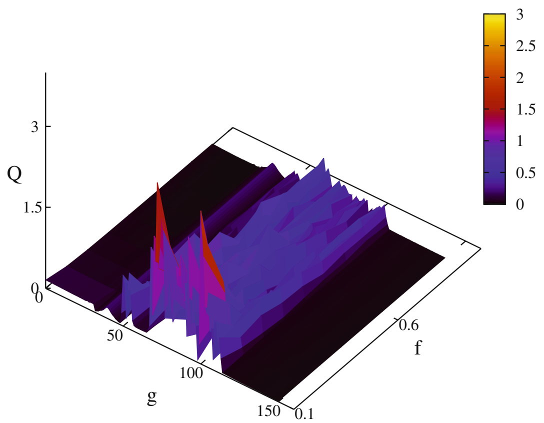

Figure 6. Variation of Q with respect to g and f for $\Omega = 10$, $\omega = 0.5$.

value of $f \ll g$. However, prominent peaks are observed in a wide range of g from 50 to 100.

Similarly, by setting $r = 28$, the enhancement in the response of chaotic convection cycle to vertical wind shear in the presence of tropical wind shear can be studied and the role of tropical waves and vertical wind shear in affecting the dynamics of a chaotic convection system is discussed. Figure 7 shows the three-dimensional representation of variation of Q, f and Ω for $r = 28$, $g = 45$ and $\omega = 0.5$. Multiple resonant peaks are observed for a wide range of f and Ω. Hence, it is inferred that in the region with high convective instability there prevails strong atmospheric disturbances under the influence of tropical waves, when compared with other regions but the lifetime of these disturbances is rather small.

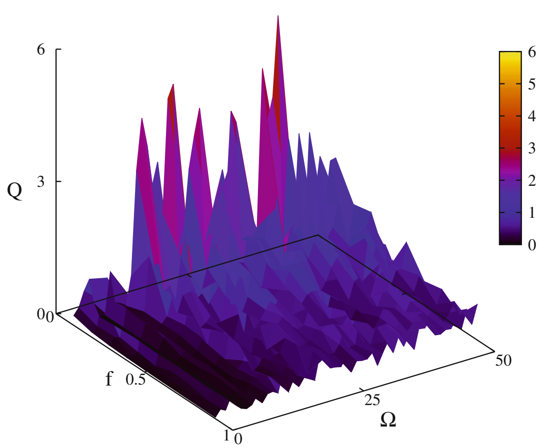

Figure 7. Variation of Q with respect to f and Ω for $g = 45$ and $\omega = 0.5$ when $r = 28$.

4. Dependence of Lyapunov dimension (λ_d) on the response amplitude (Q)

So far the enhancement in response of the system for various control parameters like ω, Ω, f and g has been discussed. The changes in the dynamical behaviour of the system on the verge of resonant value are also interesting and are explained using phase portraits in figure 8. It is found that after the addition of the tropical wave component, the system oscillates between two equilibrium points rather than one as in figure 2. This is analogous to the transition between potential wells as in the case of double well potential under the influence of high-frequency component. Also for $g = 50$ and 70, the system sets in chaotic motion, whereas for $g = 60$, the system operates in periodic region. It means that the systems behave chaotically for a narrow range in which resonant peaks are prominent.

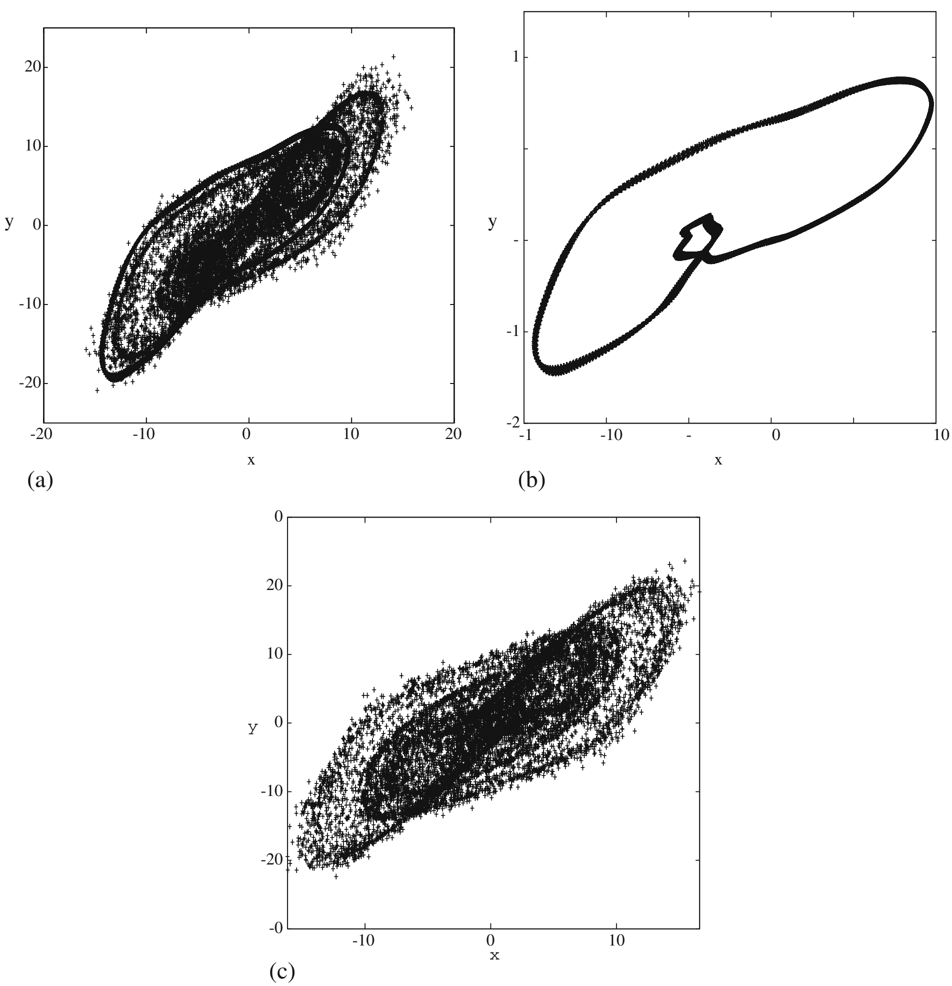

Figure 8. Phase portrait of the system (eq. (3)) with $\Omega = 10$, $\omega = 0.5$, $f = 0.5$, $b = 8/3$, $r = 20$ for (**a**) $g = 50$, (**b**) $g = 60$ and (**c**) $g = 70$.

Table 1. List of Lyapunov exponents and Lyapunov dimension (λ_d) for different values of g with $\Omega = 10$, $f = 0.5$ and $\omega = 0.5$.

g	λ_1	λ_2	λ_3	λ_d
50	0.569	−3.113	−17.34	1.18
60	−0.106	−5.265	−14.52	0.0
70	1.655	−4.074	−17.47	1.40

To characterize the nuance of the system, Kaplan Yorke conjecture has been employed and is given as

$$\lambda_d = j + \sum_{i=1}^{j} \frac{\lambda_i}{|\lambda_{j+1}|}, \tag{7}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$ are the Lyapunov exponents and j is the largest integer for which the sum of the first j exponents is positive. Lyapunov exponent values and Lyapunov dimension (λ_d) for a few selected values of g are tabulated in table 1. It can be observed from figure 9 that the Lyapunov dimension is nonzero only for a value where is there no peak.

In the periodic region, when $r = 20$, the Lyapunov dimension seems to vary along with the peaks of Q for $f = 0.4$, $\omega = 0.5$ and $g = 45$. The strangeness of the system supports the fact that due to the interaction of vertical wind shear and tropical waves, long term prediction of the system becomes tedious. Similarly, Lyapunov dimension for the system in chaotic regime ($r = 28$) is depicted in figure 10. The variables are chosen as $f = 0.4$, $\omega = 0.5$ and $g = 45$. Hence it is inferred that the presence of resonant peaks can also be identified using Lyapunov spectrum as well as Lyapunov dimension.

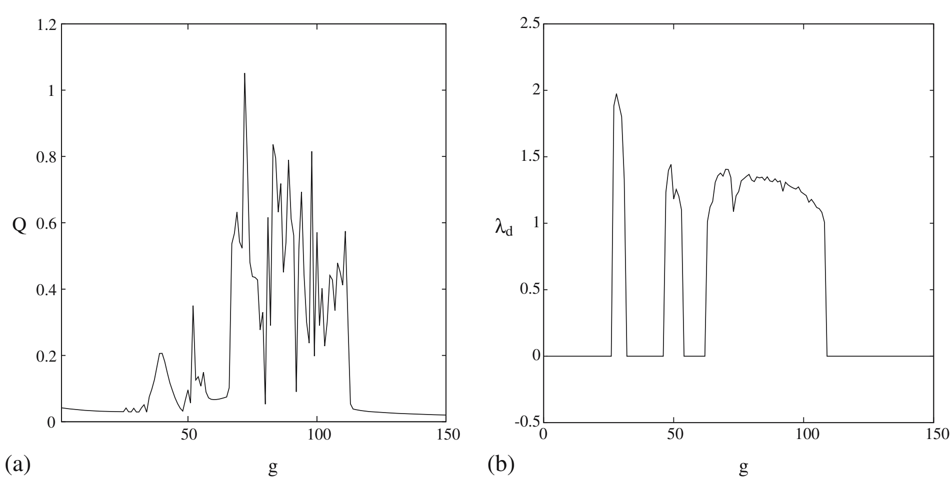

Figure 9. Variation of Q and λ_d for various values of Ω. Other parameters are fixed as $f = 0.4$, $\omega = 0.5$, $g = 45$ and $r = 20$.

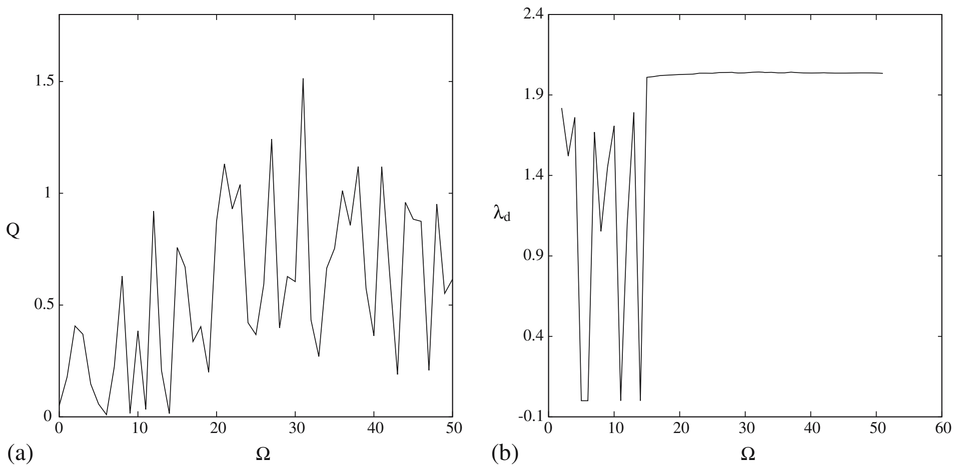

Figure 10. Variation of Q and λ_d for various values of Ω. Other parameters are fixed the same as in figure 9 except for $r = 28$.

5. Conclusions

The occurrence of vibrational resonance phenomenon has been studied in various contexts and in this attempt a systematic and logical approach has been made to describe the development of various atmospheric disturbances including TC which in turn has much relevance in understanding the atmosphere by employing vibrational resonance. The Lorenz equation with r less than the critical value, but greater than 1.345 convincingly describes convective instability and the dependence of this convective region on vertical wind shear and tropical wind is analysed using numerically computed response amplitude Q. Even though the Lorenz system is one of the simplest forms in describing convection, boundedness and sensitive dependence on initial conditions makes it suitable to mimic the atmospheric convection to a satisfactory level. When the magnitude or the frequency of vertical wind shear increases, then the response amplitude of the convective system decreases. The absence of either of the two components minimizes the response of the system towards external forcing which in turn decreases the possibility for the occurrence of atmospheric disturbances like tropical disturbances, formation of thunderstorms and so on. The response of the system drastically increases with the frequent flow and greater magnitude of tropical waves. The reason for enhanced response in the amplide Q may be attributed to the transfer of energy from high-frequency forcing into the sysem which is primarily under the action of weak periodic forcing for a particular range of g values over which the resonance takes place. Thus, the energy of tropical wind acts as the main fuel in developing tropical disturbances. The Lyapunov dimension of the system also undergoes nonmonotonic dependence on control parameters which makes it suitable for characterizing the vibrational resonance in this investigation. Thus, the phenomenon of vibrational resonance throws an insight to move further in understanding the atmospheric disturbances by considering a simple convective system.

Acknowledgements

The author P Philominathan acknowledges the University Grants Commission, New Delhi, India, for the financial support to this research work through a major research project F. No. 41-961/2012(SR) dt. 01-7-2012 and A Jeevarekha would like to thank the Department of Science and technology for the financial support provided in the form of DST INSPIRE Fellowship.

References

[1] E N Lorenz, *J. Atmos. Sci.* **20**, 130 (1963)
[2] A Tamasevicius, A Namajuna and A Cenys, *Electron. Lett.* **32**, 957 (1996)
[3] O E Rossler, *Phys. Lett. A* **71**, 155 (1979)
[4] Steven G Craig, Janet E Kohlhase and David H Papell, *Rev. Econ. Stat.* **73**, 208 (1991)
[5] Dominique Guegan, *Int. Fed. Automatic Control* **33**, 89 (2009)
[6] J E Skinner, M Molnar, T Vybiral and M Mitra, *Integr. Physiol. Behav. Sci.* **27**, 39 (1992)
[7] James N Weiss, Alan Garfinkel, Mark L Spano and William L Ditto, *J. Clin. Investigation* **93**, 1355 (1994)
[8] J P Gollub and S V Benson, *Phys. Rev. Lett.* **41**, 948 (1978)
[9] K Emanuel, *J. Geophys. Res.* **106**, 771 (2001)
[10] J H Christenson and O B Christenson, *Nature* **421**, 805 (2003)
[11] J L McGregor, *Meteorol. Atmos. Phys.* **63**, 105 (1997)
[12] M F Khairoutdinov and D A Randall, *J. Atmos. Sci.* **60**, 607 (2003)
[13] Chien Wang, *J. Geophys. Res.* **110** (2005)
[14] M D Slack, R O Prasad, A Bakker and F Boyson, *Chem. Eng. Res. Design* **78**, 1098 (2000)
[15] Thomas R Knutson, John L McBride, Johnny Chan, Kerry Emanuel, Greg Holland, Chris Landsea, Issac Held, James P Kossin, A K Srivasta and Masato Sugi, *Nature Geosci.* **3**, 157 (2010)
[16] Luca Gammaitoni, Peter Hanggi, Perter Jung and Fabio Marchesoni, *Rev. Mod. Phys.* **70**, 70 (1998)
[17] Isao T Tokuda, Cheol E Han, Kazuyuki Aihara, Mitsuo Kawato and Nicolas Schweighofer, *Neural Networks* **23**, 836 (2010)
[18] P S Landa and P V E McClintock, *J. Phys. A: Math. Gen.* **33**, L433 (2000)
[19] D Su, M Chiu and C Chen, *Prec. Eng.* **18**, 161 (1996)
[20] J D Victor and M M Conte, *Visual Neurosci.* **17**, 959 (2000)
[21] A Gherm, N Zernov, B Lundborg and A Vastberg, *J. Atmos. Solar Terrestial Phys.* **59**, 1831 (1997)
[22] S Jeyakumari, V Chinnathambi, S Rajasekar and M A F Sanjuan, *Phys. Rev. E* **80**, 046608-1-8 (2009)
[23] E Ullner, A Zaikina, J García-Ojalvo, R Báscones and J Kurths, *Phys. Lett. A* **312**, 348 (2003)
[24] A Jeevarekha, M Santhiah and P Philominathan, *Pramana – J. Phys.* **83**, 493 (2014)
[25] J P Baltanas, L Lopez, I I Blechman, P S Landa, A Zaikin, J Kurths and M A F Sanjuan, *Phys. Rev. E* **67**, 066119-1-7 (2003)
[26] S S Chen, J A Knaff and F D Jr Marks, *Mon. Weather Rev.* **134**, 3190 (2006)
[27] Tomio Asai, *J. Meteor. Soc. Jpn* **42**, 245 (1964)
[28] A Wolf, J B Swift, H L Swinney and J A Vastano, *Physica D* **16**, 285 (1985)
[29] Y Hayashi and D G Golder, *Mon. Weather Rev.* **54**, 1262 (1996)
[30] Mao Jiang-Yu and Wang Zai-Zhi, *Chin. Phys. Lett.* **25**, 1506 (2008)
[31] D E Stevens, R S Lindzen and L J Shapiro, *Dyn. Atmos. Ocean* **1**, 365 (1977)
[32] D Zurovac-Jevtić, Sandrine Bony and Kerry Emanuel, *J. Atmos. Sci.* **63**, 2140 (2005)