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Constraining the lightest neutrino mass and m_{ee} from general lepton mass matrices

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Abstract. Despite spectacular advances in fixing the neutrino mass and mixing parameters through various neutrino oscillation experiments, we still have little knowledge about the magnitudes of some vital parameters in the neutrino sector such as the absolute neutrino mass scale, effective Majorana mass m_{ee} measured in neutrinoless double beta decay. In this context, the present work aims to make an attempt to obtain some bounds for m_{ee} and the lightest neutrino mass using fairly general lepton mass matrices in the Standard Model.

Keywords. Lepton mass matrices; neutrinoless double beta decay.

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1. Introduction

In the last few years, significant developments have taken place in the context of phenomenology of neutrino oscillations, both from theoretical as well as experimental points of view. Owing to various solar, atmospheric, reactor and accelerator neutrino experiments, at present, the measurements of leptonic mixing angles and the neutrino mass squared differences have reached almost a precision level. However, despite intense experimental efforts, magnitudes of some of the key parameters still lack precise measurements. For example, neutrino oscillation experiments provide no clue regarding the absolute neutrino mass scale and the related issue of the neutrino mass hierarchy. Another important issue which needs to be taken note of is regarding the rather small neutrino masses as compared to their charged counterparts. From the theoretical point of view, the most popular explanation for this observation is the see-saw mechanism [1] which requires the neutrinos to be Majorana fermions. In this context, precise measurement of the effective Majorana mass m_{ee} in the neutrinoless double beta decay (NDBD) experiments can be pivotal in establishing or ruling out the Majorana neutrinos.

On the experimental front, the next-generation NDBD search aims at reaching the region of 10 meV for the parameter m_{ee}. For this purpose, several new NDBD experiments [2], such as NEXT, SuperNEMO, Majorana, GENIUS, CUORE etc., are in line. However, one of the most promising experiments in this context is the germanium detector array (GERDA) [3]. This experiment searches for the neutrinoless double beta decay of ^{76}Ge, in which ^{76}Ge ($Z = 32$) would decay into ^{76}Se ($Z = 34$) and two electrons. Prior to the GERDA results, the best limits for NDBD in ^{76}Ge were provided by the Heidelberg–Moscow and IGEX-enriched ^{76}Ge experiments which provided an upper limit on the effective Majorana mass, viz. $m_{ee} < 0.33$–1.35 eV; the range in mass arises from the estimated uncertainty in the nuclear matrix elements.

Using ^{76}Ge nuclei GERDA obtained an upper bound on m_{ee} of the range 0.2–0.4 eV, depending upon the nuclear matrix element used. A joint analysis of the KamLAND–Zen and the previous EXO-2000 results carried out by the KamLAND–Zen group yielded an upper limit of the effective mass $m_{ee} < 0.1$–0.25 eV at 90% CL. The plan for GERDA Phase II is to reach the target sensitivity of $T^{0\nu}_{1/2} = 1.4 \times 10^{26}$ yr, with an increased total mass of the enriched material and a reduced background level which would represent a major step on the way towards an ultimate double beta decay experiment aiming at a sensitivity in the 10 meV mass range.

Likewise, for the absolute neutrino mass scale, the most significant upper bounds on $m_{\nu e}$ have been obtained by Mainz and Triotsk neutrino mass experiments, viz. [4],

$$\text{Mainz}: m_{\nu e} < 2.30 \text{ eV}, \quad \text{Triotsk}: m_{\nu e} < 2.05 \text{ eV}, \tag{1}$$

at 95% confidence level. Currently, the most important experiment in this context is the Karlsruhe tritium neutrino experiment (KATRIN) [4] which is expected to push the sensitivity for the mass of the electron antineutrino down to a value of 200 meV (90% CL).

On the theoretical front, intense activity has taken place to develop models for explaining neutrino masses and mixings. However, despite a large number of attempts [5] in various models, we still have not been able to obtain rigorous bounds on m_{ee} and the lightest neutrino mass from the most general considerations. In this context, it would be interesting to explore the possibility of obtaining some constraints on the above-mentioned parameters from the general mass matrices within the framework of Standard Model (SM). The purpose of the present work, therefore, is to make an attempt to obtain bounds for m_{ee} and the lightest neutrino mass by considering the fairly general lepton mass matrices in the SM using the facility of weak basis (WB) transformations.

2. Methodology

The lepton mass matrices in the SM can, in general, be given as

$$M_l = \frac{\upsilon}{\sqrt{2}} Y^l_{ij}, \quad M_{\nu D} = \frac{\upsilon}{\sqrt{2}} Y^{\nu D}_{ij}, \tag{2}$$

where M_l and $M_{\nu D}$ respectively correspond to the charged lepton and Dirac neutrino mass matrices, while Y_{ij}s and υ correspond to the Yukawa couplings and the vacuum expectation value of the Higgs field respectively. To this end, using the facility of weak basis (WB) transformations [6], these mass matrices can be reduced to the form

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} C_\nu & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}. \tag{3}$$

The effective neutrino mass matrix can be obtained using the type-I see-saw relation, i.e.,

$$M_\nu = -M_{\nu D}^T (M_R)^{-1} M_{\nu D}. \tag{4}$$

Details of diagonalizing transformations for the above mass matrices and the methodology connecting the lepton mass matrices to the mixing matrix can be looked up in [7] and [8]: however we mention the essentials here. A general mass matrix M_k can be expressed as

$$M_k = Q_k M_k^r P_k, \tag{5}$$

where $Q_k,\ P_k$ are diagonal phase matrices given as Diag$(\mathrm{e}^{i\alpha_k}, 1, \mathrm{e}^{-i\beta_k})$ and Diag$(\mathrm{e}^{-i\alpha_k}, 1, \mathrm{e}^{i\beta_k})$ respectively and M_k^r is a real symmetric matrix. M_k^r can be diagonalized by an orthogonal transformation O_k, e.g.,

$$M_k^{\text{diag}} = O_k^T M_k^r O_k, \tag{6}$$

which can be rewritten as

$$M_k^{\text{diag}} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \tag{7}$$

Assuming fine tuning, the phase matrices $Q_{\nu D}^T$ and $Q_{\nu D}$ along with $-M_R$ can be taken as m_R diag(1, 1, 1). Making this assumption as well as using the orthogonality of $O_{\nu D}$, it can be shown that using the see-saw mechanism, the effective neutrino mass matrix can be expressed as

$$M_\nu = P_{\nu D} O_{\nu D} \frac{(M_{\nu D}^{\text{diag}})^2}{(m_R)} O_{\nu D}^T P_{\nu D}, \tag{8}$$

m_R being the right-handed neutrino mass scale. Further, the lepton mixing matrix can be expressed as

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \tag{9}$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as $(\mathrm{e}^{i\phi_1},\ 1,\ \mathrm{e}^{i\phi_2})$, ϕ_1 and ϕ_2 being related to the phases of mass matrices and can be treated as free parameters. Further, it needs to be mentioned that for constructing PMNS matrix, the parameters $C_k,\ D_k$ ($k = l, \nu$) of the mass matrices have been considered as free parameters. Using the mass matrix invariants, viz. Trace(M), Determinant(M), Trace(M^2), other parameters of the mass matrices can be expressed as, e.g.

$$E_k = -D_k - C_k + m_{k1} - m_{k2} + m_{k3}, \tag{10}$$

$$A_k = \sqrt{\frac{(m_{k3} - C_k)(m_{k2} + C_k)(m_{k1} - C_k)}{(E_k - C_k)}}, \tag{11}$$

$$B_k = \sqrt{\frac{(-E_k + m_{k1})(E_k + m_{k2})(E_k - m_{k3})}{(E_k - C_k)}}, \tag{12}$$

where $m_{k1},\ -m_{k2},\ m_{k3}$ are the eigenvalues of the matrix M_k.

3. Inputs used for the analysis

For the purpose of calculations, we have made use of the results of a latest global three-neutrino oscillation analysis [9]. Further, the phases ϕ_1, ϕ_2 and the elements $D_{l,\nu}$, $C_{l,\nu}$ are considered to be free parameters. For all the three possible mass hierarchies of neutrinos, the explored range of the lightest neutrino mass is taken to be between 10^{-8} and 10^{-1} eV, and our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π. Although $D_{l,\nu}$ and $C_{l,\nu}$ are free parameters, they have been constrained such that diagonalizing transformations O_l and O_ν always remain real. To facilitate the calculations as well as to find the parameter space available to various parameters, we have resorted to Monte Carlo simulations of various input parameters. For the numerical analysis, we generate 10^7 random points (10^9 when the number of allowed points is small) (table 1).

4. Results and discussions

The effective Majorana mass in the neutrinoless double beta decay can be defined as

$$\langle m_{ee}\rangle = m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2. \tag{13}$$

In the 'standard parametrization' of PMNS matrix, the expression for m_{ee} can be rewritten as

$$|m_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 \mathrm{e}^{2i\alpha} + m_3 s_{13}^2 \mathrm{e}^{2i\beta}|, \tag{14}$$

where α and β represent the Majorana phases, while c_{ij} and s_{ij} correspond to the cosine and sine of the leptonic mixing angles respectively. For the purpose of our calculations,

Table 1. Ranges of neutrino oscillation parameters [9].

Parameter	1σ range	3σ range
$\Delta m^2_{\rm sol}$ (10^{-5} eV^2)	(7.32–7.80)	(6.99–8.18)
$\Delta m^2_{\rm atm}$ (10^{-3} eV^2)	(2.33–2.49) (NH); (2.31–2.49) (IH)	(2.19–2.62) (NH); (2.17–2.61) (IH)
$\sin^2\theta_{13}$ (10^{-2})	(2.16–2.66) (NH); (2.19–2.67) (IH)	(1.69–3.13) (NH); (1.71–3.15) (IH)
$\sin^2\theta_{12}$ (10^{-1})	(2.91–3.25)	(2.59–3.59)
$\sin^2\theta_{23}$ (10^{-1})	(3.65–4.10) (NH);(3.70–4.31) (IH)	(3.31–6.37) (NH);(3.35–6.63) (IH)

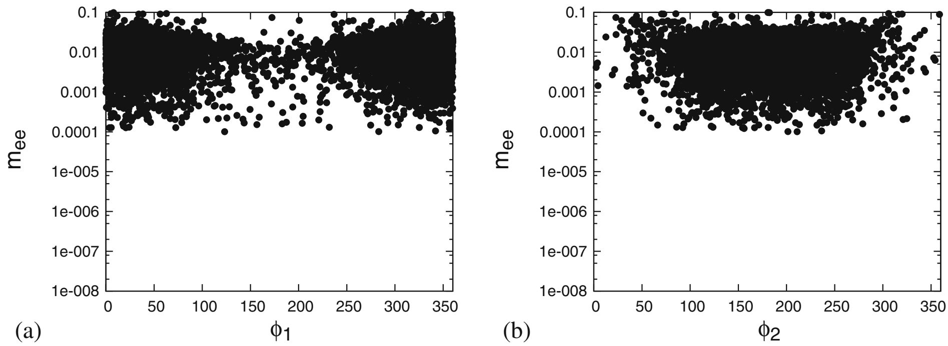

Figure 1. Plots showing the variation of m_{ee} with respect to the phases (**a**) ϕ_1 and (**b**) ϕ_2 pertaining to the normal hierarchy of neutrino masses.

both the Majorana phases have been considered as arbitrary parameters and have been allowed full variation from 0 to 2π. Using the methodology outlined in §2, we calculate m_{ee} for the most general lepton mass matrices in SM given in eq. (3) pertaining to normal as well as inverted neutrino mass orderings. To this end, in figure 1, we have plotted the parameter m_{ee} with respect to the phases ϕ_1 and ϕ_2 pertaining to normal hierarchy (NH) of neutrino masses. While plotting these figures, all the three mixing angles have been constrained by their 3σ experimental bounds. A careful look at these plots clearly establishes a lower bound ($\sim$0.1 meV) for m_{ee} in the NH scenario of neutrino masses. The corresponding plots for the inverted hierarchy scenario establish a lower bound of $\sim$1 meV which, however, are not being presented here for the sake of brevity.

As a next step, we study the dependence of the parameter m_{ee} on the lightest neutrino mass. For this purpose, in figure 2 we have presented the parameter space of m_{ee} with respect to the lightest neutrino mass for mass matrices given in eq. (3) pertaining to the normal hierarchy (NH) and inverted hierarchy (IH) of neutrino masses respectively. These plots clearly indicate that for NH, a lower bound ($\sim$2 meV) can be obtained for the lightest

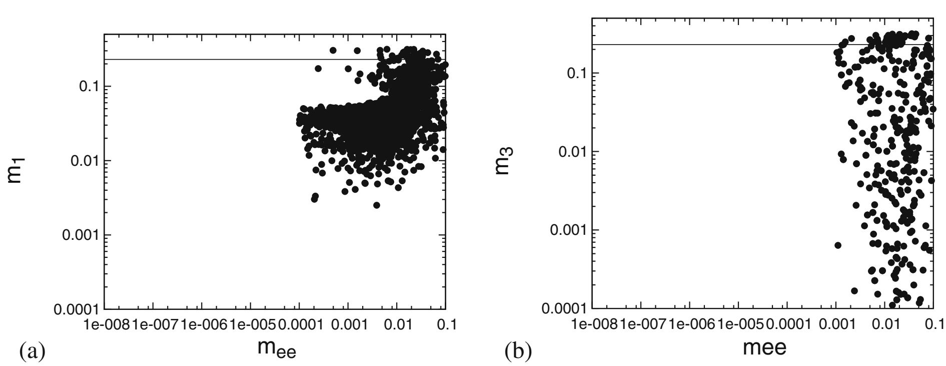

Figure 2. Plots showing the dependence of m_{ee} with the lightest neutrino mass pertaining to the (**a**) normal hierarchy and (**b**) inverted hierarchy of neutrino masses. The horizontal line in both the plots corresponds to the cosmological bound given by PLANCK [10] for the sum of neutrino masses, viz. $\sum m_i < 0.23$ eV.

neutrino mass, whereas for the IH case it remains largely unrestricted while a lower bound (∼1 meV) can be obtained for m_{ee}.

5. Summary and conclusions

This paper contains the preliminary results of our analyses wherein, starting with general lepton mass matrices within the framework of SM, using the facility of WB transformations, we have attempted to obtain bounds on m_{ee} and the lightest neutrino mass for different neutrino mass hierarchies. In the light of the bounds so obtained, the future experiments in this direction are, thus, expected to have important implications for determining the neutrino mass hierarchy as well as texture structure of lepton mass matrices.

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