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Synchronization of general complex networks via adaptive control schemes

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Abstract. In this paper, the synchronization problem of general complex networks is investigated by using adaptive control schemes. Time-delay coupling, derivative coupling, nonlinear coupling etc. exist universally in real-world complex networks. The adaptive synchronization scheme is designed for the complex network with multiple class of coupling terms. A criterion guaranteeing synchronization of such complex networks is established by employing the Lyapunov stability theorem and adaptive control schemes. Finally, an illustrative example with numerical simulation is given to show the feasibility and efficiency of theoretical results.

Keywords. Synchronization; complex network; general couple; adaptive control.

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1. Introduction

In recent years, complex networks have become considerably interesting in various science and technology fields [1–11]. The investigation on dynamical complex networks becomes more and more important with the development of industry and the growth in realization of physics, biology, and social sciences. Therefore, it is very interesting and important to investigate the synchronization dynamical behaviours of various coupled complex networks.

The synchrony of all dynamical nodes for coupled complex network is a prominent phenomenon. In a case where not all the dynamical nodes synchronize, the controllers may be designed to ensure synchronization. Some controllers have been commonly used, such as feedback and delayed feedback controllers [12,13], nonlinear adaptive feedback controllers [14–17], and so on.

Coupled linear ordinary differential equations are widely used to describe a large class of dynamical systems with continuous time and state, as well as discrete space. This class of dynamical systems has been extensively investigated as theoretical models of synchronization in complex networks [18–27]. Analytical results have shown that quite rigorous mathematical conditions are required to guarantee the synchronization of complex networks. Yet in practice, such synchronization is urgently expected [28,29]. Although pre-exist synchronization schemes are quite simple, the assumptions of network models are not always reasonable or complete. One key reason is that a huge quantity of nodes and complexity will lead to partially or completely coupling structures of complex networks.

Some authors utilized adaptive methods to deal with the synchronization problem of complex networks with nonlinear couplings [30–35]. Some others used the knowledge of nonlinearities to construct controllers for synchronization of complex networks. In this case, the nonlinear couplings have been considered [36–40].

Moreover, as we know, time-delay exists commonly in real-world complex networks, and cannot be ignored in many cases like the finite speed of transmission, long-distance communication, traffic congestion and so on. Therefore, time-delays should be modelled in the controlled network.

Furthermore, in some cases the more realistic network model should also include the past change rate information of the state variables of complex networks, such as the stock transaction system, the population ecological system, the biological system and ecosystem, where each node's state is defined by the present and historical fluctuating rate information. Recently, the synchronization problem of a general complex network with non-derivative and derivative coupling was considered [41]. Synchronization of complex networks with derivative coupling and time-delay coupling was investigated by adaptive control schemes [42].

However, our understanding of the synchronization of complex networks is still insufficient. On the one hand, there are a few results concerning nonlinear coupling, time-delay coupling and derivative coupling, simultaneously and on the other hand, no study was done on synchronization of general complex networks consisting of more models.

Motivated by the above discussions, in this paper, we shall formulate the synchronization problem for general complex networks with time-delay coupling, nonlinear coupling and derivative coupling. The most important aims of this paper are to establish a synchronization criterion and propose effective adaptive synchronization schemes for a general complex network. These criteria and schemes will be given to ensure such a network to be global synchronization.

The rest of this paper is organized as follows. In §2, a general complex network is introduced and several hypotheses and lemmas are given. In §3, the synchronization problem is investigated, and an adaptive synchroniation controller is designed. Numerical simulations for verifying the theoretical results are given in §4. Finally, conclusions are presented in §5.

2. Problem formulation

2.1 *Model description*

In ref. [41], the synchronization of complex dynamical networks with non-derivative coupling and derivative coupling was investigated, and the complex network can be represented by

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij}^{(1)} x_j(t) + \sum_{j=1}^{N} c_{ij}^{(2)} \dot{x}_j(t), \quad i \in \mathcal{I}, \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in R^n$ is the state vector of node i, $\mathcal{I} = \{1, 2, \ldots, N\}$, $f(x_i(t)) = [f_1(x_i(t)), f_2(x_i(t)), \ldots, f_n(x_i(t))]^T \in R^n$ is a smooth non-linear vector-valued function, $C^{(k)} = [c_{ij}^{(k)}]_{N\times N} \in R^{N\times N} (k = 1, 2)$ are the coupling matrices. $c_{ii}^{(k)}$ is defined as follows:

$$c_{ii}^{(k)} = -\sum_{j=1, i\neq j}^{N} c_{ij}^{(k)}, \quad k = 1, 2, \quad i \in \mathcal{I}.$$

In ref. [42], the problem of synchronization of complex networks with derivative coupling and time-varying coupling delay was investigated by using adaptive control schemes, whose networks can be described as follows:

$$\begin{aligned} x_i(t) &= Ax_i(t) + f(x_i(t)) + \sum_{j=1}^{N} c_{ij}^{(1)} \Gamma^{(1)} x_j(t - \tau(t)) \\ &\quad + \sum_{j=1}^{N} c_{ij}^{(2)} \Gamma^{(2)} \dot{x}_j(t - \tau(t)), \quad i \in \mathcal{I}, \end{aligned} \tag{2}$$

where A is a constant matrix, $\tau(t) \geq 0$ is the time-varying coupling delay, $\Gamma^{(k)}$ $(k = 1, 2)$ are the inner coupling matrices.

In this paper, we consider a general complex network consisting of N coupled identical nodes with derivative coupling and time-varying coupling delay, each node is an n-dimensional system. This network has the following form:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + f(x_i(t)) + g(x_i(t - \tau_1(t))) \\ &\quad + h_i(x_1(t), x_2(t), \ldots, x_N(t)) \\ &\quad + l_i(x_1(t - \tau_2(t)), x_2(t - \tau_2(t)), \ldots, x_N(t - \tau_2(t))) \\ &\quad + m_i(\dot{x}_1(t - \tau_3(t)), \dot{x}_2(t - \tau_3(t)), \ldots, \dot{x}_N(t - \tau_3(t)), \quad i \in \mathcal{I}, \end{aligned} \tag{3}$$

where $\tau_k(t)$ $(k = 1, 2, 3)$ are the time-varying delay in isolated node, time-varying coupling delay and time-varying derivative coupling delay, respectively. $g(\cdot)$ is a continuously differentiable nonlinear vector function, and $h_i, l_i, m_i : R^{nN} \rightarrow R^n$ are coupling functions. We assume that the complex network (3) satisfies the following initial conditions:

$$x_i(t) = \phi_i(t) \in \mathcal{L}([-\tau, 0], R^n), \quad i \in \mathcal{I},$$

where $\tau = \max\{\tau_1(t), \tau_2(t), \tau_3(t)\}$, $\mathcal{L}([-\tau, 0], R^n)$ denotes the set of all continuous functions from $[-\tau, 0]$ to R^n.

Remark 1. The complex network model (3) is very general, which includes almost all the dynamical systems studied in [43,44]. The coupling functions h_i, l_i, m_i are quite general. First of all, it can be chosen as linear combinations of the states of the nodes, that is, $h_i = \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma x_j$ [43, 45–48], where ε is the coupling strength. Secondly, we can choose delayed couplings, that is, $l_i = \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma x_j(t - \tau(t))$ [25], where $\tau(t)$ is the time-varying coupling delay. In addition, they can be chosen with derivative coupling, that is, $m_i = \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma \dot{x}_j(t - \tau(t))$ [41,42]. Moreover, they can be chosen as distributed delayed coupling, that is, $l_i = \sum_{j=1}^{N} c_{ij} \int_{-\infty}^{t} k(t-s) x_j(s) \mathrm{d}s$ [46], where $k(\cdot)$ is the weight matrix function. Last but not the least, h_i, l_i and m_i can be combinations of nonlinear function, that is, $h_i = \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma H(x_j)$ and $l_i = \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma L(x_j(t - \tau(t)))$,where $H(\cdot)$ and $L(\cdot)$ are the inner coupling functions [49].

2.2 *Control object*

In this paper, we shall investigate the synchronization problem of the complex network model (3). Let solution $s(t)$ of an isolated node satisfies

$$\dot{s}(t) = As(t) + f(s(t)) + g(s(t - \tau_1(t))), \tag{4}$$

where $s(t)$ may be an equilibrium point, a periodic orbit or even a chaotic orbit. In order to synchronize the complex network (3) to object state $s(t)$, the controllers will affect some of its node. The controlled network can be described as

$$\begin{aligned} \dot{x}_i(t) = {} & Ax_i(t) + f(x_i(t)) + g(x_i(t - \tau_1(t))) \\ & + h_i(x_1(t), x_2(t), \ldots, x_N(t)) \\ & + l_i(x_1(t - \tau_2(t)), x_2(t - \tau_2(t)), \ldots, x_N(t - \tau_2(t))) \\ & + m_i(\dot{x}_1(t - \tau_3(t)), \dot{x}_2(t - \tau_3(t)), \ldots, \dot{x}_N(t - \tau_3(t))) \\ & + u_i(t), \quad i \in \mathcal{I}, \end{aligned} \tag{5}$$

where $u_i \in R^n$ is the feedback controller which will be designed later. The general nonlinear coupling function and the input should vanish under the controlled complex network (5) achieved complete synchronization. This means that any solution $s(t)$ of any isolated node is also a solution of synchronized coupling networks.

2.3 *Preliminaries*

In order to obtain the main result, the following assumptions and lemma are needed.

Assumption 1. Functions $f(\cdot)$ and $g(\cdot)$ are Lipshitz, that is, there exist non-negative constants α, β for all $x, y \in R^n$ such that

$$\|f(x) - f(y)\| \leq \alpha\|x - y\|, \quad \|g(x) - g(y)\| \leq \beta\|x - y\| .$$

Assumption 2. For functions $h_i(\cdot)$, $l_i(\cdot)$ and $m_i(\cdot)$, when the controlled complex network (5) achieves synchronization, the general nonlinear coupling functions and the control inputs should vanish, that is, $h_i(s, s, \ldots, s) = 0$, $l_i(s, s, \ldots, s) = 0$, $m_i(\dot{s}, \dot{s}, \ldots, \dot{s}) = 0$, $u_i(t) = 0$. Additionally, there exist non-negative constants γ_{ij}, η_{ij}, ξ_{ij} $(i, j = 1, 2, \ldots, N)$ such that

$$\|h_i(x_1, x_2, \ldots, x_N) - h_i(s, s, \ldots, s)\| \leq \sum_{j=1}^{N} \gamma_{ij}\|x_j - s\|,$$

$$\|l_i(x_1, x_2, \ldots, x_N) - l_i(s, s, \ldots, s)\| \leq \sum_{j=1}^{N} \eta_{ij}\|x_j - s\|,$$

$$\|m_i(\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_N) - m_i(\dot{s}, \dot{s}, \ldots, \dot{s})\| \leq \sum_{j=1}^{N} \xi_{ij}\|\dot{x}_j - \dot{s}\| .$$

Remark 2. Assumptions 1 and 2 are quite mild. Assumption 1 is satisfied as long as $\partial f/\partial x$ and $\partial g/\partial x$ are bounded. If we choose $h_i = \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma x_j$, $l_i = \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma x_j(t-\tau(t))$ and $m_i = \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma \dot{x}_j(t-\tau(t))$ (where $c_{ii} = -\sum_{j=1, i\neq j}^{N} c_{ij}$), Assumption 2 automatically vanishes when synchronization is achieved. Therefore, the complex network (3) actually includes many dynamical networks.

Assumption 3. The time-varying coupling delay $\tau_k(t)$ $(k = 1, 2)$ is a differential function with

$$0 \leq \dot{\tau}_k(t) \leq \mu_k < 1, \quad 0 \leq \tau_k(t) \leq \bar{\tau}_k.$$

Clearly, this hypothesis is ensured if the delay $\tau_k(t)$ is a constant.

Lemma 1 (*Matrix Cauchy inequality* [50]). *For any symmetric positive definite matrix $M \in R^{n\times n}$ and $x, y \in R^n$, there is*

$$\pm 2x^T y \leq x^T M x + y^T M^{-1} y.$$

The error dynamics is defined as $e_i(t) = x_i(t) - s(t)$. Subtracting (4) from (5), yield

$$\begin{aligned}\dot{e}_i(t) = {} & Ae_i(t) + f(x_i(t)) - f(s(t)) + g(x_i(t-\tau_1(t))) \\ & -g(s(t-\tau_1(t))) + h_i(x_1, x_2, \ldots, x_N) - h_i(s, s, \ldots, s) \\ & +l_i(x_1(t-\tau_2(t)), x_2(t-\tau_2(t)), \ldots, x_N(t-\tau_2(t))) \\ & -l_i(s(t-\tau_2(t)), s(t-\tau_2(t)), \ldots, s(t-\tau_2(t))) \\ & +m_i(\dot{x}_1(t-\tau_3(t)), \dot{x}_2(t-\tau_3(t)), \ldots, \dot{x}_N(t-\tau_3(t))) \\ & -m_i(\dot{s}(t-\tau_3(t)), \dot{s}(t-\tau_3(t)), \ldots, \dot{s}(t-\tau_3(t))) + u_i(t), \quad i \in \mathcal{I}. \end{aligned} \tag{6}$$

3. Synchronization of general complex networks

In this section, the synchronization problem of the complex network (3) is investigated. The controller is designed to achieve the synchronization of controlled complex network (5).

Theorem 1. *Suppose Assumptions 1–3 hold. The controlled complex network* (5) *can achieve synchronization under the following adaptive synchronization controller*:

$$\begin{aligned}u_i(t) = {} & -b_i(t)e_i(t) \\ & -k_i(t)[m_i(\dot{x}_1(t-\tau_3(t)), \dot{x}_2(t-\tau_3(t)), \ldots, \dot{x}_N(t-\tau_3(t))) \\ & -m_i(\dot{s}(t-\tau_3(t)), \dot{s}(t-\tau_3(t)), \ldots, \dot{s}(t-\tau_3(t)))]. \end{aligned} \tag{7}$$

with the following adaptive updating laws:

$$\begin{aligned}\dot{b}_i(t) = {} & \alpha_i e_i^T(t)e_i(t), \\ \dot{k}_i(t) = {} & \beta_i e_i^T(t)[m_i(\dot{x}_1(t-\tau_3(t)), \dot{x}_2(t-\tau_3(t)), \ldots, \dot{x}_N(t-\tau_3(t))) \\ & -m_i(\dot{s}(t-\tau_3(t)), \dot{s}(t-\tau_3(t)), \ldots, \dot{s}(t-\tau_3(t)))], \end{aligned} \tag{8}$$

where α_i and β_i are arbitrary positive constants. ■

Proof. We choose a non-negative function as Lyapunov function, that is

$$\begin{aligned}V_i(e(t)) = {} & \sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N} \frac{1}{\alpha_i}(b_i(t) - h_i^*)^2 + \sum_{i=1}^{N} \frac{1}{\beta_i}(k_i(t) - 1)^2 \\ & + \frac{1}{1-\mu_1} \int_{t-\tau_1(t)}^{t} \sum_{i=1}^{N} e_i^T(s)e_i(s)\mathrm{d}s \\ & + \frac{\eta N}{1-\mu_2} \int_{t-\tau_2(t)}^{t} \sum_{i=1}^{N} e_i^T(s)e_i(s)\mathrm{d}s, \end{aligned} \tag{9}$$

where h_i^* and η are positive constants to be determined later.

The derivative of Lyapunov function (9) with respect to time t along (6) is then given by

$$\begin{aligned}
\dot{V}_i(e(t)) &= 2\sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + 2\sum_{i=1}^{N}\frac{1}{\alpha_i}(b_i(t)-h_i^*)\dot{b}_i(t) \\
&\quad +2\sum_{i=1}^{N}\frac{1}{\beta_i}(k_i(t)-1)\dot{k}_i(t) + \frac{1}{1-\mu_1}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\
&\quad -\frac{1-\dot{\tau}_1(t)}{1-\mu_1}\sum_{i=1}^{N} e_i^T(t-\tau_1(t))e_i(t-\tau_1(t)) \\
&\quad +\frac{\eta N}{1-\mu_2}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\
&\quad -\frac{1-\dot{\tau}_2(t)}{1-\mu_2}\sum_{i=1}^{N}\eta N e_i^T(t-\tau_2(t))e_i(t-\tau_2(t)) \\
&= 2\sum_{i=1}^{N} e_i^T(t)[Ae_i(t) + f(x_i(t)) - f(s(t)) \\
&\quad +g(x_i(t-\tau_1(t))) - g(s(t-\tau_1(t))) \\
&\quad +h_i(x_1(t), x_2(t), \ldots, x_N(t)) - h_i(s, s, \ldots, s) \\
&\quad +l_i(x_1(t-\tau_2(t)), x_2(t-\tau_2(t)), \ldots, x_N(t-\tau_2(t))) \\
&\quad -l_i(s(t-\tau_2(t)), s(t-\tau_2(t)), \ldots, s(t-\tau_2(t))) \\
&\quad +m_i(\dot{x}_1(t-\tau_3(t)), \dot{x}_2(t-\tau_3(t)), \ldots, \dot{x}_N(t-\tau_3(t))) \\
&\quad -m_i(\dot{s}(t-\tau_3(t)), \dot{s}(t-\tau_3(t)), \ldots, \dot{s}(t-\tau_3(t))) \\
&\quad +u_i(t)] + 2\sum_{i=1}^{N}\frac{1}{\alpha_i}(b_i(t)-h_i^*)\dot{b}_i(t) \\
&\quad +2\sum_{i=1}^{N}\frac{1}{\beta_i}(k_i(t)-1)\dot{k}_i(t) + \frac{1}{1-\mu_1}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\
&\quad -\frac{1-\dot{\tau}_1(t)}{1-\mu_1}\sum_{i=1}^{N} e_i^T(t-\tau_1(t))e_i(t-\tau_1(t)) \\
&\quad +\frac{1}{1-\mu_2}\sum_{i=1}^{N}\eta N e_i^T(t)e_i(t) \\
&\quad -\frac{1-\dot{\tau}_2(t)}{1-\mu_2}\sum_{i=1}^{N}\eta N e_i^T(t-\tau_2(t))e_i(t-\tau_2(t)). \qquad (10)
\end{aligned}$$

Using the adaptive synchronization controller (7) and the adaptive updating law (8), yield

$$\begin{aligned}\dot{V}_i(e(t)) = {} & 2\sum_{i=1}^{N} e_i^T(t)[Ae_i(t) + f(x_i(t)) - f(s(t)) \\ & +g(x_i(t-\tau_1(t))) - g(s(t-\tau_1(t))) \\ & +h_i(x_1(t), x_2(t), \cdots, x_N(t)) - h_i(s, s, \cdots, s) \\ & +l_i(x_1(t-\tau_2(t)), x_2(t-\tau_2(t)), \cdots, x_N(t-\tau_2(t))) \\ & -l_i(s(t-\tau_2(t)), s(t-\tau_2(t)), \cdots, s(t-\tau_2(t)))] \\ & -2\sum_{i=1}^{N} h_i^* e_i^T(t)e_i(t) + \frac{1}{1-\mu_1}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\ & -\frac{1-\dot{\tau}_1(t)}{1-\mu_1}\sum_{i=1}^{N} e_i^T(t-\tau_1(t))e_i(t-\tau_1(t)) \\ & +\frac{\eta N}{1-\mu_2}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\ & -\frac{1-\dot{\tau}_2(t)}{1-\mu_2}\sum_{i=1}^{N} \eta N e_i^T(t-\tau_2(t))e_i(t-\tau_2(t)).\end{aligned} \tag{11}$$

According to Assumption 1, we have

$$e_i^T(t)[f(x_i(t) - f(s(t))] \le \alpha e_i^T(t)e_i(t). \tag{12}$$

$$e_i^T(t)[g(x_i(t-\tau_1(t))) - g(s(t-\tau_1(t)))] \le \beta \|e_i^T(t)\| \|e_i(t-\tau_1(t))\| . \tag{13}$$

According to Assumption 2, we have

$$\begin{aligned} & e_i(t)\,[h_i(x_1(t), x_2(t), \ldots, x_N(t)) - h_i(s, s, \ldots, s)] \\ & \le \|e_i(t)\sum_{j=1}^{N}\gamma_{ij}e_j(t)\| .\end{aligned} \tag{14}$$

$$\begin{aligned} & e_i(t)\,[l_i(x_1(t-\tau_2(t)), x_2(t-\tau_2(t)), \ldots, x_N(t-\tau_2(t))) \\ & -l_i(s(t-\tau_2(t)), s(t-\tau_2(t)), \ldots, s(t-\tau_2(t)))] \\ & \le \| \, e_i(t)\sum_{j=1}^{N}\eta_{ij}e_j(t-\tau_2(t))\| .\end{aligned} \tag{15}$$

Let $\eta = \max_{1\le i\le N, 1\le j\le N} \eta_{ij}$, $\gamma = \max_{1\le i\le N, 1\le j\le N} \gamma_{ij}$ and $\sum_{j=1}^{n} e_{ij}^2 = e_i^T(t)e_i(t)$. According to Lemma 1 yield,

$$2\beta\|e_i^T(t)e_i(t-\tau_1(t))\| \le \beta^2 e_i^T(t)e_i(t) + e_i^T(t-\tau_1(t))e_i(t-\tau_1(t)). \tag{16}$$

$$2\left\|e_i(t)\sum_{j=1}^{N}\gamma_{ij}e_j(t)\right\| \le \gamma\sum_{j=1}^{N}\left[e_i^T(t)e_i(t) + e_j^T(t)e_j(t)\right]. \tag{17}$$

$$\begin{aligned}2\left\|e_i(t)\sum_{j=1}^{N}\eta_{ij}e_j(t-\tau_2(t))\right\| \le\ & \eta\sum_{j=1}^{N}\left[e_i^T(t)e_i(t)\right.\\ &\left.+e_j^T(t-\tau_2(t))e_j(t-\tau_2(t))\right].\end{aligned} \tag{18}$$

According to (11), (12), (16)–(18),

$$\begin{aligned}\dot{V}_i(e(t)) \le\ & 2\sum_{i=1}^{N}e_i^T(t)Ae_i(t) + 2\sum_{i=1}^{N}e_i^T(t)\alpha e_i(t)\\
&-2\sum_{i=1}^{N}h_i^* e_i^T(t)e_i(t) + \beta^2\sum_{i=1}^{N}e_i^T(t)e_i(t)\\
&+\sum_{i=1}^{N}e_i^T(t-\tau_1(t))e_i(t-\tau_1(t)) + \eta N\sum_{i=1}^{N}e_i^T(t)e_i(t)\\
&+\eta N\sum_{i=1}^{N}e_i^T(t-\tau_2(t))e_i(t-\tau_2(t)) + \gamma N\sum_{i=1}^{N}e_i^T(t)e_i(t)\\
&+\gamma N\sum_{i=1}^{N}e_i^T(t)e_i(t) + \frac{1}{1-\mu_1}\sum_{i=1}^{N}e_i^T(t)e_i(t)\\
&-\frac{1-\dot{\tau}_1(t)}{1-\mu_1}\sum_{i=1}^{N}e_i^T(t-\tau_1(t))e_i(t-\tau_1(t))\\
&+\frac{\eta N}{1-\mu_2}\sum_{i=1}^{N}e_i^T(t)e_i(t)\\
&-\frac{\eta N}{1-\mu_2}\sum_{i=1}^{N}(1-\dot{\tau}_2(t))e_i^T(t-\tau_2(t))e_i(t-\tau_2(t))\end{aligned}$$

$$\leq \sum_{i=1}^{N} e_i^T(t) \Big(\lambda_{max}(A + A^T) + 2\alpha + \beta^2 + \eta N + 2\gamma N$$

$$-2h_i^* + \frac{1}{1-\mu_1} + \frac{\eta N}{1-\mu_2}\Big) e_i(t)$$

$$+ \left(1 - \frac{1-\dot{\tau}_1(t)}{1-\mu_1}\right) \sum_{i=1}^{N} e_i^T(t-\tau_1(t)) e_i(t-\tau_1(t))$$

$$+\eta N \left(1 - \frac{1-\dot{\tau}_2(t)}{1-\mu_2}\right) \sum_{i=1}^{N} e_i^T(t-\tau_2(t)) e_i(t-\tau_2(t)). \tag{19}$$

According to Assumption 3, we have

$$1 < \frac{1-\dot{\tau}_1(t)}{1-\mu_1}, \quad 1 < \frac{1-\dot{\tau}_2(t)}{1-\mu_2}. \tag{20}$$

According to (19) and (20),

$$\dot{V}_i(e(t)) \leq \sum_{i=1}^{N} e_i^T(t) \Big[\lambda_{\max}(A + A^T) + 2\alpha + \beta^2 + \eta N + 2\gamma N$$

$$-2h_i^* + \frac{1}{1-\mu_1} + \frac{\eta N}{1-\mu_2}\Big] e_i(t). \tag{21}$$

We can choose suitable h_i^* such that

$$\lambda_{\max}(A + A^T) + 2\alpha + \beta^2 + \eta N + 2\gamma N - 2h_i^* + \frac{1}{1-\mu_1} + \frac{\eta N}{1-\mu_2} < 0. \tag{22}$$

It is easy to know that

$$\dot{V}_i(e(t)) < 0.$$

Then the error dynamics (6) is asymptotically stable. That is to say, the dynamical network (3) achieves synchronization under the adaptive control scheme (7) and the adaptive updating law (8).

The proof is thus completed. □

Remark 3. When $A = 0$, $h_i = \sum_{j=1}^{n} c_{ij} x_j(t)$, $l_i = 0$, $m_i = \sum_{j=1}^{N} d_{ij} \dot{x}_j(t)$, $g(x_i(t - \tau_1(t))) = 0$, the complex network (3) is translated into

$$x_i(t) = f(x_i(t)) + \sum_{j=1}^{n} c_{ij} x_j(t) + \sum_{j=1}^{N} d_{ij} \dot{x}_j(t), \quad i \in \mathcal{I}, \tag{23}$$

which was investigated by Xu [41]. Obviously, it is a special case of this paper.

Remark 4. When $A = 0$, $h_i = 0$, $l_i = \sum_{j=1}^{n} c_{ij} H x_j(t-\tau(t))$ and $m_i = \sum_{j=1}^{N} a_{ij} G \dot{x}_j(t-\tau(t))$, $g(x_i(t - \tau_1(t)) = 0$, the complex network (3) is translated into

$$x_i(t) = f(x_i(t)) + \sum_{j=1}^{n} c_{ij} H x_j(t-\tau(t)) + \sum_{j=1}^{N} a_{ij} G \dot{x}_j(t-\tau(t)), \quad i \in \mathcal{I}, \tag{24}$$

which was regarded as the special case of this paper and was investigated by Jian [42].

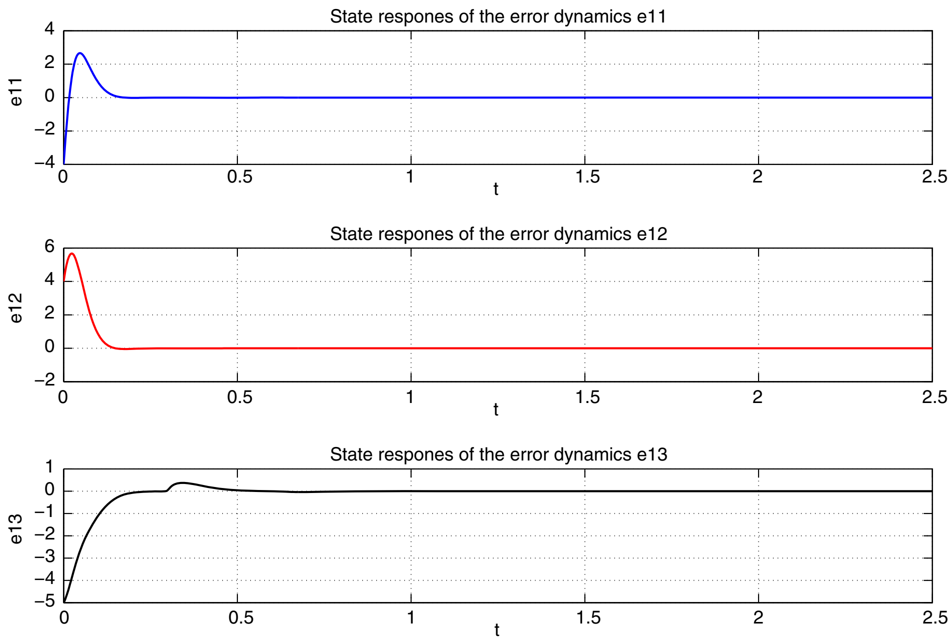

Figure 1. Adaptive synchronization errors $e_{1i}(t)$ $(i = 1, 2, 3)$ with the adaptive synchronization controllers (7) and (8).

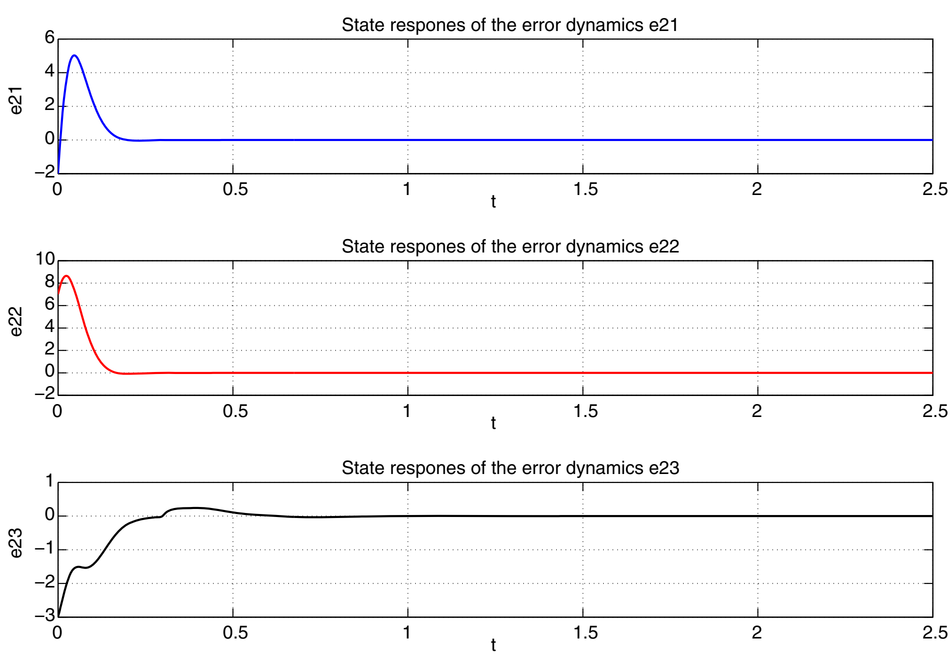

Figure 2. Adaptive synchronization errors $e_{2i}(t)$ $(i = 1, 2, 3)$ with the adaptive synchronization controllers (7) and (8).

Remark 5. If $g(x_i(t - \tau_1(t))) = 0$, $m_i = 0$, this special case was proposed by Wang [51]. Obviously, the simplified case can still ensure the stability of the network by using controllers of this paper.

Remark 6. If there is no derivative coupling, this special case was investigated by Yu [52]. Obviously, this can be regarded as the special case of this paper.

4. Numerical examples

In this section, illustrative example is provided to verify the effectiveness of the synchronization controller obtained in the previous section. Without loss of generality, we take the time-delay Chen chaotic system [53] as the local node dynamics, which can be given by

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = (c - a)x_1(t) + cx_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t) + d(x_3(t) - x_3(t - \tau_1)), \end{cases}$$

where $a = 35$, $b = 3$, $c = 18$, $d = 3.8$ and $\tau_1 = 0.3$.

The constants in Assumption 1 are calculated as $\alpha = 45$ and $\beta = 3$.

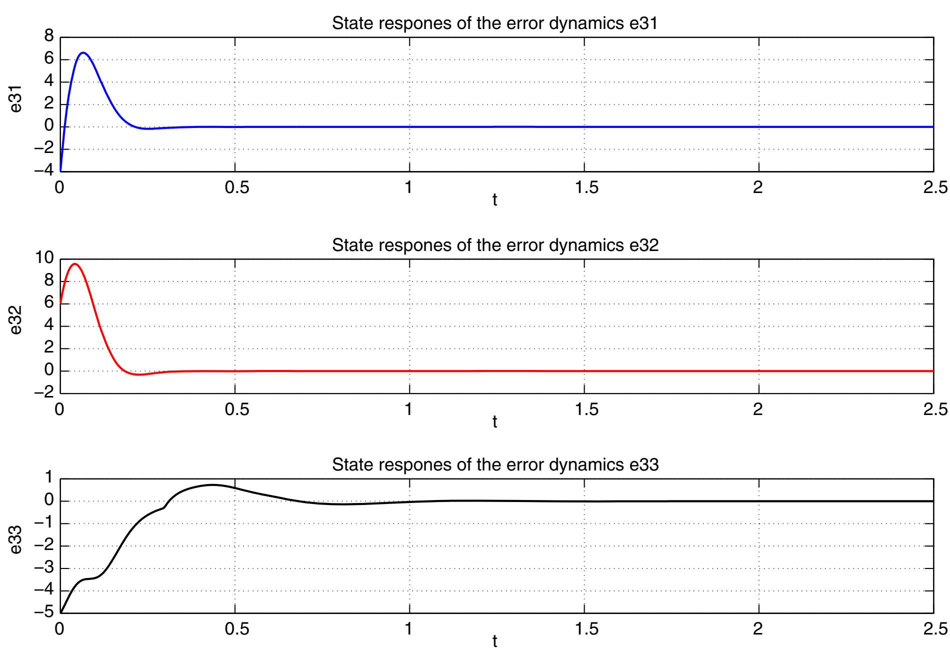

Figure 3. Adaptive synchronization errors $e_{3i}(t)$ $(i = 1, 2, 3)$ with the adaptive synchronization controllers (7) and (8).

Let

$$h_i = \varepsilon_1 \sum_{j=1}^{3} c_{ij}^{(1)} \Gamma^{(1)} \left[x_j(t) + 2\log_2 \left(|x_j| + \frac{1}{2} \right) + \frac{1}{2}\sin(t) \right],$$
$$l_i = \varepsilon_2 \sum_{j=1}^{3} c_{ij}^{(2)} \Gamma^{(2)} x_j(t - \tau_2(t)),$$
$$m_i = \varepsilon_3 \sum_{j=1}^{3} c_{ij}^{(3)} \Gamma^{(3)} \dot{x}_j(t - \tau_3(t)),$$

where

$$c^{(1)} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}, \quad c^{(2)} = \begin{bmatrix} -2 & 2 & 0 \\ 4 & -5 & 1 \\ 3 & 5 & -8 \end{bmatrix}, \quad c^{(3)} = \begin{bmatrix} 2 & -1 & -1 \\ 3 & -4 & 1 \\ -4 & -2 & 6 \end{bmatrix},$$

$$\Gamma^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma^{(2)} = \begin{bmatrix} 1 & -2 & 6 \\ 4 & 2 & 3 \\ 2 & 5 & -3 \end{bmatrix}, \quad \Gamma^{(3)} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

$$\varepsilon_1 = 1, \quad \varepsilon_2 = \frac{1}{2}, \quad \varepsilon_3 = 1, \quad \tau_2(t) = \frac{1}{2} - \frac{1}{2}e^{-t}, \quad \tau_3(t) = \frac{2}{5} - \frac{2}{5}e^{-t}.$$

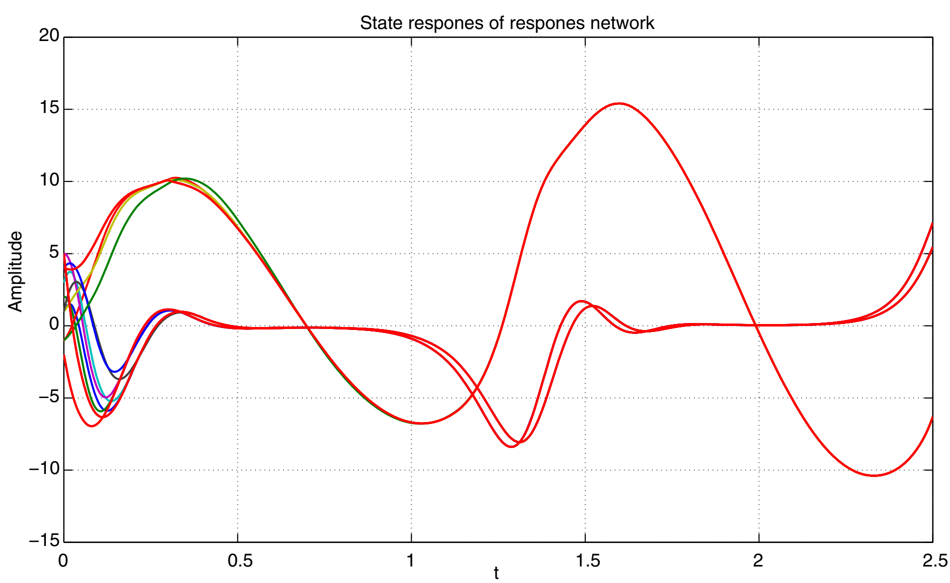

Figure 4. Synchronization state response curves of the complex network (3).

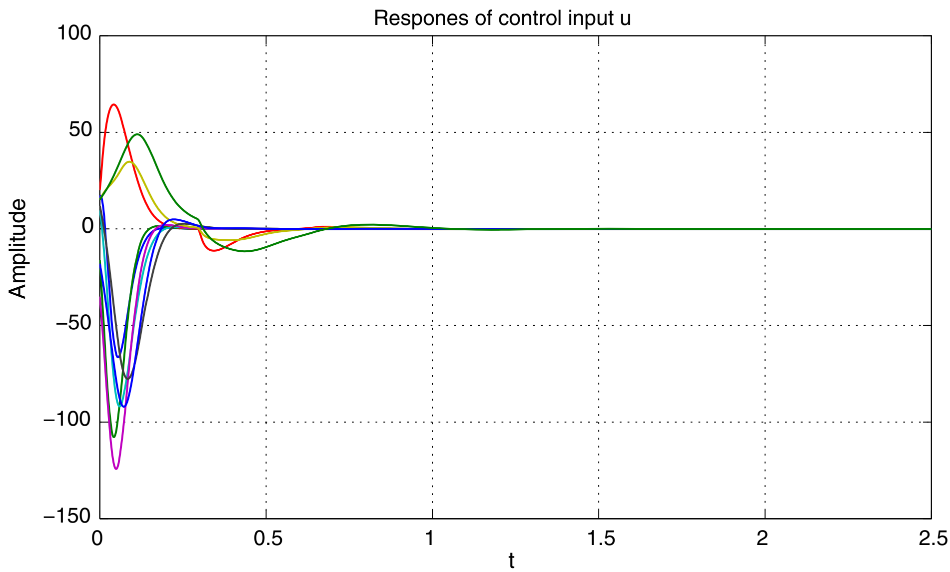

Figure 5. Response curves of adaptive synchronization control inputs (7).

Theorem 1 can be obtained, the complex network (3) can achieve complete synchronization under the adaptive synchronization controller (7) and the adaptive updating law (8). With initial conditions

$$x_1(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \qquad x_2(0) = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \qquad x_3(0) = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix},$$

$$s(0) = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}, \qquad b_i(0) = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \qquad k_i(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Let adaptive gains $\alpha_1 = 9, \alpha_2 = 3, \alpha_3 = 1$ and $\beta_1 = 2, \beta_2 = 4, \beta_3 = 8$. The numerical simulations are presented in figures 1–5. Figures 1–3 show the synchronization errors of the complex network and it can be concluded that errors can tend to be zero soon. The response curves of the complex network is given in figure 4. Figure 5 illustrates control inputs $u_i(t)$ $(i = 1, 2, 3)$ and the values of control inputs are acceptable.

From figures 1–5, it is easy to see that the controlled complex network (3) is eventually synchronized.

5. Conclusions

In this paper, we have investigated adaptive synchronization of general complex network with time-delay coupling, nonlinear coupling and derivative coupling. An effective synchronization controller and adaptive updating laws are derived for the synchronization of various delayed complex networks based on the Lyapunov functional method. Finally, one numerical example has been provided to show the effectiveness of the proposed method.

The proposed method is simple and effective, but still rather conservative due to the generality of the network model. Nevertheless, this leaves more theoretical studies of some other network models and better controller design to the future, for example, complex networks with unknown parameters and uncertainties and so on.

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