

Systematic of signature inversion in $(h_{11/2})_p \otimes (i_{13/2})_n$ for odd–odd nuclei in rare-earth nuclei

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Abstract. Systematic features of the signature inversion phenomenon in $(h_{11/2})_p \otimes (i_{13/2})_n$ in doubly-odd rare-earth nuclei are presented. These features are generally observed in high- j orbitals, mainly, in $(h_{9/2})$, $(h_{11/2})$ and $(i_{13/2})$. Calculations are carried out within the framework of the two quasiparticle plus rotor model (TQPRM) to explain strong odd–even staggering and signature inversion observed in these high- j orbitals. The shifting of point of inversion to lower/higher spin with the increase in neutron/proton numbers is well explained by the calculations. It is found that $1/2[541]$ proton orbital of $h_{9/2}$ is necessary in the lower mass region to obtain the point of inversion.

Keywords. Signature inversion; odd–even staggering; Coriolis coupling; particle–particle coupling.

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1. Introduction

Many experiments have been done in rare-earth region in the last few decades, and a large amount of data is available for comprehensive study in this region [1–5]. It is a great challenge to analyse rotational bands based on 2qp states of high- j orbitals in this region. From the available data, many interesting features like large odd–even staggering [6–8], signature inversion [9–13] and signature reversal [14,15] were observed in the rotational bands of high- j orbitals of rare earths. Recently, double signature inversion in $\{(h_{9/2})_p \otimes (i_{13/2})_n\}$ and $\{(i_{13/2})_p \otimes (i_{13/2})_n\}$ orbitals were observed in ^{184}Au [16]. Also in the Tl isotopes of $\{(h_{9/2})_p \otimes (i_{13/2})_n\}$ configuration, the low spin signature inversion [17–19] was observed which seemed to be a very challenging and an interesting

problem of odd–odd nuclei. To explain the phenomenon of signature inversion, scientists have used several models such as the angular momentum projection method [8], the interacting boson–fermion model [20], the cranked shell model [13] and the particle plus rotor model [9,10,21,22].

It is well known that the coupling of odd protons and odd neutrons gives rise to a pair of bands with band quantum number $K_+ = (\Omega_p + \Omega_n)$ and $K_- = |\Omega_p - \Omega_n|$. The $K_+ = (\Omega_p + \Omega_n)$ bands of odd–odd nuclei generally exhibit very smooth behaviour in contrast to the $K_- = |\Omega_p - \Omega_n|$ bands which exhibit odd–even staggering. However, the K_+ bands composed of high- j configuration displays a stronger odd–even staggering than K_- bands in their rotational energy spacing, implying a signature dependence and also signature inversion. These features are generally observed in high- j orbitals like $h_{9/2}$, $g_{7/2}$, $h_{11/2}$ and $i_{13/2}$.

At high rotational frequencies, signature and parity are the only two good quantum numbers available to label a state. Signature quantum number is denoted by α . From the rotational properties of a wave function, it follows that $\alpha = 0$ for even-angular momentum states of a rotational band and $\alpha = 1$ for odd-angular momentum states. We have analysed the $(h_{11/2})_p \otimes (i_{13/2})_n$ orbitals of double-odd nuclei in rare-earth region. The favoured signature in odd–odd rotational bands is given by $\alpha_f = 1/2 (-1)^{j_p - (1/2)} + 1/2 (-1)^{j_n - (1/2)}$. For $(h_{11/2} \otimes i_{13/2})$ configuration bands $\alpha_f = 0$, the even-spin members should lie lower in energy. It has, however, been observed that the odd-spin members lie lower in energy upto a certain spin I_c and the normal signature dependence is restored later. This anomalous feature is termed as signature inversion. Study of high- j orbitals has been done in the past by Goel *et al* which was well explained within the framework of two-quasiparticle rotor model (TQPRM). Over the years, sufficient data have been accumulated by researchers to undertake a meaningful study of the systematic of the signature inversion phenomenon. So, here we have revisited the experimental data of high- j orbitals for odd–odd nuclei in rare earths.

We have analysed the data for all the 41 experimentally known cases ($Z = 61$ –75) for all available isotopes of Pm to Re. The large odd–even staggering and signature inversion are observed in high- j $((h_{11/2})_p \otimes (i_{13/2})_n)$ orbitals in rare earths. The main features of the systematics of high- j $\{(h_{11/2})_p \otimes (i_{13/2})_n\}$ orbitals are: (i) the point of signature inversion I_c shifts towards lower spin with increasing neutron number in a chain of isotopes, (ii) the point of signature inversion I_c shifts towards higher spin with increasing proton number in a chain of isotones and the magnitude of staggering before the inversion point becomes smaller with increasing neutron number in a chain of isotopes.

We have carried out the TQPRM calculations to produce large odd–even staggering and shifting in the point of inversion I_c towards lower/higher spin with increasing neutron/proton number. We have chosen $_{67}\text{Ho}$ chain to show shifting in the point of inversion I_c towards lower spin with increasing neutron number from ^{158}Ho to ^{164}Ho and also the point of signature inversion I_c shifts towards higher spin with increasing proton number in a chain of isotones in ^{156}Tb , ^{158}Ho , ^{160}Tm . It is found that TQPRM calculations can reproduce the point of inversion and magnitude of odd–even staggering in Ho isotopes and ^{156}Tb , ^{160}Tm . It is very important to mention that inclusion of $1/2[541]$ orbital of $h_{9/2}$ is necessary to obtain the point of inversion in lower mass region as in ^{152}Eu and ^{156}Tb [10]. To achieve this, we have only carried out schematic calculations without the actual fitting of the input parameters of the model.

A brief detail of the model and methodology is given in §2. Section 3 presents the results and discussion. This section contains detailed analysis of the 41 experimentally known cases to show the systematic change in magnitude of odd–even staggering and point of inversion as the proton/neutron number increases. Experimental γ energies E_γ (keV) vs. angular momentum (I) are plotted to show systematic change in point of inversion and large magnitude of staggering. The results obtained by TQPRM calculations for $^{158-164}_{67}\text{Ho}$ chain, $^{156}_{65}\text{Tb}$ and $^{160}_{69}\text{Tm}$ are presented in the same section. Section 4 contains the conclusion.

2. Model and methodology

We have used an axially symmetric TQPRM for calculations. A detailed description of the model may be found in other works [7]. A brief description is, however, presented here.

The total Hamiltonian is divided into two parts, the intrinsic and the rotational,

$$H = H_{\text{intr}} + H_{\text{rot}}. \quad (1)$$

The intrinsic part consists of a deformed axially symmetric average field H_{av} , a short-range residual interaction H_{pair} and a short-range neutron–proton interaction V_{np} , so that

$$H_{\text{intr}} = H_{\text{av}} + H_{\text{pair}} + V_{\text{np}}. \quad (2)$$

The vibrational part has been neglected in this formulation. For an axially symmetric reflection-symmetric rotor

$$H_{\text{rot}} = \hbar^2/2\Im(I^2 - I_3^2) + H_{\text{cor}} + H_{\text{ppc}} + H_{\text{irrot}}, \quad (3)$$

where

$$\begin{aligned} H_{\text{cor}} &= -\hbar^2/2\Im(I_+j_- + I_-j_+), \\ H_{\text{ppc}} &= \hbar^2/2\Im(j_{p+}j_{n-} + j_{p-}j_{n+}), \\ H_{\text{irrot}} &= \hbar^2/2\Im\left[(j_p^2 - j_{pz}^2) + (j_n^2 - j_{nz}^2)\right]. \end{aligned}$$

The particle angular momentum j is given by the sum of the angular momentum of the odd proton j_p and the odd neutron j_n . The operators $I_\pm = I_1 \pm iI_2$, $j_\pm = j_1 \pm ij_2$, $j_{n\pm} = j_{n1} \pm ij_{n2}$ and $j_{p\pm} = j_{p1} \pm ij_{p2}$ are the usual shifting operators. $\Im$ is the moment of inertia with respect to the rotational axis. The set of basis eigenfunctions of $H_{\text{av}} + \hbar^2/2\Im(I^2 - I_3^2)$ may be written in the form of symmetrized product of the rotational wave function D_{MK}^I and the intrinsic wave function $|K\alpha_\rho\rangle$ can be written as

$$|IMk\alpha_\rho\rangle = \left[\frac{2I+1}{16\pi^2(1+\delta_{KO})} \right]^{1/2} [D_{MK}^I |K\alpha_\rho\rangle + (-1)^{1+K} D_{M-K}^I R_i |K\alpha_\rho\rangle], \quad (4)$$

where the index α_ρ characterizes the configuration ($\alpha_\rho = \rho_p\rho_n$) of the odd neutron and the odd proton.

Diagonalization of the total Hamiltonian matrix for each value of the angular momentum I gives the energies $E_{\text{th}}(I, \alpha_\rho, \sigma)$ for all bands built on the two-quasiparticle

configuration $|K\alpha\rho\sigma\rangle$ present in the basis set of the eigenfunctions. The Newby shift [24] is considered as a parameter along with other parameters such as the quasiparticle energies E_α , the moment of inertia $\mathfrak{J}$ and the single-particle matrix elements (j_+). The symbol $\sigma = \pm$ is used to denote the two types of bands K_+ and K_- . The correct choice of the set of basis function is very important as all the states which may couple together and influence each other should be included in the calculations. We have included the orbitals of $(h_{11/2})_p$ and $(i_{13/2})_n$ for calculations. In addition we have also included $1/2[541]_p$ of $h_{9/2}$ orbitals, as these orbitals play important roles in the signature phenomenon in lower mass region [10].

The Nilsson model parameters κ and μ are taken as 0.0637 and 0.614 for protons and 0.0637 and 0.393 for neutrons [25]. The single-particle energies are estimated from the neighbouring odd- A nuclei [26]. Then, the single-particle energies are used to calculate the one-quasiparticle (1qp) energies (E_{qp}) for all cases by the formula, $E_{qp}(\Omega) = \sqrt{(\epsilon_\Omega - \lambda)^2 + \Delta^2} - \Delta$ with pairing gap $\Delta = 1$ MeV. These 1qp energies are used to calculate the two-quasiparticle (2qp) excitation energies by using the formula

$$E_{qp}(\Omega_p, \Omega_n) = E_{qp}(\Omega_p) + E_{qp}(\Omega_n) + (\hbar^2/2\mathfrak{J})K \pm \frac{1}{2}E_{GM} - \delta_{K,0}E_N.$$

E_{GM} is the Gallagher–Mosczkowski (GM) splitting energy and GM splitting of ± 100 keV is used here for the calculations, E_N is the Newby shift parameter for $K_- = 0$ bands. Throughout these calculations, value of the rotational parameter ($\hbar^2/2\mathfrak{J}$) is fixed at 10.0 and 10.5 keV for K_+ and K_- bands, respectively. All the above written parameters are adjusted during the calculations. The deformation parameters for each nucleus are taken from Mollar and Nix [27].

3. Results and discussion

We have analysed 41 cases of $(h_{11/2})_p \otimes (i_{13/2})_n$ orbitals in rare-earth region ($Z = 61-75$) [28]. The summary of the compiled data is given in table 1. From the systematics we have also estimated the configuration which was not known experimentally. The experimental staggering plot of γ energies E_γ (keV) with angular momentum (I) are shown in figures 1 and 2. These plots show the variation in magnitude of odd–even staggering and signature inversion observed in rare earths. The figures also show shifting in the point of inversion with varying proton/neutron number.

The results of the schematic TQPRM calculations with $\Delta E(I \rightarrow I-1)/2I$ vs. I are presented in figures 3a and 3b. These plots show the shifting in the point of inversion as neutron/proton number increases. The critical spin I_c of inversion is indicated by arrows in the plots. We have done two sets of calculations: one for ${}_{67}\text{Ho}$ isotopes to show shift in the point of inversion towards lower spin as neutron number increases. The results are shown in figure 3a. In the second set of calculations we have chosen ${}^{156}\text{Tb}$, ${}^{158}\text{Ho}$ and ${}^{160}\text{Tm}$ with the same neutron number ($N = 91$) in which the point of inversion shifts to higher spin with increase in proton number. The magnitude of the odd–even staggering is very large as actual fitting of the experimental data is not done. The calculations are done to show variation in the point of inversion with neutron and proton numbers. We

Table 1. The experimental data are taken from nndc site. In the table $K_+ = (\Omega_p + \Omega_n)$ bands of high j -orbitals $\{(h_{11/2})_p \otimes (i_{13/2})_n\}$ for rare earths are reported. Each box lists the following information: Top row: K (observed), I_c (inversion), Middle row: K and configuration from systematic, Bottom row: lowest observed spin and its energy in keV. xxx denotes unknown energy. Configurations are abbreviated as: protons: $A = 5/2[532]$, $B = 7/2[523]$, $C = 9/2[514]$, neutrons: $U = 1/2[660]$, $V = 3/2[651]$, $W = 5/2[642]$, $X = 7/2[633]$, $Y = 9/2[624]$, $Z = 11/2[615]$. Complete study of all the nuclei with their different isotopes is done by plotting graphs between gamma energies and angular momenta. The analysis is shown in the table.

	87	89	91	93	95	97	99	101	103	105	107	109
⁷⁵ Re								? N 8, CX 8, NC**	8, SR 8, CX 8, NC**	9, ? 9, CY 9, xxx	9, N 9, CY 9, 443	10, NS* 10, CZ 10, 917
⁷³ Ta			? SR 5, CU 11, NC**	? 20 6, CV 9, NC**	? 18 6, CV 10, NC**	? ? 7, CW 10, NC**	8, ? 8, CX 8, xxx					
⁷¹ Lu		? SR 4, BU 9, NC**	? 20 5, BW 10, NC**	? 18 7, CW 7, NC**	7, 16 7, CW 8, 287	7, ? 7, ? 7, CW 9, NC**	? N 7, CX 8, NC**					
⁶⁹ Tm		? >27 4, BU 9, xxx	? 18 6, BW 9, NC**	6, 16 6, BW 6, NC**	6, 15 6, CW 6, NC**	6, 13 6, BW 6, NC**	? N 7, BW 7, NC**					
⁶⁷ Ho	? N 4, BU 11, NC**	? 19 4, BU 9, xxx	5, 16 5, BV 8, 208	6, 15 6, BW 6, 118	6, N 6, BW 6, 106	6, NS* 6, BW 6, 140						
⁶⁵ Tb	? N 3, AU 9, 1212	? 16 3, AU 9, NC**	4, 12 4, AV 6, 379									
⁶³ Eu		5, 14 4, AU Or AV 5, 181										

For ¹⁷²Ta and ¹⁷⁸Re, no information is available. We have given from systematic.

SR signifies abnormal behaviour, i.e., odd spin is favoured throughout.

N signifies no inversion, only odd-even staggering.

*NS signifies smooth behaviour, no odd-even staggering.

**NC signifies unconfirmed energy, in the form of W+, X+, Y+, Z+.

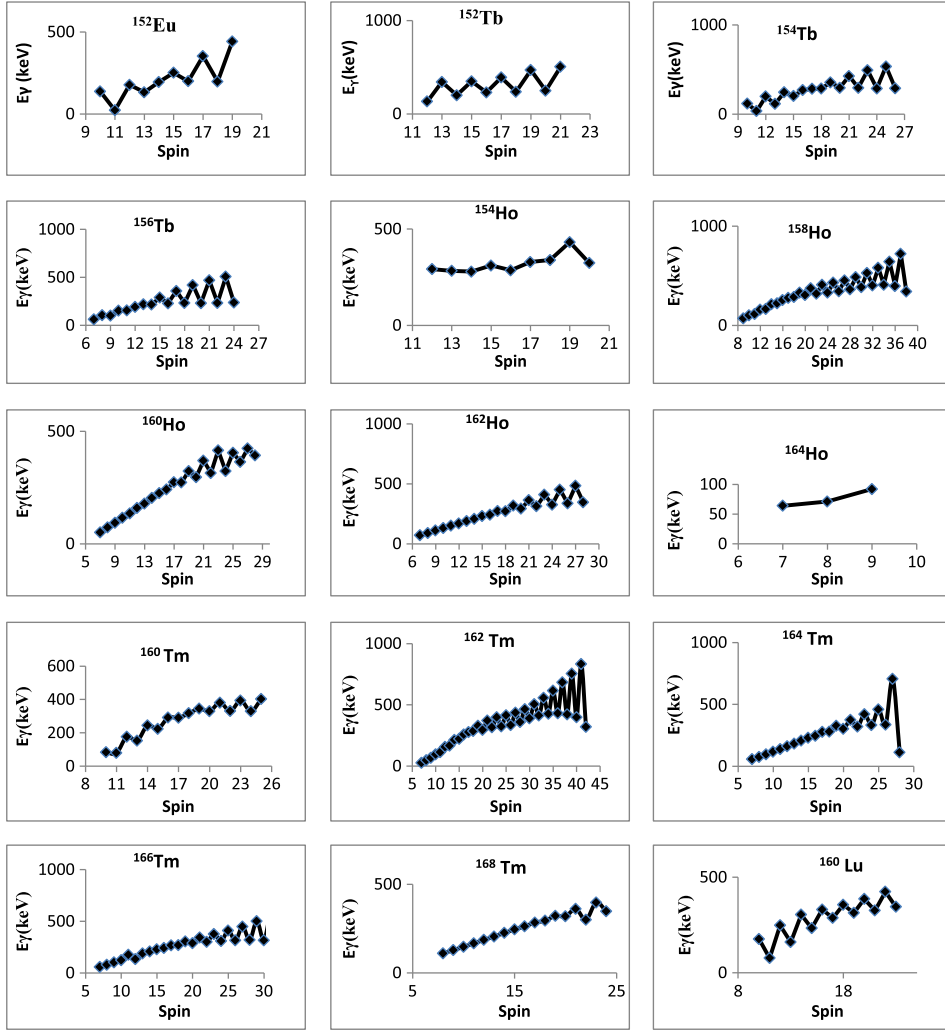


Figure 1. Experimental plots for E_γ (keV) vs. I for rare in earth elements from $Z = {}_{61}\text{Pm}$ to ${}_{75}\text{Re}$ for high- j orbitals.

have adjusted the single-particle matrix element for $(h_{11/2})$ proton and $(i_{13/2})$ neutron to obtain the required pattern. But actual fitting has not been done. Table 2 represents the results of TQPRM calculations for ${}_{67}\text{Ho}$ and ${}^{156}\text{Tb}$, ${}^{160}\text{Tm}$ isotopes along with the shift in point of inversion.

The large odd-even staggering originates from the odd-even effects present in the $K_- = 0, \{1/2[550]_p \otimes 1/2[660]_n\}$ bands. The odd-even staggering of $K_- = 0$ band originates from the large decoupling term in the diagonal matrix element and the Newby shift [23,24]. Newby shift is defined as

$$C_\alpha = \langle \rho_p \Omega_p; \rho_n - \Omega_n | V_{np} | \rho_p - \Omega_p; \rho_n \Omega_n \rangle,$$

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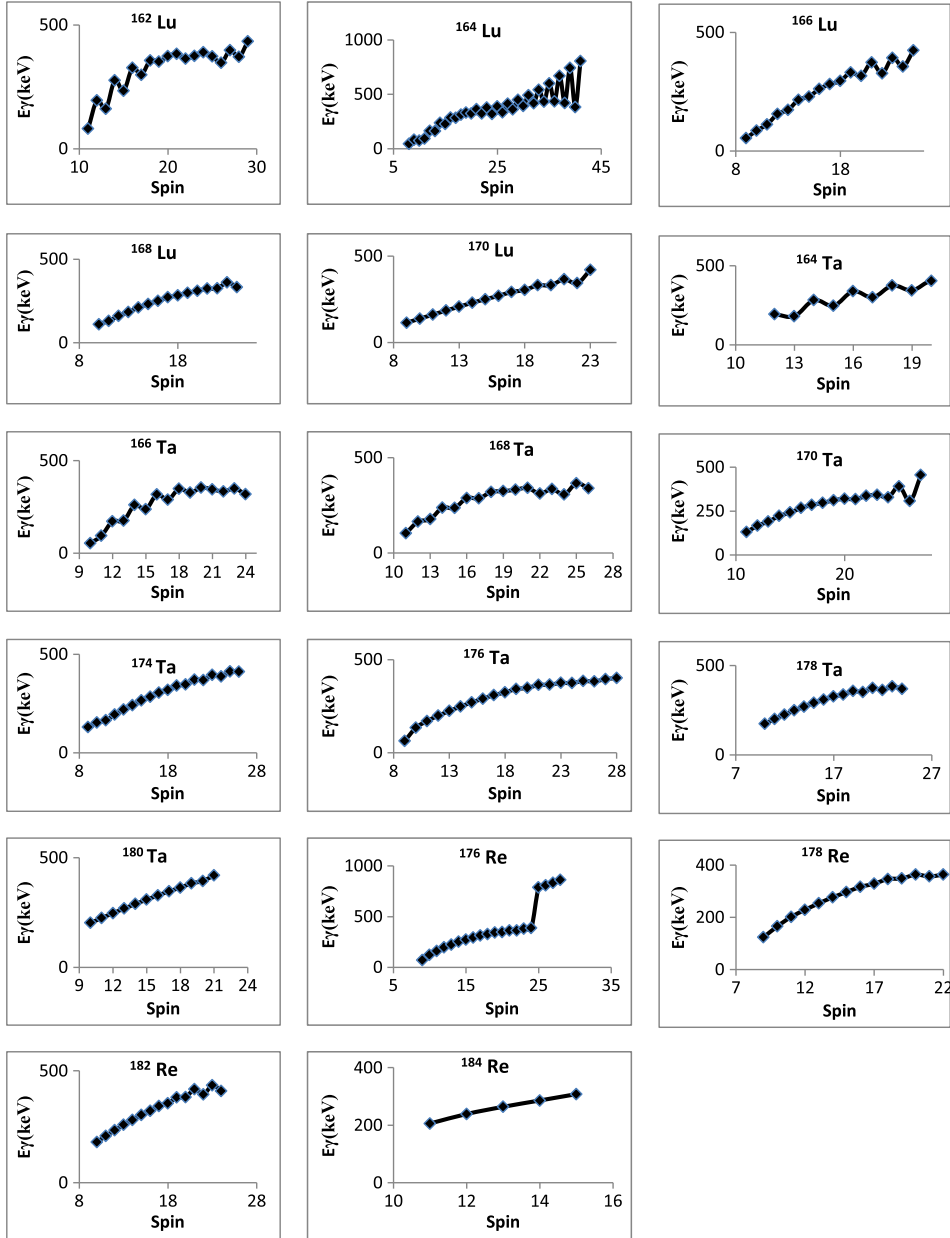


Figure 2. Same as figure 1, but for higher Z elements of rare earths.

where ρ_p and ρ_n are the quasiproton and quasineutron states, respectively and the decoupling term in the diagonal matrix element is given as

$$(-\hbar^2/2\mathfrak{I}) [I(I+1)]^{1/2} [a_p + a_n (-1)^{I+1}].$$

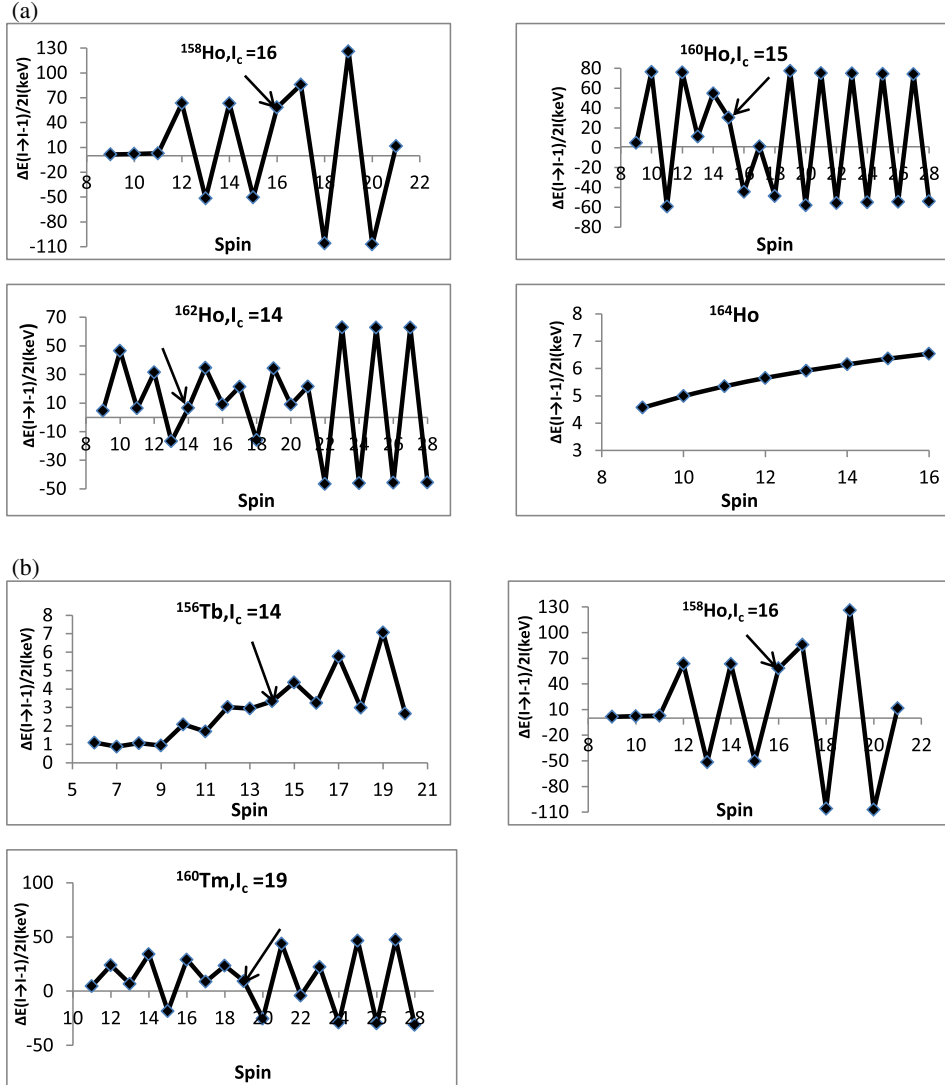


Figure 3. (a) The TQPRM plots of $\Delta E(I \rightarrow I-1)/2I$ vs. I for ${}_{67}\text{Ho}$ isotopic chain to show shift in point of inversion to a lower spin as neutron number increases. (b) The TQPRM plots of $\Delta E(I \rightarrow I-1)/2I$ vs. I for ${}^{156}\text{Tb}$, ${}^{158}\text{Ho}$, ${}^{160}\text{Tm}$ to show shift in point of inversion to a higher spin as proton number ($N = 91$) increases. The critical spin I_c is indicated by the arrow. The calculations are only to show systematic change in the point of inversion. Actual fitting of experimental data have not been done.

This odd-even staggering in $K_- = 0$ bands is transmitted to higher K bands through Coriolis coupling and particle-particle coupling. The inversion is due to reverse behaviour of $K_- = 0$ and $K_+ = 1$ bands. The $K_- = 0$, $\{1/2[550]_p \otimes 1/2[660]_n\}$ band favours even spin

Table 2. The calculations for $^{156}_{65}\text{Tb}$, $^{67}_{67}\text{Ho}$ ($^{158}_{67}\text{Ho}$, $^{160}_{67}\text{Ho}$, $^{162}_{67}\text{Ho}$, $^{164}_{67}\text{Ho}$), $^{160}_{69}\text{Tm}$. N denotes the neutron number and I_c denotes the point of inversion obtained by calculations.

$^{160}_{69}\text{Tm}$ $K = 6, I_c = 19$				
$^{67}_{67}\text{Ho}$	$K = 5, I_c = 16$	$K = 6, I_c = 15$	$K = 6, I_c = 14$	$K = 6, \text{No staggering}$
N	91	93	95	97
$^{156}_{65}\text{Tb}$ $K = 4, I_c = 14$				

The following points can be well observed from this table:

- (i) When we move from ^{158}Ho to ^{164}Ho , the point of inversion shifts towards lower spin. It shifts from 16 (in ^{158}Ho) to 14 (in ^{162}Ho). ^{164}Ho signifies no inversion but only smooth behaviour.
- (ii) When we move from lower to higher Z element with the same neutron number, then the point of inversion shifts to higher spin. For ^{156}Tb , ^{158}Ho , ^{160}Tm neutron number is 91 and point of inversion increases to 14, 16 and 19, respectively.

while $K_+ = 1$ favours odd spin. The reverse behaviour of the $K_- = 0$ and $K_+ = 1$ bands is transmitted to higher K bands through Coriolis coupling and particle–particle coupling.

The deformation parameters ($\varepsilon_2, \varepsilon_4$) used in the calculations are: (0.233, -0.013), (0.250, -0.013), (0.250, -0.007), (0.258, 0.007) for Ho isotopes, (0.233, -0.040) for ^{156}Tb and (0.208, -0.007) for ^{160}Tm [27]. As the deformation changes with isotopes, the Fermi energy is accordingly shifted to represent the increase in neutron number in a chain of isotopes and increase in proton number in a chain of isotones. The suggested Nilsson configurations for ^{158}Ho , ^{160}Ho , ^{162}Ho , ^{164}Ho are $\{7/2[523]_p \otimes 3/2[651]_n\}$, $\{7/2[523]_p \otimes 5/2[642]_n\}$, $\{7/2[523]_p \otimes 5/2[642]_n\}$, $\{7/2[523]_p \otimes 5/2[642]_n\}$, respectively. It is clear from the configurations that ($h_{11/2}$), $7/2[523]$ proton orbital is closest to the Fermi energy in all the Ho Isotopes. As we move from ^{156}Ho to ^{164}Ho , the Fermi level of neutron shifts from $3/2[651]_n$ to $5/2[642]_n$. These orbitals belong to ($i_{13/2}$) which favours even angular momenta. That is why the normal feature (favours even spin) is restored at the lower spin with increase in neutron number. So, the point of inversion shifts to lower spin with increase in neutron number in a chain of isotopes. Similarly, in ^{156}Tb , ^{158}Ho , ^{160}Tm (neutron number $N = 91$) the Fermi level of neutron is the same, i.e., $3/2[651]_n$ but Fermi level of proton shifts from $5/2[532]_p$ to $7/2[523]_p$. The proton orbitals belong to ($h_{11/2}$) and it favours odd angular momenta. So the effect of proton orbitals dominate and odd spin favours upto the higher spin after normal feature is restored. Therefore, in a chain of isotones the point of inversion shifts to higher spin with increase in proton numbers.

3.1 Signature inversion in ^{67}Ho isotopes

We have done TQPRM calculations for ^{158}Ho , ^{160}Ho , ^{162}Ho and ^{164}Ho . The details of ^{160}Ho calculation are presented here. The reason for signature inversion in ^{160}Ho lies in the odd–even staggering pattern of $K_- = 0$ and $K_+ = 1$ with configuration $\{1/2[550]_p \otimes 1/2[660]_n\}$. The behaviour of $K_- = 0$ and $K_+ = 1$ for ^{160}Ho is shown in figure 4, which

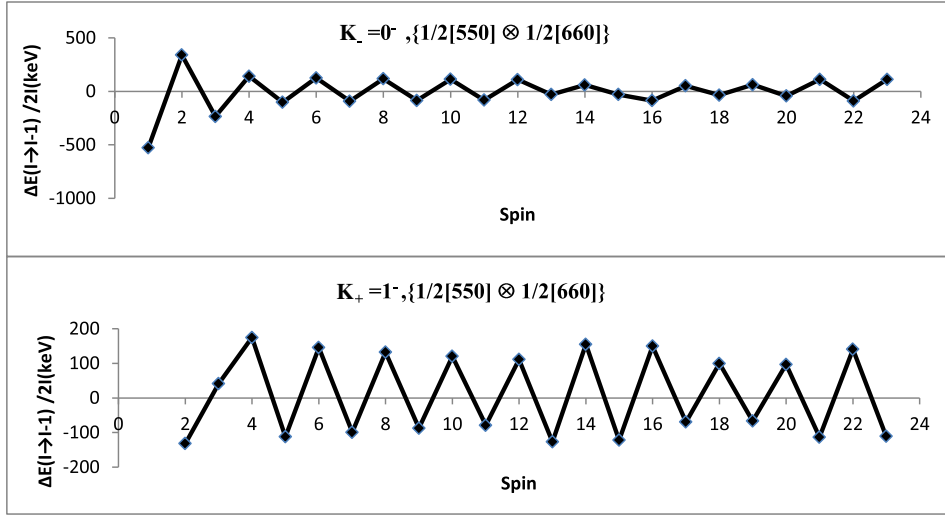


Figure 4. The staggering plots of the $K_+ = 1$ and the $K_- = 0$ bands used in the calculations of ^{160}Ho .

shows that the $K_+ = 1$ band favours odd spin and $K_- = 0$ band favours odd spin in the lower region and after a particular spin, even spins are favoured. The opposite signature effect of $K_- = 0$ and $K_+ = 1$ bands are transmitted to the $K_+ = 6$ band through Coriolis coupling and particle-particle coupling producing odd-even staggering and signature inversion. The mixed feature of both $K_- = 0$ and $K_+ = 1$ bands is transmitted to $K_+ = 6$ band. The most important chain involved in the mixing of ^{160}Ho is shown in figure 5. The Newby shift for $K_- = 0$ of $\{1/2[550]_p \otimes 1/2[660]_n\}$ is assigned a value of -200 keV in the calculations but it is not effective. The decoupling parameter plays an important role as on changing its value the odd-even staggering and point of inversion are changing. The point of inversion is obtained at $I_c = 15$ by adjusting the decoupling parameters. These calculations suggest that the Coriolis mixing can reproduce the odd-even staggering as well as the signature inversion in ^{160}Ho .

3.2 Signature inversion in ^{160}Tm

Similarly in ^{160}Tm for $K_+ = 6$, $\{7/2[523]_p \otimes 5/2[642]_n\}$, the signature inversion can be obtained as in ^{160}Ho . The reverse behaviour of $K_- = 0$ and $K_+ = 1$ with configuration $\{1/2[550]_p \otimes 1/2[660]_n\}$ is also responsible for the odd-even staggering and signature inversion in $K_+ = 6$. The point of inversion shifts to higher spin ($I_c = 19$) in ^{160}Tm due to change in Fermi energy of proton.

3.3 Signature inversion in ^{156}Tb

The signature inversion phenomenon in ^{156}Tb is not achieved by the basis set of $\{(h_{11/2})_p \otimes (i_{13/2})_n\}$ orbitals only, but it requires inclusion of $1/2[541]$ of proton orbital of

Signature inversion for odd-odd nuclei in rare-earth nuclei

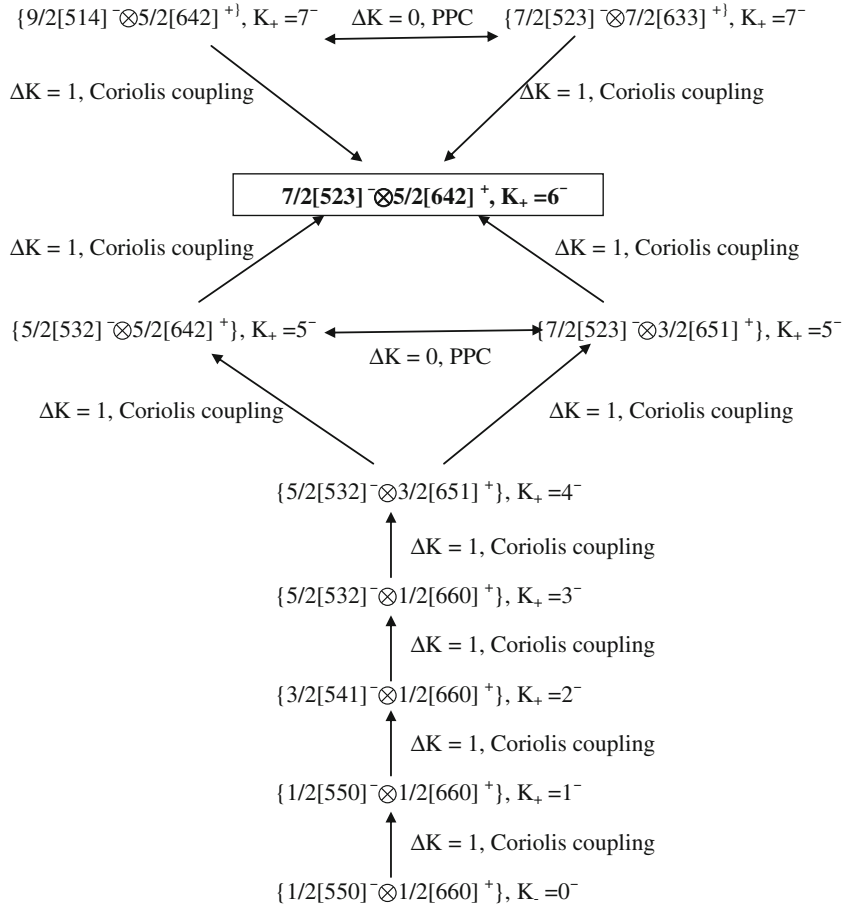


Figure 5. The chain diagram of K_+ and K_- bands responsible for the signature inversion observed in the $K_+ = 6^-$ band of ^{160}Ho .

$h_{9/2}$. As the $K_- = 0$, $\{1/2[541]_p \otimes 1/2[660]_n\}$ band is closer to the Fermi energy in the lighter isotopes, the signature inversion effect has to be taken into account. We focus our attention on the behaviour of $K_- = 0$ of $\{1/2[541]_p \otimes 1/2[660]_n\}$ band. The behaviour of $K_- = 0$ and $K_+ = 1$ of $\{1/2[541]_p \otimes 1/2[660]_n\}$ is opposite to $K_- = 0$ and $K_+ = 1$ of $\{1/2[550]_p \otimes 1/2[660]_n\}$. Even angular momenta are favoured for $K_- = 0$ of $\{1/2[541]_p \otimes 1/2[660]_n\}$ and odd angular momenta are favoured for $K_+ = 1$ of $\{1/2[541]_p \otimes 1/2[660]_n\}$, while in $\{1/2[550]_p \otimes 1/2[660]_n\}$ the odd angular momenta are favoured for $K_- = 0$ and even angular momenta in $K_+ = 1$ GM partner. The reverse behaviour of the two sets of $K_- = 0$ and $K_+ = 1$ is responsible for the signature inversion in ^{156}Tb with configuration $\{5/2[532]_p \otimes 3/2[651]_n\}$ for $K_+ = 4$. The Newby shift for $K_- = 0$ of $\{1/2[541]_p \otimes 1/2[660]_n\}$ is +10 keV, having no effect on the staggering pattern but the point of inversion varies as the value of decoupling parameter of $1/2[541]$ changes. The signature inversions in ^{156}Tb and ^{152}Eu are obtained by including the $1/2[541]$ proton orbital of $h_{9/2}$ referred in [11] and supports our results.

From the above discussion, it is clear that the point of inversion shifts to lower spin with the increase in neutron number in a chain of isotopes and point of inversion shifts to higher spin with an increase in proton number in a chain of isotones. The importance of $1/2[541]$ proton orbital of $h_{9/2}$ in lower mass region is justified by the above calculations.

4. Conclusion

We have presented the systematics of high- j orbitals in odd–odd nuclei of rare earths where the point of inversion and magnitude of odd–even staggering change with varying neutron/proton number. The point of signature inversion I_c shifts towards lower spin with increasing neutron number in a chain of isotopes as in Ho chain from ^{158}Ho to ^{164}Ho . The point of signature inversion I_c shifts towards higher spin with increasing proton number in a chain of isotones as seen in ^{156}Tb , ^{158}Ho , ^{160}Tm . It has been proved by TQPRM calculations that inclusion of $1/2[541]$ proton orbital of $h_{9/2}$ is necessary in lower mass region of rare earths as in ^{152}Eu and ^{156}Tb to obtain exact point of signature inversion. From the calculations, we have shown the variation of point of inversion towards lower/higher spin with increasing neutron/proton number. These calculations are purely schematic and exact fitting of the parameters is not done. The matrix elements and band head energies of different bands are reduced manually to obtain the odd–even staggering feature.

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