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Frequency-domain criterion for the chaos synchronization of time-delay power systems under linear feedback control

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Abstract. This paper studies the global synchronization of non-autonomous, time-delay, chaotic power systems via linear state-error feedback control. The frequency domain criterion and the LMI criterion are proposed and applied to design the coupling matrix. Some algebraic criteria via a single-variable linear coupling are derived and formulated in simple algebraic inequalities. The effectiveness of the new criteria is illustrated with numerical examples.

Keywords. Chaos, synchronization; frequency-domain criterion; linear state-error feedback control; time-delay; power system.

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1. Introduction

Chaotic dynamics for the time-delay systems has been extensively investigated in the past [1–5]. An interesting phenomenon for such a class of chaotic systems is that the arbitrary dimension of the time-delay systems can keep their chaotic characteristic as long as their delayed time is large enough [2]. From this point of view, it is possible to improve the security of chaotic secure communication based on a chaotic key produced by the time-delay chaotic systems.

As a basic theory on chaotic secure communication, synchronization of time-delay chaotic system is an issue worthy to be considered. From the theoretical point of view, research on synchronization of time-delay chaotic systems presented by the delay differential equations (DDE) may be more difficult and challenging than those presented by the ordinary differential equations (ODE). Sun [6,7] has investigated time-delay effects on the master–slave synchronization scheme and obtained some synchronization criteria expressed by means of linear matrix inequalities (LMI). However, the frequency-domain criterion, which can also be applied conveniently to design the controller and analyse system parameters, has received less attention.

The single-machine-infinite-bus (SMIB) system is a type of non-autonomous electrical power system which can be presented by either ODE or DDE [8]. Very recently, the synchronization of ODE power systems via a single-variable linear coupling has been investigated in [9]. However, the method of applying the Lyapunov stability theory to study the sufficient criterion for the synchronization of DDE power systems still remains an issue that needs to be solved.

We are thus motivated to study the frequency-domain criterion for the synchronization of DDE power systems via liner state-error feedback control. We verify that a very simple controller can be obtained using frequency-domain criterion and through an example we show that the controller obtained by the frequency-domain criterion is sharper than that obtained by the LMI criterion.

The paper is organized as follows. In §2, some definitions and lemmas are given. In §3, a master–slave feedback-controlled synchronization scheme is constructed and the global synchronization of such a scheme is converted to the global asymptotical stability of a corresponding dynamical error system. In §4, the new frequency-domain criterion and the LMI criterion are presented and applied to deduce the algebraic criterion. In §5, numerical examples are given to verify the effectiveness of the obtained results. Finally, §6 concludes the paper.

2. Definition and lemmas

DEFINITION 1 [10]

The complex-valued function $T(z)$ belongs to a class of strictly positive real functions (SPR), $T(z) \in \{\mathrm{SPR}\}$, if for any real value of z, this function is real and if $\mathrm{Re}\, z \geq 0$, then this function satisfies $\mathrm{Re}\, T(z) > 0$.

Lemma 1 [11]. *The complex-valued function* $T(z) \in \{\mathrm{SPR}\}$, *if and only if the following conditions are satisfied*:

- *(i) for real values of z, the function* $T(z)$ *takes on real values only*;
- *(ii) the function* $T(z)$ *has no poles in* $\mathrm{Re}\, z > 0$;
- *(iii) on the imaginary semi-axis, the function* $T(z)$ *can have only simple poles with positive residues*;
- *(iv) inequality* $\mathrm{Re}\, T(j\omega) > 0$ *holds for* $\forall \omega \in R \cup \{\infty\}$, *where* $j = \sqrt{-1}$.

Lemma 2 *(MKY)* [10]. *Suppose* $A \in R^{n\times n}$, $B \in R^n$, $F \in R^n$, $\mathrm{Re}\, \lambda(A) < 0$, *and the constant* $r > 0$. *Then for any real symmetric matrices* $P \in R^{n\times n}$ *and* $D \in R^{n\times n}$, *and any vector* $q \in R^n$, *the following matrix equations*

$$\begin{cases} PA + A^T P = -qq^T - D, \\ PB - F = \sqrt{r}q, \end{cases}$$

have solutions $P > 0$ *and* $D > 0$ *if and only if*

$$T(z) = r + 2F^T A(z)^{-1} B \in \{\mathrm{SPR}\},$$

where $A(z) = zI - A$.

Lemma 3. *Let x, y, m, n be real vectors of appropriate dimensions and the constant $a > 0$, we have*

$$2x^T mn^T y \leq ax^T mm^T x + y^T nn^T y/a.$$

3. The global synchronization scheme

The DDE power system can be described by [12]

$$\dot{x} = Ax(t) + f(x(t), x(t-\tau)) + m(t), \tag{1}$$

where $x = (x_1, x_2)^T \in R^2$ and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -c \end{bmatrix}, \quad m(t) = \begin{bmatrix} 0 \\ h \sin \omega t \end{bmatrix},$$

$$f(x(t), x(t-\tau)) = \begin{bmatrix} 0 \\ -\beta \sin x_1 + \xi \sin(Rx_1(t-\tau)) \end{bmatrix}. \tag{2}$$

Here, all the parameters c, β, ω, ξ, h, R are positive constants. It is well known that such a DDE power system demonstrates the complex dynamics including chaos [12], as shown in figure 1.

Now, a master–slave synchronization scheme by using a linear state-error feedback controller $u(t)$ to couple two SMIB power systems is constituted and formulated as

$$\begin{cases} \text{Master: } \dot{x} = Ax(t) + f(x(t), x(t-\tau)) + m(t), \\ \text{Slave: } \dot{z} = Az(t) + f(z(t), z(t-\tau)) + m(t) + u(t), \\ \text{Controller: } u(t) = K(x(t) - z(t)), \end{cases} \tag{3}$$

where the state variables $x, z \in R^2$ and $K \in R^{2\times 2}$ is a constant matrix, referred to as the coupling matrix.

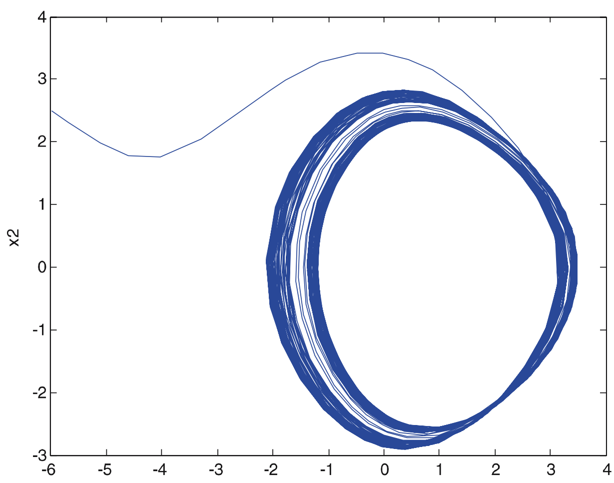

Figure 1. The master system with the parameters $c = 0.5$, $\beta = 1$, $\omega = 1$, $h = 2.34$, $\xi = 0.03$, $\tau = 1$, $R = 2.65$ and the initial condition $x(0) = (-6, 2.5)^T$.

The task here is to design a constant coupling matrix K such that for any initial conditions $x(0)$ of the chaotic master system and any initial conditions $z(0)$ of the slave system, the trajectories $x(t)$ and $z(t)$ satisfy

$$\lim_{t\to\infty} \| x(t) - z(t)\| = 0. \tag{4}$$

The description above is called the global chaos synchronization.

Defining a state-error vector $e(t) = x(t) - z(t)$, one can obtain a dynamical error system for scheme (3), as follows:

$$\dot{e}(t) = (A - K + Q(t))e(t) + Q_1(t-\tau)e(t-\tau), \tag{5}$$

where

$$Q(t) = \begin{bmatrix} 0 & 0 \\ q(t) & 0 \end{bmatrix}, \tag{6}$$

$$q(t) = -\beta(\sin x_1 - \sin z_1)/(x_1 - z_1) \tag{7}$$

and

$$Q_1(t-\tau) = \begin{bmatrix} 0 & 0 \\ q_1(t-\tau) & 0 \end{bmatrix}, \tag{8}$$

$$\begin{aligned} q_1(t-\tau) = {} & \xi(\sin(Rx_1(t-\tau)) \\ & - \sin(Rz_1(t-\tau)))/(x_1(t-\tau) - z_1(t-\tau)). \end{aligned} \tag{9}$$

Clearly, the master–slave synchronization scheme (3) achieves global chaos synchronization in the sense of (4) provided that the error system (5) is globally asymptotically stable at $e = 0$.

4. Main results

First, for $q(t)$ defined by (7) and $q_1(t-\tau)$ defined by (9), one has [13]

$$|q(t)| \le \beta, \quad \forall t \ge 0 \tag{10}$$

and

$$|q_1(t-\tau)| \le \xi R, \quad \forall t \ge 0 \quad \text{and} \quad \tau \ge 0. \tag{11}$$

Theorem 1. *Define the complex matrices*

$$G(z) = zI - \left(A - K + HF^T\right),$$

$$W(z) = F^T G^{-1}(z) H,$$

where z is a complex variable, the vectors

$$F = \left(\sqrt{\beta^2 + a^{-1}\xi^2 R^2}, 0\right)^T, \quad H = \left(0, \sqrt{1+a}\right)^T$$

and a is a positive constant. Then the master–slave synchronization scheme (3) *achieves the global chaos synchronization provided that*

$$\mathrm{Re}\,\lambda(A - K + HF^T) < 0 \tag{12}$$

and

$$1 + 2\,\mathrm{Re}W(j\omega) > 0, \quad \forall\omega \in R \cup \{\infty\}, \tag{13}$$

where $\lambda(\cdot)$ *denotes any eigenvalue of a real matrix,* $\mathrm{Re}(\cdot)$ *is the real part of a complex variable and* $j = \sqrt{-1}$.

Proof. Let $L(t-\tau) = (q_1(t-\tau),\ 0)$, $B = (0,\ 1)^T$. Then $Q_1(t-\tau) = BL(t-\tau)$.

The error system (5) can thus be rewritten as

$$\dot{e}(t) = (A - K + Q(t))e(t) + BL(t-\tau)e(t-\tau). \tag{14}$$

Choose a Lyapunov–Krasovskii function candidate:

$$V = \mathrm{e}^T Pe + a^{-1}\int_{t-\tau}^{t} \mathrm{e}^T(s)L^T(t-\tau)L(t-\tau)e(s)\mathrm{d}s,$$

where $P = \mathrm{diag}\{p_1,\ p_2\}$ with $p_1 > 0$ and $p_2 > 0$.

By Lemma 3, the time derivative of V along the orbit of error system (14) equals

$$\begin{aligned}
\dot{V} &= \mathrm{e}^T(A-K+Q(t))^T Pe + \mathrm{e}^T P(A-K+Q(t))e \\
&\quad +2\mathrm{e}^T PBL(t-\tau)e(t-\tau) + a^{-1}\mathrm{e}^T L^T(t-\tau)L(t-\tau)e \\
&\quad -a^{-1}\mathrm{e}^T(t-\tau)L^T(t-\tau)L(t-\tau)e(t-\tau) \\
&\le \mathrm{e}^T\left[(A-K+Q(t))^T P + P(A-K+Q(t))\right]e \\
&\quad +a\mathrm{e}^T PB(PB)^T e + a^{-1}\mathrm{e}^T L^T(t-\tau)L(t-\tau)e \\
&\quad +a^{-1}\mathrm{e}^T(t-\tau)L^T(t-\tau)L(t-\tau)e(t-\tau) \\
&\quad -a^{-1}\mathrm{e}^T(t-\tau)L^T(t-\tau)L(t-\tau)e(t-\tau) \\
&= \mathrm{e}^T\left[(A-K)^T P + P(A-K) + Q^T(t)P + PQ(t)\right. \\
&\quad \left. +aPB(PB)^T + a^{-1}L^T(t-\tau)L(t-\tau)\right]e.
\end{aligned}$$

It follows from (10) that

$$\begin{aligned}
&\mathrm{e}^T[Q^T(t)P + PQ(t)]e = 2p_2q(t)e_1e_2 \le q^2(t)e_1^2 + (p_2e_2)^2 \\
&\le \beta^2 e_1^2 + (p_2e_2)^2 = \mathrm{e}^T\left[a^{-1}EE^T + PB(PB)^T\right]e,
\end{aligned}$$

where

$$E = \left(\sqrt{a}\beta,\ 0\right)^T.$$

It follows from (11) that

$$\mathrm{e}^T\left[a^{-1}L^T(t-\tau)L(t-\tau)\right]e \le \mathrm{e}^T\left[a^{-1}CC^T\right]e,$$

where

$$C = (\xi R,\ 0)^T .$$

Thus, for any $e \neq 0$, $\dot{V}(e) < 0$ provided that

$$(A-K)^T P + P(A-K) + a^{-1}EE^T \\ +PB(PB)^T + aPB(PB)^T + a^{-1}CC^T < 0. \tag{15}$$

Therefore, inequality (15) is sufficient for the stability of the error system (14) at $e = 0$.

Transform (15) as

$$(A-K)^T P + P(A-K) + (1+a)PB(PB)^T \\ +a^{-1}(EE^T + CC^T) = -D, \tag{16}$$

where $D \in R^{2\times 2}$ is a symmetric positive definite matrix.

It is easily verified that $a^{-1}(EE^T + CC^T) = FF^T$ and $(1+a)PB(PB)^T = PH(PH)^T$. Equality (16) can thus be rewritten as

$$(A-K+HF)^T P + P(A-K+HF) = -(PH-F)(PH-F)^T - D,$$

or, equivalently,

$$\begin{cases} P(A-K+HF^T) + \left(A-K+HF^T\right)^T P = -qq^T - D, \\ PH - F = q. \end{cases} \tag{17}$$

By Lemma 2, one knows that the matrix eqs (17) have resolutions $P > 0$ and $D > 0$ if and only if the function

$$T(z) = 1 + 2FG^{-1}(z)H \in \{SPR\}.$$

It follows from (12) that the function $T(z)$ does not have any pole in Re $z > 0$. By Lemma 1 we know that $T(z) \in \{SPR\}$, if and only if, inequality (13) is satisfied. This completes the proof. □

The following theorem also gives a criterion for the global synchronization of the master–slave synchronization scheme (3), which is formulated in linear matrix inequality (LMI).

Theorem 2 [**6**]. *If there exists a symmetric positive definite matrix $P \in R^{2\times 2}$, a constant $a > 0$ and a constant coupling matrix $K \in R^{2\times 2}$ such that for any $t \geq 0$ and any $\tau > 0$,*

$$(A-K+Q(t))^T P + P(A-K+Q(t)) \\ +aP^T P + a^{-1}Q_1^T(t-\tau)Q_1(t-\tau) < 0, \tag{18}$$

then the master–slave synchronization scheme (3) *achieves global chaos synchronization*

Remark 1. Both the frequency domain criteria (12) and (13) and the LMI criterion (18) can be applied to design the coupling matrix $K \in R^{2\times 2}$ for global chaos synchronization in the sense of (4). In what follows, some synchronization criteria of the scale inequalities will be derived from (12), (13) and (18) respectively for the coupling matrix $K = \text{diag}\{k,\ 0\}$, which implies that only a pair of state variables (x_1, z_1) of the master–slave systems are linearly fed into the first differential equation of the slave system.

PROPOSITION 1

If there exists a single-variable coupling matrix $K = \mathrm{diag}\{k,\ 0\}$ *such that*

$$k > g(a) = \begin{cases} \max\left\{\sqrt{2(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right) - 2\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} - c^2},\right. \\ \left.\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)}/c\right\}, \quad \text{if } 2(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right) \\ \qquad\qquad -2\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} \geq c^2, \\ \sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)}/c, \qquad \text{others,} \end{cases} \tag{19}$$

then the master–slave synchronization scheme (3) *achieves global chaos synchronization.*

Proof. Choosing $K = \mathrm{diag}\{k,\ 0\}$, the eigenvalues λ of the matrix $A - K + HF^T$ can be obtained from

$$\begin{aligned} &\left|\lambda I - \left(A - K + HF^T\right)\right| \\ &= \begin{vmatrix} \lambda + k & -1 \\ -\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} & \lambda + c \end{vmatrix} \\ &= \lambda^2 + (k+c)\lambda + kc - \sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} = 0. \end{aligned}$$

By Hurwitz criterion, we know that $\mathrm{Re}\,\lambda\left(A - K + HF^T\right) < 0$, if and only if,

$$\begin{cases} k + c > 0, \\ kc - \sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} > 0. \end{cases} \tag{20}$$

Obviously, inequalities (20) hold if

$$k > \frac{1}{c}\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)}, \tag{21}$$

is satisfied.

Again,

$$\begin{aligned} W(z) &= F^T G^{-1}(z) H \\ &= \left(\sqrt{\beta^2 + a^{-1}\xi^2 R^2}\ \ 0\right)\begin{pmatrix} z+k & -1 \\ -\sqrt{(1+a)(\beta^2 + a^{-1}\xi^2 R^2)} & z+c \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ \sqrt{1+a} \end{pmatrix} \\ &= \frac{(1+a)(\beta^2 + a^{-1}\xi^2 R^2)}{z^2 + (k+c)z + kc - \sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)}\beta}. \end{aligned}$$

By letting

$$\Omega = \left[kc - \sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} - \omega^2\right]^2 + (k+c)^2\omega^2,$$

one can obtain that

$$
\begin{aligned}
&1 + 2\,\mathrm{Re}\, W(j\omega)\\
&\quad = \frac{1}{\Omega} \times \Big(\omega^4 + \Big(k^2 + c^2 - 2(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)\\
&\quad +2\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)}\Big)\,\omega^2\\
&\quad + \left(kc - \sqrt{(1+a)(\beta^2 + a^{-1}\xi^2 R^2)}\right)^2\\
&\quad +2(1+a)(\beta^2 + a^{-1}\xi^2 R^2)\left(kc - \sqrt{(1+a)(\beta^2 + a^{-1}\xi^2 R^2)}\right)\Big).
\end{aligned}
$$

It is clear that $1 + 2\,\mathrm{Re}\, W(j\omega) > 0,\ \forall \omega \in R \cup \{\infty\}$, if

$$k^2 + c^2 - 2(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right) + 2\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} > 0 \quad (22)$$

and

$$kc - \sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} > 0. \quad (23)$$

Therefore, it follows from Theorem 1 that the master–slave synchronization scheme (3) achieves global chaos synchronization if the inequalities (21) and (22) are satisfied simultaneously.

Again, the inequalities (21) and (22) hold if the condition (19) is satisfied. This completes the proof. □

Remark 2. The synchronization condition (19) depends on the constant $a > 0$, which needed to be selected in applications. As presented in [14], a sharp criterion, which means the coupling coefficient k is as close as possible to the values determined by the necessary synchronization conditions (if any), is more valuable. Thus, to improve the sharpness of criterion (19), let us select the constant a such that $g(a)$ defined by (19) is as small as possible.

Based on Lemma 3, we have

$$
\begin{aligned}
\sqrt{(1+a)\left(\beta^2 + a^{-1}\xi^2 R^2\right)} &= \sqrt{\beta^2 + a^{-1}\xi^2 R^2 + a\beta^2 + \xi^2 R^2}\\
&\geq \sqrt{\beta^2 + 2\xi R\beta + \xi^2 R^2} = \beta + \xi R.
\end{aligned}
$$

Thus, the function $g(a)$ defined by (9) can be minimized, which leads to a sharper criterion as follows.

PROPOSITION 2

If there exists a single-variable coupling matrix $K = diag\{k,\ 0\}$ *such that*

$$
k > \begin{cases} \max\left\{\sqrt{2(\beta+\xi R)^2 - 2(\beta+\xi R) - c^2},\right. \\ \quad \left.(\beta+\xi R)/c\right\}, \quad \text{if } 2(\beta+\xi R)^2 - 2(\beta+\xi R) \geq c^2, \\ (\beta+\xi R)/c, \qquad\qquad\qquad \text{others}, \end{cases} \quad (24)
$$

then the master–slave synchronization scheme (3) *achieves global chaos synchronization.*

The following algebraic criterion is derived from LMI criterion (18).

PROPOSITION 3

If the single-variable coupling matrix $K = \text{diag}\{k, 0\}$ *is selected such that for the constants* $a > 0$ *and* $0 < \mu < 2c/a$

$$k > g(\mu, a) = a/2 + \left(\xi^2 R^2\right)/(2a) + (1 + \mu\beta)^2/\left(4c\mu - 2a\mu^2\right), \tag{25}$$

then the master–slave synchronization scheme (3) *achieves global synchronization.*

Proof. Take a positive definite matrix

$$P = \text{diag}\{1,\ \mu\} \quad \text{with} \quad a > 0 \quad \text{and} \quad 0 < \mu < 2c/a, \tag{26}$$

and then one has

$$\begin{aligned}
&(A - K + Q(t))^T P + P(A - K + Q(t)) + aP^T P + a^{-1} Q_1(t-\tau)^T Q_1(t-\tau) \\
&= \begin{bmatrix} -2k + a + a^{-1} q_1^2(t-\tau) & 1 + \mu q(t) \\ 1 + \mu q(t) & -2c\mu + a\mu^2 \end{bmatrix}.
\end{aligned}$$

The above symmetric matrix is negative definite if and only if, for any $t \geq 0$ and any $\tau > 0$,

$$-2k + a + a^{-1} q_1^2(t-\tau) < 0 \tag{27}$$

and

$$\left(-2k + a + a^{-1} q_1^2(t-\tau)\right)\left(-2c\mu + a\mu^2\right) - (1 + \mu q(t))^2 > 0. \tag{28}$$

It follows from (10) and (11) that inequalities (27) and (28) hold, provided

$$-2k + a + a^{-1}\xi^2 R^2 < 0 \tag{29}$$

and

$$\left(-2k + a + a^{-1}\xi^2 R^2\right)\left(-2c\mu + a\mu^2\right) - (1 + \mu\beta)^2 > 0. \tag{30}$$

It is easily verified that for $0 < \mu < 2c/a$, inequalities (29) and (30) hold provided condition (25) is satisfied. This completes the proof. □

In order to improve the sharpness of criterion (25), let us select the constants μ and a such that $g(\mu, a)$ defined by (25) achieves the minimal value. To do so, calculate the first partial derivative of $g(\mu, a)$ with respect to μ and a as follows:

$$\partial g\,(\mu, a)/\partial\mu = (1 + \mu\beta)\,(\mu\beta c + a\mu - c)/\left(\mu^2 (a\mu - 2c)^2\right),$$

$$\partial g\,(\mu, a)/\partial a = 1/2 - \xi^2 R^2/2a^2 + \mu^2\,(1 + \mu\beta)^2/\left(2(2c\mu - a\mu)^2\right).$$

Let

$$\left(\mu^*,\ a^*\right) = \left(\sqrt{c^2+1}/\left(\xi R + \beta\sqrt{c^2+1}\right),\ \xi Rc/\sqrt{c^2+1}\right). \tag{31}$$

It can be verified that μ^* and a^* satisfy constraint (26), and

$$(\partial g(\mu, a)/\partial\mu)(\mu^*, a^*) = (\partial g(\mu, a)/\partial a), (\mu^*, a^*) = 0.$$

Let us further observe that the corresponding second partial derivative:

$$\left(\partial^2 g/\partial^2 a\right)\left(\mu^*, a^*\right) = \left(c^2 a^* + 2a^* + 2c^3\beta + 2c\beta\right)/\left(c^2 a^*\left(a^* + 2c\beta\right)\right) > 0,$$

$$\left(\partial^2 g/\partial^2 \mu\right)\left(\mu^*, a^*\right) = \left(a^* + c\beta\right)^2 (c^3\beta^2 + c\beta^2 + \xi^2 R^2 c + 2a^*\beta c^2 + 2a^*\beta)/((c^5 + c^3)(a^* + 2c\beta)) > 0,$$

$$(\partial^2 g/\partial^2 a)(\mu^*, a^*) \times (\partial^2 g/\partial^2 \mu)(\mu^*, a^*) - (\partial^2 g/\partial a \partial \mu)(\mu^*, a^*)$$
$$= (a^* + c\beta)^2 (c\xi^2 R^2 + 2a^*\beta c^2 + 2a^*\beta + \beta^2 c^3 + \beta^2 c)/(c^5 a^*(a^* + c\beta)) > 0.$$

Thus, the stationary point (μ^*, a^*) is a unique minimal point of $g(\mu, a)$ subject to conditions (26), and the corresponding minimal value $g(\mu^*, a^*)$ is

$$g(\mu^*, a^*) = \left(\xi R\sqrt{c^2 + 1} + \beta\right)/c,$$

where μ^* and a^* are determined by (31).

The above discussion can be summarized to yield a sharper algebraic synchronization criterion as follows.

PROPOSITION 4

If the single-variable coupling matrix $K = \mathrm{diag}\{k, 0\}$ *is selected such that*

$$k > \left(\xi R\sqrt{c^2 + 1} + \beta\right)/c, \tag{32}$$

then the master–slave synchronization scheme (3) *achieves global synchronization*

Note that if $\xi = 0$ the DDE power system can be presented by the ordinary differential equations (ODE) as follows:

$$\dot{x} = Ax + f(x) + m(t), \tag{33}$$

where $x = (x_1, x_2)^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -c \end{bmatrix}, \quad f(x) = \begin{bmatrix} 0 \\ -\beta \sin x_1 \end{bmatrix}, \quad m(t) = \begin{bmatrix} 0 \\ h \sin \omega t \end{bmatrix}.$$

Form Proposition 4, the following corollary can be easily derived.

COROLLARY 1

If the single-variable coupling matrix $K = \mathrm{diag}\{k, 0\}$ *is selected such that*

$$k > \beta/c, \tag{34}$$

then two identical ODE power systems can achieve global chaos synchronization.

Obviously, criterion (34) is the same as Theorem 3 in [9].

5. Numerical example

Consider the master–slave synchronization scheme (3) where the parameters of the DDE power system are $c = 0.5$, $\beta = 1$, $\omega = 1$, $h = 2.45$, $\xi = 0.03$, $\tau = 1$ and $R = 2.65$.

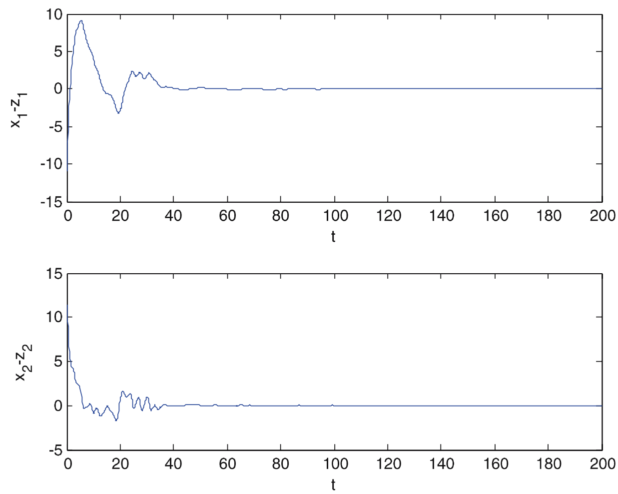

Figure 2. The synchronization evolution for $K = \mathrm{diag}\{k,\ 0\}$ with $k = 2.16$, where initial conditions are $x(0) = (-6, 2.5)^T$ and $z(0) = (5, -9)^T$.

The initial conditions of the master and slave DDE power systems are freely chosen as $x(0) = (-6, 2.5)^T$ and $z(0) = (5, -9)^T$. Simply computing by the algebraic criteria (24) and (32), we obtain the feedback matrix $K = \mathrm{diag}\{k,\ 0\}$ of the synchronization controller as follows:

By the algebraic criterion (24): $k > 2.159$;

By the algebraic criterion (31): $k > 2.178$.

It is clear that the synchronizing condition obtained by the algebraic criterion (24) is weaker than that obtained by the algebraic criterion (32). Hence the frequency-domain criteria (12) and (13) are sharper than the LMI criteria (18) in this example. We take $k = 2.16$, illustrating the synchronization results in figure 2.

6. Conclusion

This paper proposes the frequency-domain criteria for the global synchronization of the time-delay chaotic power systems coupled by the linear state-error feedback control. Based on the new frequency-domain criterion and the LMI criterion, some sufficient algebraic synchronization criteria for the single-variable linear coupling have been derived and formulated in simple algebraic inequalities, facilitating the design of synchronization controller. An example shows that the controller obtained by the frequency-domain criterion is sharper than that obtained by the LMI criterion.

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