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Rogue wave solutions of the nonlinear Schrödinger equation with variable coefficients

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Abstract. In this paper, a unified formula of a series of rogue wave solutions for the standard (1+1)-dimensional nonlinear Schrödinger equation is obtained through exp-function method. Further, by means of an appropriate transformation and previously obtained solutions, rogue wave solutions of the variable coefficient Schrödinger equation are also obtained. Two free functions of time t and several arbitrary parameters are involved to generate a large number of wave structures.

Keywords. Nonlinear Schrödinger equation; exp-function method; breather soliton; rogue wave.

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1. Introduction

The (1+1)-dimensional variable coefficient nonlinear Schrödinger equation (VCNLSE) is written as [1–3]

$$iu_t + \kappa(t)u_{xx} + \chi(t)|u|^2u = i\Gamma(t)u, \tag{1}$$

where $u = u(x,t)$ is a complex-valued function of two real variables (x,t), $\kappa(t)$ represents the group velocity dispersion, $\chi(t)$ is the nonlinearity parameter which represents the self-focussing ($\chi > 0$) or de-focussing ($\chi < 0$) cubic nonlinearity, and $\Gamma(t)$ denotes the amplification ($\Gamma > 0$) or absorption ($\Gamma < 0$) coefficient. There are three nonzero real functions. This equation describes the evolution of modulations of dispersive waves with weak nonlinearity. It occurs in various areas of physics, including nonlinear optics, plasma physics, superconductivity and quantum mechanics.

In nonlinear optics, the variable t generally represents propagation distance and x represents the retarded time in eq. (1). In this case, it describes the amplification or absorption of pulses propagating in a single-mode optical fibre with distributed dispersion and nonlinearity. In practical applications, the model is of primary interest not only in the amplification and compression of optical solitons in inhomogeneous systems, but also in the stable transmission of managed soliton. Recently, the applications of eq. (1) with various forms have been studied by many researchers. In these literatures, the analytic solution of the harmonic form and Jacobian elliptic function solution are obtained by Hao *et al* by employing perturbation and F-expansion methods [1,2]. Exact rogue wave solutions of the (2+1)-dimensional nonlinear Schrödinger equation with varying coefficients are obtained by Zhang *et al* [3]. When $\kappa(t) = \alpha,\ \chi(t) = \beta$, eq. (1) reduces to the standard nonlinear Schrödinger equation (NLSE) [4]

$$iu_t + \alpha u_{xx} + \beta|u|^2 u = 0, \tag{2}$$

where α and β are two nonzero real constants. Depending on the character of its solutions, it is called the 'self-focussing' ($\alpha > 0, \beta > 0$) and 'de-focussing' ($\alpha > 0, \beta < 0$) NLSE, respectively. In this study, we use NLSE$^+$ and NLSE$^-$ to denote them.

In recent years, rogue wave phenomenon has become a hot topic for many researchers. They found that rogue waves appear not only in oceanic conditions [5–7] but also in plasmon [8], optics [9–14], superfluids [15], Bose–Einstein condensates [16,17] and in the form of capillary waves [18]. Moslem *et al* found that the electrostatic surface plasma rogue waves can be excited and propagated along a plasma–vacuum interface due to the nonlinear coupling between high-frequency surface plasmons and low-frequency ion oscillations. They also discussed the nonlinear pulse propagation condition and its behaviour. They believed that the nonlinear structures may be useful for controlling and maximizing plasmonic energy along the plasma surface [8]. Solli *et al* introduced the concept of optical rogue waves, a counterpart of the infamous rare water waves. Using a new real-time detection technique, they studied a system that exposes extremely steep, large waves as rare outcomes from an almost identically prepared initial population of waves. Specifically, they reported the observation of rogue waves in an optical system, based on a microstructured optical fibre, near the threshold of soliton-fission supercontinuum generating a noise-sensitive nonlinear process in which extremely broadband radiations are produced from a narrow-band input. They modelled the formation of these rogue waves using the generalized nonlinear Schrödinger equation and demonstrated that they arise infrequently from initially smooth pulses owing to power transfer seeded by a small noise perturbation [9]. Bludov *et al* predicted the existence of rogue waves in Bose–Einstein condensates either loaded into a parabolic trap or embedded in an optical lattice. In the latter case, rogue waves can be observed in condensates with positive scattering length and they are immensely enhanced by the lattice. Local atomic density may increase up to ten times. They provided the initial conditions necessary for the experimental observation of the phenomenon. Numerical simulations illustrated the process of rogue wave creation [16].

Recently, the structure of the rogue waves and its behaviour have attracted the attention of a large number of researchers [19–24]. For describing the natural nonlinear phenomenon, the nonlinear Schrödinger equation is a fundamental model which is widely applied in nonlinear science and is also widely used in studying the existence of rogue waves and their structures [16,17,19–33].

Some researchers have investigated eq. (2), derived its breather solutions, such as the Akhmediev breather soliton [21], the Ma breather soliton [22] and the Peregrine breather soliton [4]. These solutions are suggested as models for a class of freak wave events [23]. In order to obtain these breather solitons, different methods were applied by researchers in [4,29,30,32,33].

In this work, we apply the exp-function method to NLSE [34,35], and obtain a solution containing the three parameters. Through this unified formula in which different values of the parameters, one can obtain the above-mentioned series of breather solitons (rogue waves). Then, by using the similarity transformation [36–38], VCNLSE can be transformed into NLSE, thereby obtaining rogue wave solutions of VCNLSE. These rich and explicit rogue wave structures will help us to understand the diversity of rogue wave dynamics for VCNLSE.

2. Exp-function method to construct a series of rogue waves for NLSE

By using the transformation

$$u(x,t) = r\mathrm{e}^{(ir^2\beta t)}\left(1+\frac{A(x,t)+iB(x,t)}{F(x,t)}\right), \tag{3}$$

eq. (2) can be transformed into the following trilinear equation:

$$\begin{aligned}
&2\beta r^2 A(x,t)F(x,t)^2 + 2\alpha A(x,t)F_x(x,t)^2 + \beta r^2 A(x,t)^3\\
&\quad - 2\alpha A_x(x,t)F_x(x,t)F(x,t) + \alpha A_{xx}(x,t)F(x,t)^2\\
&\quad - \alpha A(x,t)F_{xx}(x,t)F(x,t) + B(x,t)F(x,t)F_t(x,t)\\
&\quad + 3\beta r^2 A(x,t)^2 F(x,t) + \beta r^2 B(x,t)^2 F(x,t) - B_t(x,t)F(x,t)^2\\
&\quad + \beta r^2 A(x,t)B(x,t)^2 + i(\beta r^2 A(x,t)^2 B(x,t) - A(x,t)F(x,t)F_t(x,t)\\
&\quad - \alpha B(x,t)F_{xx}(x,t)F(x,t) + \beta r^2 B(x,t)^3\\
&\quad + \alpha B_x(x,t)F(x,t)^2 + A_t(x,t)F(x,t)^2 + 2\alpha B(x,t)F_x(x,t)^2\\
&\quad + 2\beta r^2 A(x,t)B(x,t)F(x,t) - 2\alpha B_x(x,t)F_x(x,t)F(x,t)) = 0,
\end{aligned} \tag{4}$$

where r is the real constant, $A(x,t)$, $B(x,t)$ and $F(x,t)$ are real functions. Separating the real and imaginary parts, we have

$$\begin{aligned}
&2\beta r^2 A(x,t)F(x,t)^2 + 2\alpha A(x,t)F_x(x,t)^2 + \beta r^2 A(x,t)^3\\
&\quad - 2\alpha A_x(x,t)F_x(x,t)F(x,t) + \alpha A_{xx}(x,t)F(x,t)^2\\
&\quad - \alpha A(x,t)F_{xx}(x,t)F(x,t) + B(x,t)F(x,t)F_t(x,t)\\
&\quad + 3\beta r^2 A(x,t)^2 F(x,t) + \beta r^2 B(x,t)^2 F(x,t) - B_t(x,t)F(x,t)^2\\
&\quad + \beta r^2 A(x,t)B(x,t)^2 = 0,\\
&\beta r^2 A(x,t)^2 B(x,t) - A(x,t)F(x,t)F_t(x,t) - \alpha B(x,t)F_{xx}(x,t)F(x,t)\\
&\quad + \beta r^2 B(x,t)^3 + \alpha B_x(x,t)F(x,t)^2 + A_t(x,t)F(x,t)^2\\
&\quad + 2\alpha B(x,t)F_x(x,t)^2 + 2\beta r^2 A(x,t)B(x,t)F(x,t)\\
&\quad - 2\alpha B_x(x,t)F_x(x,t)F(x,t) = 0.
\end{aligned} \tag{5}$$

Suppose $A(x,t)$, $B(x,t)$ and $F(x,t)$ are the following exponential functions:

$$\begin{aligned} A(x,t) &= a_1 \mathrm{e}^{p(Vx+Kt)} + a_2 \mathrm{e}^{-p(Vx+Kt)} + a_3 \mathrm{e}^{q(Wx+Lt)} + a_4 \mathrm{e}^{-q(Wx+Lt)}, \\ B(x,t) &= b_1 \mathrm{e}^{p(Vx+Kt)} + b_2 \mathrm{e}^{-p(Vx+Kt)} + b_3 \mathrm{e}^{q(Wx+Lt)} + b_4 \mathrm{e}^{-q(Wx+Lt)}, \\ F(x,t) &= c_1 \mathrm{e}^{p(Vx+Kt)} + c_2 \mathrm{e}^{-p(Vx+Kt)} + c_3 \mathrm{e}^{q(Wx+Lt)} + c_4 \mathrm{e}^{-q(Wx+Lt)}, \end{aligned} \tag{6}$$

where $a_i, b_i, c_i (i = 1,\ldots,4)$, p, q, W, V, K and L are constants to be determined. Substituting eq. (6) into eq. (5) yields two algebraic equations with respect to $\mathrm{e}^{mp(Vx+Kt)}\mathrm{e}^{nq(Wx+Lt)}$ $(m, n = -3,\ldots,3)$. Equating all coefficients of $\mathrm{e}^{mp(Vx+Kt)}\mathrm{e}^{nq(Wx+Lt)} (m, n = -3,\ldots,3)$ to zero yields a set of algebraic equations for $a_i, b_i, c_i (i = 1,\ldots,4)$, p, q, W, V, K and L. Solving them using Maple, one can obtain the following results:

$$\begin{aligned} a_1 &= 0, \quad a_2 = 0, \quad a_3 = \frac{b_4^2}{\sqrt{4c_2^2 - b_4^2}}, \quad a_4 = \frac{1}{\sqrt{4c_2^2 - b_4^2}} b_4^2, \\ b_1 &= 0, \quad b_2 = 0, \quad b_3 = -b_4, \quad c_1 = c_2, \quad c_3 = -\frac{2c_2^2}{\sqrt{4c_2^2 - b_4^2}}, \\ c_4 &= -\frac{2c_2^2}{\sqrt{4c_2^2 - b_4^2}}, \quad W = 0, \quad L = -\frac{\beta r^2 b_4}{2c_2^2 q}\sqrt{4c_2^2 - b_4^2}, \\ K &= 0, \quad V = \frac{r b_4}{2pc_2}\sqrt{-2\frac{\beta}{\alpha}}, \end{aligned} \tag{7}$$

where c_2 and b_4 are arbitrary constants.

Substituting eqs (7) and (6) into eq. (3), solutions of eq. (2) can be expressed as

$$u(x,t) = r\mathrm{e}^{(ir^2\beta t)}\left(1 + \frac{b_4^2 \cosh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t\right) + ib_4\sqrt{4c_2^2 - b_4^2}\sinh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t\right)}{c_2\sqrt{4c_2^2 - b_4^2}\cosh\left(\frac{r\sqrt{-2\frac{\beta}{\alpha}}b_4}{2c_2}x\right) - 2c_2^2\cosh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t\right)}\right). \tag{8}$$

Equation (8) is a unified formula which can produce a series of rogue wave solutions. Obviously, when $b_4 = 0$, eq. (8) becomes a plane-wave solution of eq. (2) (NLSE$^+$ and NLSE$^-$) which can be denoted as

$$u(x,t) = r\mathrm{e}^{(ir^2\beta t)}. \tag{9}$$

Case 1. Rogue wave solutions of the self-focussing NLSE ($\alpha > 0, \beta > 0$) are described as follows:

(a) *The Akhmediev breather soliton* [21]. If $4c_2^2 - b_4^2 > 0$, then eq. (8) takes the following form (figure 1a):

$$u(x,t) = r\mathrm{e}^{(ir^2\beta t)}\left(1 + \frac{b_4^2 \cosh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t\right) + ib_4\sqrt{4c_2^2 - b_4^2}\sinh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t\right)}{c_2\sqrt{4c_2^2 - b_4^2}\cos\left(\frac{r\sqrt{2\frac{\beta}{\alpha}}b_4}{2c_2}x\right) - 2c_2^2\cosh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t\right)}\right). \tag{10}$$

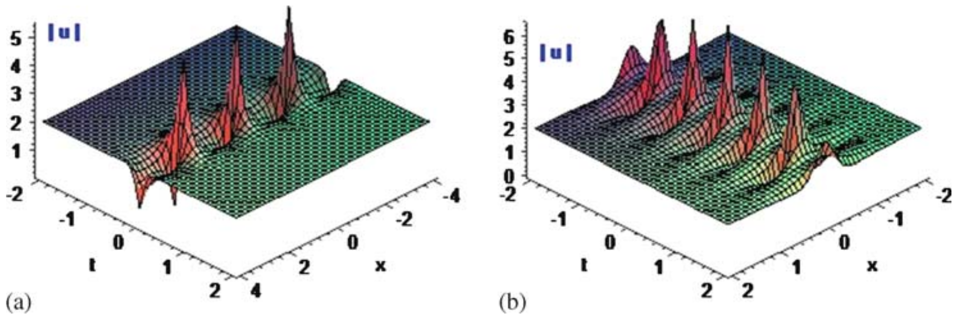

Figure 1. (**a**) The Akhmediev breather soliton with $\alpha = \frac{1}{2}, \beta = 2, r = 2, b_4 = 2, c_2 = 2$ (eq. (10)) and (**b**) the Ma breather soliton with $\alpha = \frac{1}{2}, \beta = 2, r = 2, b = 2, c_2 = 2$ (eq. (11)).

(b) *The Ma breather soliton* [22]. Setting $b_4 = ib$, eq. (8) can be given as (figure 1b)

$$u(x,t) = r\mathrm{e}^{(ir^2\beta t)}\left(1 - \frac{b^2\cos\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}t\right) + ib\sqrt{4c_2^2+b^2}\sin\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}t\right)}{c_2\sqrt{4c_2^2+b^2}\cosh\left(\frac{rb\sqrt{2\frac{\beta}{\alpha}}}{2c_2}x\right) - 2c_2^2\cos\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}t\right)}\right). \tag{11}$$

(c) *The Peregrine breather soliton* [4]. Setting $c_2 > 0$ and $b_4 \to 0$, eq. (8) takes the Peregrine breather form which is given as (figure 2a)

$$u(x,t) = r\mathrm{e}^{(ir^2\beta t)}\left(1 - \frac{4\alpha(1 + i2r^2\beta t)}{\alpha + 2\beta r^2 x^2 + 4\alpha\beta^2 r^4 t^2}\right). \tag{12}$$

We have verified that eqs (10)–(12) are solutions of NLSE$^+$, where c_2 and b are arbitrary constants. Analysis of the behaviour of these solutions can be found in [4,22,24,25].

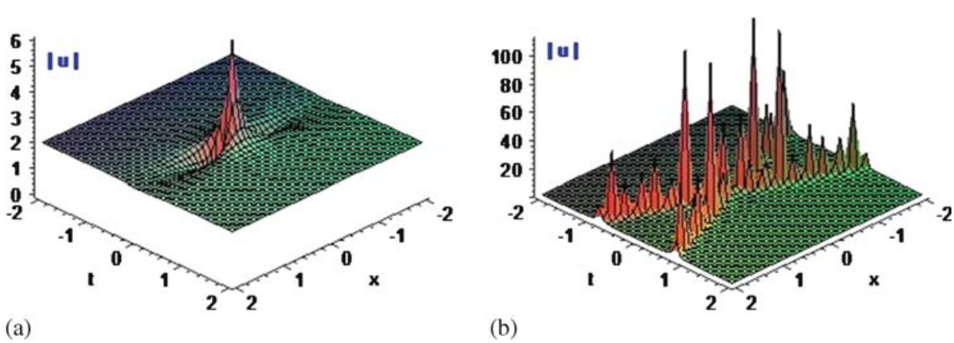

Figure 2. (**a**) The Peregrine breather soliton with $\alpha = \frac{1}{2}, \beta = 2, r = 2$ (eq. (12)) and (**b**) the bisoliton with $\alpha = \frac{1}{2}, \beta = -2, r = 2, b_4 = 2, c_2 = 2$ (eq. (8)).

Case 2. Similarly, the de-focussing NLSE ($\alpha > 0$, $\beta < 0$) has the following solutions:

(a) *The bi-soliton solution*. When $4c_2^2 - b_4^2 > 0$, eq. (8) behaves as a bi-soliton solution for NLSE$^-$ (figure 2b).

(b) *The periodic solution*. Setting $b_4 = ib$, eq. (8) takes following form:

$$u(x,t) = re^{(ir^2\beta t)}\left(1 - \frac{b^2\cos\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}t\right) + ib\sqrt{4c_2^2+b^2}\sin\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}t\right)}{c_2\sqrt{4c_2^2+b^2}\cos\left(\frac{rb\sqrt{-2\frac{\beta}{\alpha}}}{2c_2}x\right) - 2c_2^2\cos\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}t\right)}\right). \tag{13}$$

Equation (13) was verified to be the periodic solution of NLSE$^-$ and meanwhile, eq. (12) is also found to be solution of NLSE$^-$.

3. Rogue wave solutions of VCNLSE

First, we use a transformation to convert the variable coefficient nonlinear Schrödinger equation into the standard nonlinear Schrödinger equation. Assume

$$u(x,t) = Q(X(x,t), T(t))P(x,t)e^{iw(x,t)}, \tag{14}$$

to transform VCNLSE into NLSE, i.e.,

$$iQ_T + \alpha Q_{XX} + \beta|Q|^2 Q = 0. \tag{15}$$

To achieve this transformation, substitute eq. (14) into eq. (1). We find that only if $\Gamma(t) = \frac{1}{2}[\kappa'(t)\chi(t) - \kappa(t)\chi'(t)]/\kappa(t)\chi(t)$, eq. (1) can be converted into eq. (15). Then, we have

$$\begin{aligned}
P(x,t) &= \frac{\sqrt{C_1\beta\kappa(t)\chi(t)}}{\chi(t)}, \quad w(x,t) = C_0 x - C_0^2\int\kappa(t)\mathrm{d}t + C_2,\\
T(t) &= C_1\int\kappa(t)\mathrm{d}t + C_2,\\
X(x,t) &= -\sqrt{C_1\alpha}x + 2C_0\sqrt{C_1\alpha}\int\kappa(t)\mathrm{d}t + C_2,
\end{aligned} \tag{16}$$

where C_0, C_1 and C_2 are arbitrary constants. Therefore, when $\Gamma(t) = \frac{1}{2}[\kappa'(t)\chi(t) - \kappa(t)\chi'(t)]/\kappa(t)\chi(t)$, $\kappa(t) > 0$, $\chi(t) > 0$ and $\alpha > 0$, $\beta > 0$, rogue wave solutions of eq. (1) can be expressed as

$$\begin{aligned}
u(x,t) &= r\frac{\sqrt{C_1\beta\kappa(t)\chi(t)}}{\chi(t)}e^{(ir^2\beta T(t))}\\
&\times\left(1 + \frac{b_4^2\cosh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}T(t)\right) + ib_4\sqrt{4c_2^2-b_4^2}\sinh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}T(t)\right)}{c_2\sqrt{4c_2^2-b_4^2}\cos\left(\frac{r\sqrt{2\frac{\beta}{\alpha}}b_4}{2c_2}X(x,t)\right) - 2c_2^2\cosh\left(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}T(t)\right)}\right)e^{iw(x,t)},
\end{aligned} \tag{17}$$

$$u(x,t) = r\frac{\sqrt{C_1\beta\kappa(t)\chi(t)}}{\chi(t)}\mathrm{e}^{(ir^2\beta T(t))} \times \left(1 - \frac{b^2\cos\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}T(t)\right) + ib\sqrt{4c_2^2+b^2}\sin\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}T(t)\right)}{c_2\sqrt{4c_2^2+b^2}\cosh\left(\frac{rb\sqrt{2\frac{\beta}{\alpha}}}{2c_2}X(x,t)\right) - 2c_2^2\cos\left(\frac{\beta r^2 b\sqrt{4c_2^2+b^2}}{2c_2^2}T(t)\right)}\right)\mathrm{e}^{iw(x,t)} \tag{18}$$

and

$$u(x,t) = r\frac{\sqrt{C_1\beta\kappa(t)\chi(t)}}{\chi(t)}\mathrm{e}^{(ir^2\beta T(t))} \times \left(1 - \frac{4\alpha(1+i2r^2\beta T(t))}{\alpha + 2\beta r^2 X(x,t)^2 + 4\alpha\beta^2 r^4 T(t)^2}\right)\mathrm{e}^{iw(x,t)}, \tag{19}$$

where $X(x,t)$, $T(t)$ and $w(x,t)$ are given by eq. (16). When $\alpha = \frac{1}{2}, \beta = 2, \kappa(t) = (1+\cos(t)^2)^2$, $\chi(t) = 1+\cos(t)^2$, $C_0 = 2$, $C_1 = 4$, $C_2 = 1$, $r = 2$, $b = 2$, $b_4 = 2$ and $c_2 = 2$ their images are displayed by figure 3.

When $\alpha = \frac{1}{2}$, $\beta = 2$, $\kappa(t) = (1+\sin(t)^2)^2$, $\chi(t) = 1+\sin(t)^2$, $C_0 = 2$, $C_1 = 4$, $C_2 = 1$, $r = 2$, $b = 2$, $b_4 = 2$ and $c_2 = 2$, their images are displayed by figure 4.

Secondly, similar to the previous case, if $\kappa(t) > 0$ and $\chi(t) < 0$, we can apply solutions of NLSE$^-$ (solutions (8) and (13)) to eq. (1)

Third, when $\alpha = \frac{1}{2}$ and $\beta = 1$, higher-order rational solution of eq. (2) are given by Akhmediev [21,24]. Based on [21,24], the higher-order rational solution of eq. (15) are obtained as follows:

$$Q(X,T) = \sqrt{\frac{1}{\beta}}\left(1 - \frac{G+iH}{D}\right)\mathrm{e}^{iT}, \qquad \beta > 0,$$

$$G = \frac{4X^4 + 12\alpha X^2 + 72\alpha^2 T^2 + 48\alpha T^2 X^2 + 80\alpha^2 T^4 - 3\alpha^2}{16\alpha^2},$$

$$H = \frac{T(4X^4 + 8\alpha^2 T^2 + 16\alpha T^2 X^2 + 16\alpha^2 T^4 - 15\alpha^2 - 12\alpha X^2)}{8\alpha^2},$$

$$D = \frac{8X^6 + \alpha(12+48T^2)X^4 + 6\alpha^2(4T^2-3)^2X^2 + \alpha^3(9+64T^6+432T^4+396T^2)}{192\alpha^3}. \tag{20}$$

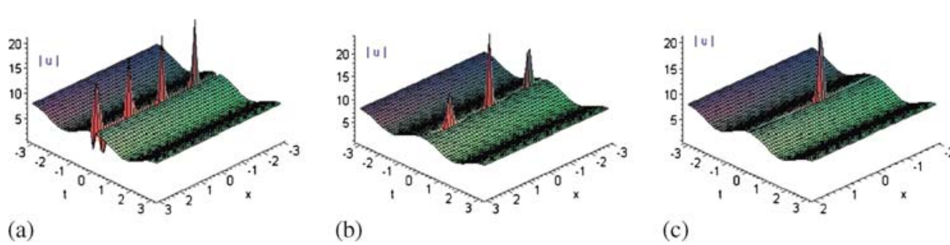

Figure 3. (**a**) The Akhmediev breather soliton, (**b**) the Ma breather soliton and (**c**) the Peregrine breather soliton.

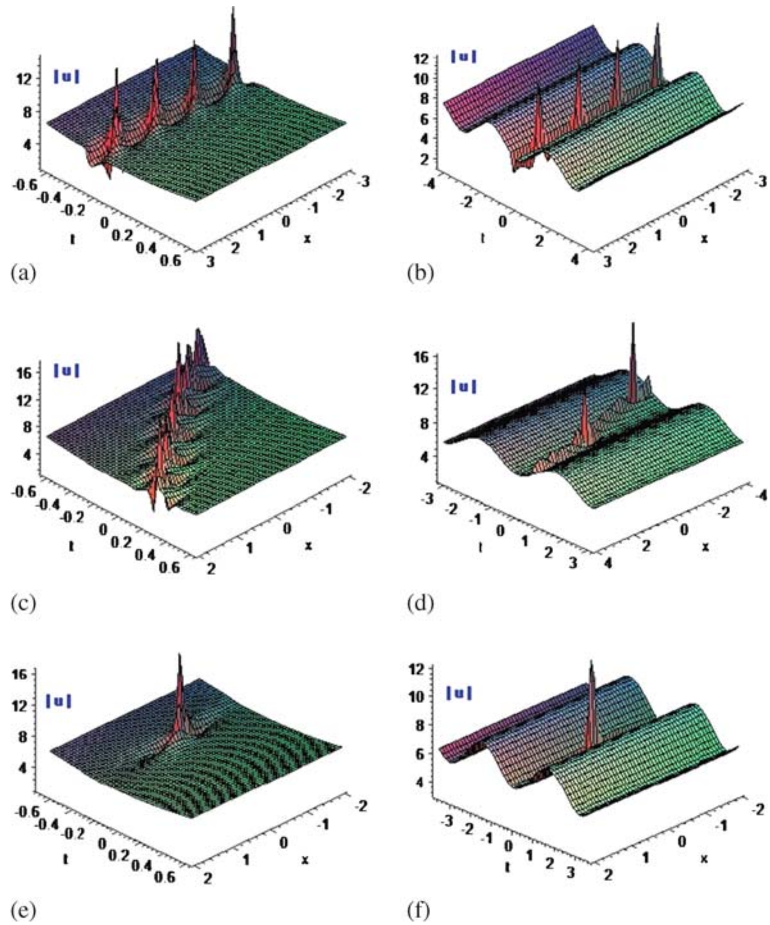

Figure 4. (**a**, **b**) The Akhmediev breather soliton, (**c**, **d**) the Ma breather soliton and (**e**, **f**) the Peregrine breather soliton.

Therefore, substituting eq. (20) into eq. (14), the obtained higher-order rational solution of eq. (1) can be written as

$$u(x,t) = Q(X,T)P(x,t)\mathrm{e}^{iw(x,t)}, \tag{21}$$

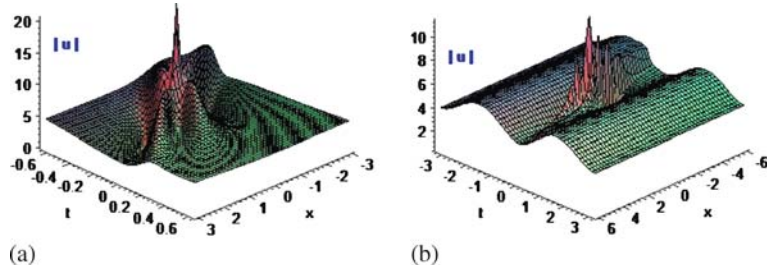

Figure 5. The higher-order rational rogue waves (solution (21)).

where $X = X(x,t)$, $T = T(t)$, $P(x,t)$ and $w(x,t)$ are provided by eq. (16) and $Q(X,T)$ is given by eq. (20). When $\alpha = \frac{1}{2}, \beta = 2, r = 2, \kappa(t) = (1+\sin(t)^2)^2, \chi(t) = 1 + \sin(t)^2$, $C_0 = 2$, $C_1 = 4$ and $C_2 = 1$, their images are displayed by figure 5.

4. Conclusion

In this paper, a unified formula solution of the standard (1+1)-dimensional NLSE, which yields a series of breather solitons (rogue waves) was obtained based on exp-function method. At the same time, by using appropriate transformation, one can transform VCNLSE into NLSE, thereby obtaining rogue wave solutions of VCNLSE which contain the Akhmediev breather soliton, the Ma breather soliton and the Peregrine breather soliton. The figures show that surface waves in the propagation process randomly generate high amplitude breather waves. The amplitude of each of these solutions depends primarily on the initial conditions and $\kappa(t)$, $\chi(t)$. The two free functions $\kappa(t)$, $\chi(t)$ and several arbitrary parameters were involved in generating a large number of wave structures.

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