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Charge radii of octet and decuplet baryons in chiral constituent quark model

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Abstract. The charge radii of the spin-$\frac{1}{2}^+$ octet and spin-$\frac{3}{2}^+$ decuplet baryons have been calculated in the framework of chiral constituent quark model (χCQM) using a general parametrization method (GPM). Our results are not only comparable with the latest experimental studies but also agree with other phenomenological models. The effects of $SU(3)$ symmetry breaking pertaining to the strangeness contribution and GPM parameters pertaining to the one-, two- and three-quark contributions have also been investigated in detail and are found to be the key parameters in understanding the non-zero values for the neutral octet (n, Σ^0, Ξ^0, Λ) and decuplet (Δ^0, Σ^{*0}, Ξ^{*0}) baryons.

Keywords. Electromagnetic moments; phenomenological quark models.

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1. Introduction

The internal structure of baryons is determined in terms of electromagnetic Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ or equivalently in terms of the electric and magnetic Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$ [1]. The electromagnetic form factors are the fundamental quantities of theoretical and experimental interest which are further related to the static low-energy observables of charge radii and magnetic moments. Although quantum chromodynamics (QCD) is accepted as the fundamental theory of strong interactions, the direct prediction of these kinds of observables from the first principle of QCD still remains a theoretical challenge as they lie in the nonperturbative regime of QCD. At present, electromagnetic form factors at low energy have been precisely measured for nucleons whereas, for other baryons, the experimental data are available only for magnetic moments.

The mean square charge radius (r_B^2), giving the possible 'size' of baryon, has been investigated experimentally with the advent of new facilities at JLAB, SELEX

Collaborations [2–4]. Several measurements have been made for the charge radii of p, n, and Σ^- in electron–baryon scattering experiments [4,5] giving $r_{\mathrm{p}} = 0.877 \pm 0.007$ fm ($r_{\mathrm{p}}^2 = 0.779 \pm 0.025$ fm^2 [6]) and $r_{\mathrm{n}}^2 = -0.1161 \pm 0.0022$ fm^2 [3]. The recent measurement of $r^2_{\Sigma^-}$ [4,5] is particularly interesting as it gives the first estimate for the charge form factor of a strange baryon at low momentum transfer. These measurements suggest the possibility of measuring the charge radii of other long-lived strange baryons such as Λ, Σ (Σ^*), and Ξ (Ξ^*) in the near future.

The naive quark model (NQM) [7–10] is quite successful in explaining many of the low-energy baryon matrix elements. For calculations pertaining to the baryon charge radii, NQM leads to vanishing charge radii for the neutral baryons like n, Σ^0, Ξ^0, and Λ. This is in contradiction with the experimental data. The inclusion of quark spin–spin interactions in NQM modify the baryon wave function to some extent leading to the dynamical breaking of the $SU(3)$ symmetry and a non-vanishing neutron charge mean square radius [9]. However, the measurements in the deep inelastic scattering (DIS) experiments [11] indicate that the valence quarks of the proton carry only about 30% of its spin and also establishes the asymmetry of the quark distribution functions [12,13]. This is referred to as the 'proton spin problem' in NQM. Several effective and phenomenological models have been developed to explain the 'proton spin problem' by including spontaneous breaking of chiral symmetry and have been further applied to study the electromagnetic properties of baryons.

Some of the important models measuring the charge radii of octet baryons are the Skyrme model with bound state approach [14] and slow-rotor approach [15], $SU(3)$ NJL model [16], cloudy bag model [17], variants of constituent quark models [18–23], $1/N_{\mathrm{c}}$ expansion approach [24,25], perturbative chiral quark model (PχQM) [26], heavy-baryon chiral perturbation theory (HBχPT) [27], chiral perturbation theory (χPT) [28], lattice QCD [29] etc. The charge radii of the decuplet baryons have been studied within the framework of quark model [30], lattice QCD [31], $1/N_{\mathrm{c}}$ expansion [24], chiral perturbation theory [32], etc. The results for different theoretical models are however not consistent with each other.

As the hadron structure is sensitive to the pion cloud in the low-energy regime, a coherent understanding is necessary as it will provide a test for the QCD-inspired effective field theories. One of the important non-perturbative approach which finds its application in the low-energy regime is the chiral constituent quark model (χCQM) [33]. The QCD Lagrangian is not invariant under the chiral transformation. If the mass terms in the QCD Lagrangian are neglected, the Lagrangian will have global chiral symmetry of the $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}$ group. The chiral symmetry is believed to be spontaneously broken around a scale of 1 GeV, much larger than the QCD confinement scale ($\Lambda_{\mathrm{QCD}} \simeq 0.1 - 0.3$ GeV) to $SU(3)_{\mathrm{L+R}}$ by forming a quark condensate. As a consequence, there exists a set of massless particles, referred to as the Goldstone bosons (GBs), which are identified with the observed (π, K, η mesons). Within the region of Λ_{QCD} and $\Lambda_{\chi\mathrm{SB}}$, the constituent quarks, the octet of GBs (π, K, η mesons) are the appropriate degrees of freedom.

The χCQM coupled with the 'quark sea' generation through the chiral fluctuation of a constituent quark GBs [34–37], successfully explains the 'proton spin problem' [37], hyperon β decay parameters [38], strangeness content in the nucleon [39], magnetic moments of octet and decuplet baryons including their transitions [40], magnetic moments of $\frac{1}{2}^-$ octet baryon resonances [41], magnetic moments of $\frac{1}{2}^-$ and $\frac{3}{2}^-$ Λ

resonances [42], charge radii [43], quadrupole moment [44], etc. The model can be extended to predict the important role played by the small intrinsic charm (IC) content in the nucleon spin in the $SU(4)$ χCQM [45] and to calculate the magnetic moment of spin-$\frac{1}{2}^+$ and spin-$\frac{3}{2}^+$ charm baryons including their radiative decays [46]. In view of the above developments in the χCQM, it become desirable to extend the model to calculate the charge radii and higher-order moments in the multipole expansion of charge density.

The purpose of the present communication is to calculate the charge radii of the spin-$\frac{1}{2}^+$ octet and spin-$\frac{3}{2}^+$ decuplet baryons within the framework of χCQM using the general parametrization method (GPM). In order to understand the role of pseudoscalar mesons in the baryon charge radii, we shall compare our results with NQM as well as other phenomenological models. The detailed analysis of $SU(3)$ symmetry breaking would also be carried out in the χCQM. Further, we aim to discuss the implications of GPM parameters by calculating the extent to which the three-quark term contributes.

2. Charge radii

The mean square charge radius, r_{B}^2, of a given baryon, is a scalar under spatial rotation defined as

$$\langle r^2 \rangle = \int \mathrm{d}^3 r \rho(\mathbf{r}) \mathbf{r}^2, \tag{1}$$

where $\rho(\mathbf{r})$ is the charge density. Charge radius is the first non-trivial moment of a Coulomb monopole $G_{\mathrm{C0}}(q^2)$ transition amplitude. In order to obtain information on charge radii, we have used GPM developed by Morpurgo *et al* [47].

The most general form of the multipole expansion of the nucleon charge density ρ in the spin-flavour space can be expressed as

$$\begin{aligned}\rho = A' \sum_{i=1}^{3} e_i 1 - B' \sum_{i \neq j}^{3} e_i [2\sigma_i \cdot \sigma_j - (3\sigma_{iz}\sigma_{jz} - \sigma_i \cdot \sigma_j)] \\ -C' \sum_{i \neq j \neq k}^{3} e_i [2\sigma_j \cdot \sigma_k - (3\sigma_{jz}\sigma_{kz} - \sigma_j \cdot \sigma_k)].\end{aligned} \tag{2}$$

The charge radii operator composed of the sum of one-, two-, and three-quark terms is expressed as

$$\widehat{r^2} = A \sum_{i=1}^{3} e_i 1 + B \sum_{i \neq j}^{3} e_i \, \sigma_i \cdot \sigma_j + C \sum_{i \neq j \neq k}^{3} e_i \, \sigma_j \cdot \sigma_k, \tag{3}$$

whereas the quadrupole moment operator composed of a two- and three-quark terms can be expressed as

$$\widehat{Q} = B' \sum_{i \neq j}^{3} e_i \big(3\sigma_{iz}\sigma_{jz} - \sigma_i \cdot \sigma_j\big) + C' \sum_{i \neq j \neq k}^{3} e_i \big(3\sigma_{jz}\sigma_{kz} - \sigma_j \cdot \sigma_k\big). \tag{4}$$

The GPM parameters of the charge radii and quadrupole moments are related to each other as $A = A'$, $B = -2B'$, and $C = -2C'$ [48]. These GPM constants are to be further determined from the experimental observations on charge radii and quadrupole moment.

Before calculating the matrix elements corresponding to the charge radii of the octet and decuplet baryons, it is essential to simplify various operator terms involved in eq. (3). It can be easily shown that

$$\sum_{i \neq j} e_i(\sigma_i \cdot \sigma_j) = 2J \cdot \sum_i e_i \sigma_i - 3 \sum_i e_i, \tag{5}$$

$$\sum_{i \neq j \neq k} e_i(\sigma_j \cdot \sigma_k) = \pm 3 \sum_i e_i - \sum_{i \neq j} e_i(\sigma_i \cdot \sigma_j), \tag{6}$$

where $+$ve sign holds for $J = \frac{3}{2}$ and $-$ve sign for $J = \frac{1}{2}$ states leading to different operators for spin-$\frac{1}{2}^+$ and spin-$\frac{3}{2}^+$ baryons

Operator	$\sum_{i \neq j} e_i(\sigma_i \cdot \sigma_j)$	$\sum_{i \neq j \neq k} e_i(\sigma_j \cdot \sigma_k)$
$J = \frac{1}{2}$	$3\sum_i e_i\sigma_{iz} - 3\sum_i e_i$	$-3\sum_i e_i\sigma_{iz}$
$J = \frac{3}{2}$	$5\sum_i e_i\sigma_{iz} - 3\sum_i e_i$	$6\sum_i e_i - 5\sum_i e_i\sigma_{iz}$

(7)

The charge radii operators for the spin-$\frac{1}{2}^+$ octet and spin-$\frac{3}{2}^+$ decuplet baryons can now be expressed as

$$\widehat{r_B^2} = (A - 3B)\sum_i e_i + 3(B - C)\sum_i e_i\sigma_{iz}, \tag{8}$$

$$\widehat{r_{B*}^2} = (A - 3B + 6C)\sum_i e_i + 5(B - C)\sum_i e_i\sigma_{iz}. \tag{9}$$

It is clear from the above equations that the determination of charge radii basically reduces to the evaluation of the flavour ($\sum_i e_i$) and spin ($\sum_i e_i\sigma_{iz}$) structure of a given baryon. The charge radii squared $r^2_{B(B^*)}$ for the octet (decuplet) baryons can now be calculated by evaluating matrix elements corresponding to the operators in eqs (8) and (9) and are given as

$$r_B^2 = \langle B|\widehat{r_B^2}|B\rangle, \qquad r_{B^*}^2 = \langle B^*|\widehat{r_{B^*}^2}|B^*\rangle. \tag{10}$$

Here $|B\rangle$ and $|B^*\rangle$ respectively, denote the spin-flavour wave functions for the spin-$\frac{1}{2}^+$ octet and the spin-$\frac{3}{2}^+$ decuplet baryons.

3. Naive quark model (NQM)

The appropriate operators for the spin and flavour structure of baryons in NQM are defined as

$$\sum_i e_i = \sum_{q=u,d,s} n_q^B q + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} n_{\bar{q}}^B \bar{q} = n_u^B u + n_d^B d$$
$$+ n_s^B s + n_{\bar{u}}^B \bar{u} + n_{\bar{d}}^B \bar{d} + n_{\bar{s}}^B \bar{s}, \tag{11}$$

and

$$\sum_i e_i \sigma_{iz} = \sum_{q=u,d,s} (n^B_{q_+} q_+ + n^B_{q_-} q_-) = n^B_{u_+} u_+ + n^B_{u_-} u_- + n^B_{d_+} d_+ + n^B_{d_-} d_- + n^B_{s_+} s_+ + n^B_{s_-} s_-, \tag{12}$$

where n^B_q ($n^B_{\bar{q}}$) is the number of quarks with charge q ($\bar{q}$) and $n^B_{q_+}$ ($n^B_{q_-}$) is the number of polarized quarks q_+ (q_-). For a given baryon $u = -\bar{u}$ and $u_+ = -u_-$, with similar relations for the d and s quarks. The general expression for the charge radii of any of the spin-$\frac{1}{2}^+$ octet baryon in eq. (3) can be expressed as

$$\widehat{r^2_B} = (A - 3B) \left(\sum_{u,d,s} n_q - \sum_{\bar{u},\bar{d},\bar{s}} n_{\bar{q}} \right) q + 3(B - C) \left(\sum_{u,d,s} n_{q_+} - \sum_{u,d,s} n_{q_-} \right) q_+. \tag{13}$$

Before we discuss the details of the charge radii calculations, it is essential to define configuration mixing generated by the spin–spin forces in the case of octet baryons [10,49] which improves the predictions of the various spin-related properties [37]. The 'mixed' state octet baryon wave function is expressed as

$$|B\rangle \equiv \left| 8, \frac{1}{2}^+ \right\rangle = \cos\theta |56, 0^+\rangle_{N=0} + \sin\theta |70, 0^+\rangle_{N=2}, \tag{14}$$

with

$$|56, 0^+\rangle_{N=0} = \frac{1}{\sqrt{2}} (\varphi' \chi' + \varphi'' \chi'') \psi^s(0^+),$$

$$|70, 0^+\rangle_{N=2} = \frac{1}{2} [(\varphi' \chi'' + \varphi'' \chi') \psi'(0^+) + (\varphi' \chi' - \varphi'' \chi'') \psi''(0^+)]. \tag{15}$$

Here θ is the mixing angle and χ, φ, and ψ are the spin, isospin, and spatial wave functions. For details of the wave function, we refer the readers to [49]. Using the 'mixed' wave function (eq. (14)), the charge radii for p and Σ^+ from eq. (13) can now be expressed as

$$r^2_{\mathrm{p}} = (A - 3B)(2u + d) + 3(B - C) \left[\cos^2\theta \left(\frac{4}{3} u_+ - \frac{1}{3} d_+ \right) + \sin^2\theta \left(\frac{2}{3} u_+ + \frac{1}{3} d_+ \right) \right], \tag{16}$$

$$r^2_{\Sigma^+} = (A - 3B)(2u + s) + 3(B - C) \left[\cos^2\theta \left(\frac{4}{3} u_+ - \frac{1}{3} s_+ \right) + \sin^2\theta \left(\frac{2}{3} u_+ + \frac{1}{3} s_+ \right) \right]. \tag{17}$$

Table 1. Charge radii of octet baryons in $\mathrm{NQM_{config}}$ in terms of the GPM parameters. The results in NQM without configuration can easily be calculated by substituting $\theta = 0$.

r_B^2	$\mathrm{NQM_{config}}$
r_{p}^2	$(A-3B)[2u+d] + (B-C)\left[\cos^2\theta(4u_+ - d_+) + \sin^2\theta\,(2u_+ + d_+)\right]$
r_{n}^2	$(A-3B)[u+2d] + (B-C)\left[\cos^2\theta(-u_+ + 4d_+) + \sin^2\theta\,(u_+ + 2d_+)\right]$
$r_{\Sigma^+}^2$	$(A-3B)[2u+s] + (B-C)\left[\cos^2\theta(4u_+ - s_+) + \sin^2\theta\,(2u_+ + s_+)\right]$
$r_{\Sigma^-}^2$	$-(A-3B)[2d+s] - (B-C)\left[\cos^2\theta(4d_+ - s_+) + \sin^2\theta\,(2d_+ + s_+)\right]$
$r_{\Sigma^0}^2$	$(A-3B)[u+d+s] + (B-C)\left[\cos^2\theta(2u_+ + 2d_+ - s_+) + \sin^2\theta\,(u_+ + d_+ + s_+)\right]$
$r_{\Xi^0}^2$	$(A-3B)[u+2s] + (B-C)\left[\cos^2\theta(-u_+ + 4s_+) + \sin^2\theta\,(u_+ + 2s_+)\right]$
$r_{\Xi^-}^2$	$-(A-3B)[d+2s] + (B-C)\left[\cos^2\theta(-d_+ + 4s_+) + \sin^2\theta\,(d_+ + 2s_+)\right]$
r_{Λ}^2	$(A-3B)[u+d+s] + (B-C)\left[\cos^2\theta(3s_+) + \sin^2\theta\,(u_+ + d_+ + s_+)\right]$
$r_{\Sigma\Lambda}^2$	$(A-3B)[u+d+s] + \sqrt{3}(B-C)\left[u_+ - d_+\right]$

The expressions for the charge radii of other octet baryons in NQM with configuration mixing ($\mathrm{NQM_{config}}$) are presented in table 1. The results without configuration mixing can easily be obtained by taking the mixing angle $\theta = 0$.

Configuration mixing generated by the spin–spin forces does not affect the spin-$\frac{3}{2}^+$ decuplet baryons. The wave function in this case is given as

$$|B^*\rangle \equiv \left|10, \frac{3}{2}^+\right\rangle = |56, 0^+\rangle_{N=0} = \chi^s\varphi^s\psi^s(0^+). \tag{18}$$

Using the baryon wave function from the above equation and the charge radii operator from eq. (9), the general expression for the charge radii of spin-$\frac{3}{2}^+$ baryons can be expressed as

$$\begin{aligned}\widehat{r^2_{B^*}} &= (A-3B+6C)\left(\sum_{u,d,s} n_q - \sum_{\bar{u},\bar{d},\bar{s}} n_{\bar{q}}\right) q \\ &\quad +5(B-C)\left(\sum_{u,d,s} n_{q_+} - \sum_{u,d,s} n_{q_-}\right) q_+ \,.\end{aligned} \tag{19}$$

As an example, the charge radii for Δ^+ baryon can be expressed as

$$r^2_{\Delta^+} = (A-3B+6C)(2u+d) + 5(B-C)(2u_+ + d_+). \tag{20}$$

The expressions for the charge radii of other decuplet baryons in NQM are presented in table 2.

Table 2. Charge radii of decuplet baryons in NQM in terms of GPM parameters.

$r^2_{B^*}$	NQM
$r^2_{\Delta^{++}}$	$\frac{1}{2}\left[(A-3B+6C)(3u)+5(B-C)(3u_+)\right]$
$r^2_{\Delta^{+}}$	$(A-3B+6C)(2u+d)+5(B-C)(2u_++d_+)$
$r^2_{\Delta^{0}}$	$(A-3B+6C)(u+2d)+5(B-C)(u_++2d_+)$
$r^2_{\Delta^{-}}$	$-(A-3B+6C)(3d)-5(B-C)(3d_+)$
$r^2_{\Sigma^{*+}}$	$(A-3B+6C)(2u+s)+5(B-C)(2u_++s_+)$
$r^2_{\Sigma^{*-}}$	$-(A-3B+6C)(2d+s)-5(B-C)(2d_++s_+)$
$r^2_{\Sigma^{*0}}$	$(A-3B+6C)(u+d+s)+5(B-C)(u_++d_++s_+)$
$r^2_{\Xi^{*0}}$	$(A-3B+6C)(u+2s)+5(B-C)(u_++2s_+)$
$r^2_{\Xi^{*-}}$	$-(A-3B+6C)(d+2s)-5(B-C)(d_++2s_+)$
$r^2_{\Omega^{-}}$	$-(A-3B+6C)(3s)-5(B-C)(3s_+)$

4. Chiral constituent quark model (χCQM)

The basic process in the χCQM is the GB emission by a constituent quark which further splits into a $q\bar{q}$ pair as

$$q_\pm \rightarrow \mathrm{GB}^0 + q'_\mp \rightarrow (q\bar{q}') + q'_\mp\,, \tag{21}$$

where $q\bar{q}' + q'$ constitute the 'quark sea' [34–36].

The effective Lagrangian describing the interaction between quarks and a nonet of GBs is

$$\mathcal{L} = g_8 \bar{q}\Phi q, \tag{22}$$

with

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \tag{23}$$

$$\Phi = \begin{pmatrix} \frac{\pi^o}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^o}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha\bar{K}^0 & -\beta\frac{2\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} \end{pmatrix}, \tag{24}$$

where $\zeta = g_1/g_8$, g_1 and g_8 are the coupling constants for the singlet and octet GBs, respectively. If the parameter $a(=|g_8|^2)$ denotes the transition probability of chiral fluctuation of the splitting $u(d) \rightarrow d(u) + \pi^{+(-)}$, then $\alpha^2 a$, $\beta^2 a$, and $\zeta^2 a$ respectively, denote the probabilities of transitions of $u(d) \rightarrow s + K^{-(o)}$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + \eta'$.

$SU(3)$ symmetry breaking is introduced by considering $M_s > M_{u,d}$ as well as by considering the masses of GBs to be non-degenerate ($M_{K,\eta} > M_\pi$ and $M_{\eta'} > M_{K,\eta}$) [34–37].

In terms of the quark contents, the GB field can be expressed as

$$\Phi = \begin{pmatrix} \phi_{uu}u\bar{u} + \phi_{ud}d\bar{d} + \phi_{us}s\bar{s} & \varphi_{ud}u\bar{d} & \varphi_{us}u\bar{s} \\ \varphi_{du}d\bar{u} & \phi_{du}u\bar{u} + \phi_{dd}d\bar{d} + \phi_{ds}s\bar{s} & \varphi_{ds}d\bar{s} \\ \varphi_{su}s\bar{u} & \varphi_{sd}s\bar{d} & \phi_{su}u\bar{u} + \phi_{sd}d\bar{d} + \phi_{ss}s\bar{s} \end{pmatrix},$$

where

$$\begin{aligned} &\phi_{uu} = \phi_{dd} = \frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \quad \phi_{ss} = \frac{2\beta}{3} + \frac{\zeta}{3}, \\ &\phi_{us} = \phi_{ds} = \phi_{su} = \phi_{sd} = -\frac{\beta}{3} + \frac{\zeta}{3}, \\ &\phi_{du} = \phi_{ud} = -\frac{1}{2} + \frac{\beta}{6} + \frac{\zeta}{3}, \\ &\varphi_{ud} = \varphi_{du} = 1, \;\; \varphi_{us} = \varphi_{ds} = \varphi_{su} = \varphi_{sd} = \alpha. \end{aligned} \tag{25}$$

In χCQM, we have introduced the exact spin and flavour symmetry breaking. A redistribution of flavour and spin structure takes place in the interior of baryon due to the chiral symmetry breaking and the modified flavour and spin content of the baryon can be calculated by substituting for every constituent quark

$$q \to P_q q + |\psi(q)|^2, \tag{26}$$

$$q_\pm \to P_q q_\pm + |\psi(q_\pm)|^2. \tag{27}$$

Here, $P_q = 1 - \sum P_q$ is the transition probability of no emission of GB from any of the q quark with

$$\begin{aligned} \sum P_u &= a\big(\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \varphi_{ud}^2 + \varphi_{us}^2\big), \\ \sum P_d &= a\big(\phi_{du}^2 + \phi_{dd}^2 + \phi_{ds}^2 + \varphi_{du}^2 + \varphi_{ds}^2\big), \\ \sum P_s &= a\big(\phi_{su}^2 + \phi_{sd}^2 + \phi_{ss}^2 + \varphi_{su}^2 + \varphi_{sd}^2\big), \end{aligned} \tag{28}$$

and $|\psi(q)|^2$ ($|\psi(q_\pm)|^2$) are the transition probabilities of the emission of q ($q_\pm$) quark

$$\begin{aligned} |\psi(u)|^2 = a[&\big(2\phi_{uu}^2 + \phi_{ud}^2 + \phi_{us}^2 + \varphi_{ud}^2 + \varphi_{us}^2\big)u + \phi_{uu}^2\bar{u} \\ &+(\phi_{ud}^2 + \varphi_{ud}^2)(d + \bar{d}) + (\phi_{us}^2 + \varphi_{us}^2)(s + \bar{s})], \end{aligned} \tag{29}$$

$$\begin{aligned} |\psi(d)|^2 = a[&\big(\phi_{du}^2 + 2\phi_{dd}^2 + \phi_{ds}^2 + \varphi_{du}^2 + \varphi_{ds}^2\big)d + \phi_{dd}^2\bar{d} \\ &+(\phi_{du}^2 + \varphi_{du}^2)(u + \bar{u}) + (\phi_{us}^2 + \varphi_{us}^2)(s + \bar{s})], \end{aligned} \tag{30}$$

$$\begin{aligned} |\psi(s)|^2 = a[&\big(\phi_{su}^2 + \phi_{sd}^2 + 2\phi_{ss}^2 + \varphi_{su}^2 + \varphi_{sd}^2\big)s \\ &+ \phi_{ss}^2\bar{s} + (\phi_{su}^2 + \varphi_{su}^2)(u + \bar{u}) + (\phi_{sd}^2 + \varphi_{sd}^2)(d + \bar{d})], \end{aligned} \tag{31}$$

$$
\begin{aligned}
|\psi(u_\pm)|^2 &= a[(\phi_{uu}^2+\phi_{ud}^2+\phi_{us}^2)u_\mp+\varphi_{ud}^2 d_\mp+\varphi_{us}^2 s_\mp],\\
|\psi(d_\pm)|^2 &= a[\varphi_{du}^2 u_\mp+(\phi_{du}^2+\phi_{dd}^2+\phi_{ds}^2)d_\mp+\varphi_{ds}^2 s_\mp],\\
|\psi(s_\pm)|^2 &= a[\varphi_{su}^2 u_\mp+\varphi_{sd}^2 d_\mp+(\phi_{su}^2+\phi_{sd}^2+\phi_{ss}^2)s_\mp]. \qquad (32)
\end{aligned}
$$

After the inclusion of 'quark sea', the charge radii for the spin-$\frac{1}{2}^+$ octet baryons, in χCQM with configuration mixing ($\chi\text{CQM}_{\text{config}}$), can be obtained by substituting eqs (26) and (27) for each quark in eqs (16) and (17). The charge radii for p and Σ^+ are now expressed as

$$
\begin{aligned}
r_{\rm p}^2 =\ & (A-3B)(2P_u u+2|\psi(u)|^2+P_d d+|\psi(d)|^2)+3(B-C)\\
&\times\left[\cos^2\theta\left(\frac{4}{3}P_u u_+ +\frac{4}{3}|\psi(u_+)|^2-\frac{1}{3}P_d d_+ -\frac{1}{3}|\psi(d_+)|^2\right)\right.\\
&\left.+\sin^2\theta\left(\frac{2}{3}P_u u_+ +\frac{2}{3}|\psi(u_+)|^2+\frac{1}{3}P_d d_+ +\frac{1}{3}|\psi(d_+)|^2\right)\right], \qquad (33)\\
r_{\Sigma^+}^2 =\ & (A-3B)(2P_u u+2|\psi(u)|^2+P_s s+|\psi(s)|^2)+3(B-C)\\
&\times\left[\cos^2\theta\left(\frac{4}{3}P_u u_+ +\frac{4}{3}|\psi(u)|^2-\frac{1}{3}P_s s_+ -\frac{1}{3}|\psi(s)|^2\right)\right.\\
&\left.+\sin^2\theta\left(\frac{2}{3}P_u u_+ +\frac{2}{3}|\psi(u_+)|^2+\frac{1}{3}P_s s_+ +\frac{1}{3}|\psi(s_+)|^2\right)\right]. \qquad (34)
\end{aligned}
$$

The charge radii in the $\chi\text{CQM}_{\text{config}}$ for other spin-$\frac{1}{2}^+$ octet baryons are presented in table 3. The results without configuration mixing can easily be obtained by taking the mixing angle $\theta=0$.

Table 3. Charge radii of octet baryons in $\chi\text{CQM}_{\text{config}}$ in terms of $SU(3)$ symmetry breaking parameters and GPM parameters. These results are obtained by substituting $q\to P_q q+|\psi(q)|^2$ and $q_\pm\to P_q q_\pm+|\psi(q_\pm)|^2$ for every constituent quark in NQM. Results in χCQM without configuration mixing can easily be obtained by substituting the mixing angle $\theta=0$.

r_B^2	$\chi\text{CQM}_{\text{config}}$
$r_{\rm p}^2$	$A-3B+(B-C)\left[\cos^2\theta\left(3-a\left(4+2\alpha^2+\beta^2+2\zeta^2\right)\right)+\sin^2\theta\left(1-\frac{a}{3}\left(6+\beta^2+2\zeta^2\right)\right)\right]$
$r_{\rm n}^2$	$(B-C)\left[\cos^2\theta\left(-2+\frac{a}{3}\left(3+9\alpha^2+2\beta^2+4\zeta^2\right)\right)+a\sin^2\theta\left(-1+\alpha^2\right)\right]$
$r_{\Sigma^+}^2$	$A-3B+(B-C)\left[\cos^2\theta\left(3-\frac{a}{3}\left(12+5\alpha^2+4\beta^2+6\zeta^2\right)\right)+\sin^2\theta\left(1-\frac{a}{3}\left(6+\alpha^2+2\zeta^2\right)\right)\right]$
$r_{\Sigma^-}^2$	$A-3B+(B-C)\left[\cos^2\theta\left(-1+\frac{a}{3}\left(7\alpha^2+2\zeta^2\right)\right)+\sin^2\theta\left(-1+\frac{a}{3}\left(5\alpha^2+2\beta^2+2\zeta^2\right)\right)\right]$
$r_{\Sigma^0}^2$	$(B-C)\left[\cos^2\theta\left(1-\frac{a}{3}\left(6-\alpha^2+2\beta^2+2\zeta^2\right)\right)+\frac{a}{3}\sin^2\theta\left(-3+2\alpha^2+\beta^2\right)\right]$
$r_{\Xi^0}^2$	$(B-C)\left[\cos^2\theta\left(-2+\frac{a}{3}\left(3+5\alpha^2+6\beta^2+4\zeta^2\right)\right)+\frac{a}{3}\sin^2\theta\left(-3+\alpha^2+2\beta^2\right)\right]$
$r_{\Xi^-}^2$	$A-3B+(B-C)\left[\cos^2\theta\left(-1+\frac{a}{3}\left(2\alpha^2+5\beta^2+2\zeta^2\right)\right)+\sin^2\theta\left(-1+\frac{a}{3}\left(4\alpha^2+3\beta^2+2\zeta^2\right)\right)\right]$
r_Λ^2	$(B-C)\left[\cos^2\theta\left(-1+\frac{a}{3}(3\alpha^2+4\beta^2+2\zeta^2)\right)+\frac{a}{3}\sin^2\theta\left(-3+2\alpha^2+\beta^2\right)\right]$
$r_{\Sigma\Lambda}^2$	$(B-C)\left[\sqrt{3}-\frac{a}{\sqrt{3}}(3+3\alpha^2+\beta^2+2\zeta^2)\right]$

Table 4. Charge radii of decuplet baryons in χCQM in terms of $SU(3)$ symmetry breaking parameters and GPM parameters. These results are obtained by substituting $q \rightarrow P_q q + |\psi(q)|^2$ and $q_\pm \rightarrow P_q q_\pm + |\psi(q_\pm)|^2$ for every constituent quark in NQM.

$r^2_{B^*}$	χCQM
$r^2_{\Delta^{++}}$	$A + 2B + C - \frac{5a}{6}(B - C)(9 + 3\alpha^2 + 2\beta^2 + 4\zeta^2)$
$r^2_{\Delta^{+}}$	$A + 2B + C - \frac{5a}{3}(B - C)(6 + \beta^2 + 2\zeta^2)$
$r^2_{\Delta^{0}}$	$5a(B - C)(-1 + \alpha^2)$
$r^2_{\Delta^{-}}$	$A + 2B + C - \frac{5a}{3}(B - C)(6\alpha^2 + \beta^2 + 2\zeta^2)$
$r^2_{\Sigma^{*+}}$	$A + 2B + C - \frac{5a}{3}(B - C)(6 + \alpha^2 + 2\zeta^2)$
$r^2_{\Sigma^{*-}}$	$A + 2B + C - \frac{5a}{3}(B - C)(5\alpha^2 + 2\beta^2 + 2\zeta^2)$
$r^2_{\Sigma^{*0}}$	$\frac{5a}{3}(B - C)(-3 + 2\alpha^2 + \beta^2)$
$r^2_{\Xi^{*0}}$	$\frac{5a}{3}(B - C)(-3 + \alpha^2 + 2\beta^2)$
$r^2_{\Xi^{*-}}$	$A + 2B + C - \frac{5a}{3}(B - C)(4\alpha^2 + 3\beta^2 + 2\zeta^2)$
$r^2_{\Omega^{-}}$	$A + 2B + C - \frac{5a}{3}(B - C)(3\alpha^2 + 4\beta^2 + 2\zeta^2)$

Similarly, for the spin-$\frac{3}{2}^+$ decuplet baryons, the charge radii is modified on substituting for each quark from eqs (26) and (27). For example, the charge radii for Δ^+ in χCQM can be expressed as

$$r^2_{\Delta^+} = (A - 3B + 6C)(2P_u u + 2|\psi(u)|^2 + P_d d + |\psi(d)|^2)$$

$$+ 5(B - C)\left(2P_u u_+ + 2|\psi(u_+)|^2 + P_d d_+ + |\psi(d_+)|^2\right). \tag{35}$$

The charge radii of the other decuplet baryons can be calculated similarly and are detailed in table 4.

5. Results and discussion

The charge radii calculations of octet and decuplet baryons involve two sets of parameters, the $SU(3)$ symmetry breaking parameters of χCQM and the GPM parameters. The χCQM parameters, a, $a\alpha^2$, $a\beta^2$, and $a\zeta^2$ represent respectively, the probabilities of fluctuations to pions, K, η, and η'. A best fit of χCQM parameters can be obtained by carrying out a fine grained analysis of the spin and flavour distribution functions [37] leading to

$$a = 0.12, \quad \alpha = 0.7, \quad \beta = 0.4, \quad \zeta = -0.15. \tag{36}$$

The mixing angle θ is fixed from the consideration of neutron charge radius [8]. This set of parameters has already been tested for a wide variety of low-energy matrix elements and have been able to give a simultaneous fit to the quantities describing proton spin and flavour structure [37], weak vector–axial vector form factors [38], strangeness content in the nucleon [39], magnetic moments of octet and decuplet baryons [40] etc.

The order of GPM parameters corresponding to the one-, two-, and three-quark terms, decreases with the increasing complexity of terms and obeys the hierarchy $A > B > C$ [50]. These are fitted by using the available experimental values for the charge radii and quadrupole moment of nucleon as input. In the present case, we have used $r_{\rm p} = 0.877 \pm 0.007$ fm [3], $r_{\rm n}^2 = -0.1161 \pm 0.0022$ fm^2 [3], and $Q_{\Delta^+ N} = -0.0846 \pm 0.0033$ fm^2 [51]. The set of GPM parameters obtained after χ^2 minimization are as follows:

$$A = 0.879, \quad B = 0.094, \quad C = 0.016. \tag{37}$$

Since we also intend to investigate the extent to which the three-quark term contributes, we calculate the charge radii corresponding to the one- and two-quark terms only by taking $C = 0$. Similarly, if we intend to calculate the charge radii corresponding to just the one-quark term, we can take $B = C = 0$.

Using the set of parameters discussed above, we have calculated the numerical values for the charge radii of octet and decuplet baryons in χCQM$_{\rm config}$ and presented the results in tables 5 and 6, respectively. To understand the implications of chiral symmetry breaking and 'quark sea', we have also presented the results of NQM and we have compared our results with the predictions of other available phenomenological models. As the calculations in χCQM have been carried out using the GPM, the NQM results have also been presented by including the one-, two-, and three-quark contributions of the GPM parameters. It is clear from tables 1 and 2 that if we consider the contribution coming from one-quark term only, the charge radii of the charged baryons are equal whereas all neutral baryons have zero charge radii. These predictions are modified by including two- and three-quark terms of GPM in NQM and are further modified by including 'quark sea' and $SU(3)$ symmetry breaking effects. Thus, it seems that the GPM parameters alone are able to explain the experimentally observed non-zero charge radii of the neutral baryons. However, NQM is unable to account for the 'proton spin problem' and other related quantities, the results have been presented for χCQM. The importance of strange quark mass has been investigated by comparing the χCQM results with and without $SU(3)$ symmetry breaking. The $SU(3)$ symmetry results can be easily derived from tables 3 and 4 by considering $\alpha = \beta = 1$ and $\zeta = -1$. The $SU(3)$ breaking results are in general higher in magnitude than the $SU(3)$ symmetric results and the values obtained are also in agreement with the other models.

For octet baryons, it can be easily seen from table 5 that, in the $SU(3)$ symmetric limit, octet baryon charge radii can be expressed in terms of the nucleon charge radii leading to the following relations:

$$r_{\Sigma^+}^2 = r_{\rm p}^2, \quad r_{\Xi^-}^2 = r_{\Sigma^-}^2 = r_{\rm p}^2 + r_{\rm n}^2. \tag{38}$$

The inclusion of $SU(3)$ symmetry breaking changes this pattern considerably and we get

$$r_{\Sigma^+}^2 > r_{\rm p}^2, \quad r_{\Xi^-}^2 > r_{\Sigma^-}^2 > r_{\rm p}^2 + r_{\rm sn}^2. \tag{39}$$

Also we have

$$2r_{\Lambda}^2 = -2r_{\Sigma^0}^2 = r_{\Xi^0}^2 = r_{\rm n}^2, \tag{40}$$

which has its importance in the isospin limit where the three-quark core in neutral baryons does not contribute to the charge radii. In the limit of $SU(3)$ symmetry breaking, a

Table 5. Charge radii of octet baryons calculated in χCQM in comparison with other phenomenological models (in units of fm^2).

Charge radii	Data [3]	NQM	HBχPT [27]	χPT [28]	$1/N_c$ [25]	Lattice [29]	χCQM$_{\text{config}}$ With $SU(3)$ symmetry	χCQM$_{\text{config}}$ With $SU(3)$ symmetry breaking $A=0.879$, $B=0.094$, $C=0.0$	χCQM$_{\text{config}}$ With $SU(3)$ symmetry breaking $A=0.879$, $B=0.094$, $C=0.016$
r^2_p	$r_p = 0.877 \pm 0.007$	0.813	0.735	0.717	0.779	0.685	0.732	0.801	0.766
r^2_n	-0.1161 ± 0.0022	−0.138	−0.113	−0.113	−0.116	−0.158	−0.087	−0.140	−0.116
$r^2_{\Sigma^+}$	–	0.813	1.366	0.60 ± 0.02	0.928	0.749	0.732	0.802	0.767
$r^2_{\Sigma^-}$	0.61 ± 0.21 [4] $0.78 \pm 0.08 \pm 0.06$	0.675	0.798	0.67 ± 0.03	0.672	0.657	0.646	0.678	0.664
$r^2_{\Sigma^0}$	–	0.069	–	-0.03 ± 0.01	0.128	–	0.043	0.062	0.052
$r^2_{\Xi^0}$	–	−0.138	−0.122	0.13 ± 0.03	0.132	−0.082	−0.087	−0.145	−0.120
$r^2_{\Xi^-}$	–	0.675	0.997	0.49 ± 0.05	0.520	0.502	0.646	0.683	0.669
r^2_{Λ}	–	−0.069	−0.284	0.11 ± 0.02	0.050	0.010	−0.042	−0.076	−0.063
$r^2_{\Sigma\Lambda}$	–	0.135	0.074	0.03 ± 0.01	−0.066	–	0.085	0.132	0.109

Table 6. Charge radii of decuplet baryons calculated in χCQM in comparison with other phenomenological models (in units of fm^2).

Charge radii	NQM	FTQM [30]	CCQM [22]	$1/N_c$ [25]	χCQM: With $SU(3)$ symmetry	χCQM: With $SU(3)$ symmetry breaking, $A=0.879$, $B=0.094$, $C=0.0$	χCQM: With $SU(3)$ symmetry breaking, $A=0.879$, $B=0.094$, $C=0.016$
$r^2_{\Delta^{++}}$	1.084	1.18	0.43	1.011	0.938	0.961	0.996
$r^2_{\Delta^{+}}$	1.084	0.82	0.43	1.011	0.938	0.946	0.983
$r^2_{\Delta^{0}}$	0.0	0.16	0.00	0.0	0.0	−0.030	−0.025
$r^2_{\Delta^{-}}$	1.084	0.84	0.43	1.011	0.938	1.006	1.033
$r^2_{\Sigma^{*+}}$	1.084	0.97	0.42	1.086	0.938	0.940	0.978
$r^2_{\Sigma^{*-}}$	1.084	0.84	0.37	0.845	0.938	1.013	1.038
$r^2_{\Sigma^{*0}}$	0.0	0.34	0.03	0.127	0.0	−0.036	−0.030
$r^2_{\Xi^{*0}}$	0.0	0.49	0.06	0.244	0.0	−0.043	−0.035
$r^2_{\Xi^{*-}}$	1.084	0.82	0.33	0.692	0.938	1.019	1.043
$r^2_{\Omega^{-}}$	0.390	0.78	0.29	0.553	0.245	0.429	0.355

non-vanishing value for the neutral baryons charge radii is generated by the 'quark sea' through the chiral fluctuations of constituent quarks leading to

$$r^2_{\Lambda} > -r^2_{\Sigma^0}, \quad r^2_{\Xi^0} > r^2_{\mathrm{n}}. \tag{41}$$

The exact order of $SU(3)$ symmetry breaking effects can be easily found from table 3. As experimental information is not available for some of these octet charge radii, the accuracy of these relations can be tested by future experiments. It is interesting to note that, the relation for the Σ baryon charge radii

$$r^2_{\Sigma^+} - 2r^2_{\Sigma^0} - r^2_{\Sigma^-} = 0, \tag{42}$$

holds good even after incorporating $SU(3)$ symmetry breaking. Since this relation is independent of $SU(3)$ symmetry breaking parameters, any refinement in the Σ baryon charge radii data would have important implications for $SU(3)$ symmetry breaking.

The $SU(3)$ symmetry breaking corrections are of the order of 5% for p, Σ^+, Σ^-, and Ξ^- baryons whereas this contribution is more than 20% for the neutral octet baryons. A closer look at the results reveal several interesting points. The $SU(3)$ symmetry breaking is expected to reduce the charge radii with increasing strangeness content of the baryon with p, Σ^-, and Ξ^- having successively smaller charge radii. However, this strangeness suppression is not dominant in the χCQM because of the presence of 'quark sea'. As a result, the charge radii of Σ^- and Ξ^- turn out to be as large as that of p. Our predicted value $r^2_{\Sigma^-} = 0.664$ is clearly of the order of proton charge radius and is also in agreement with the recent SELEX Collaboration experimental results [3].

In the table, we have presented the results for the case with configuration mixing generated by the spin–spin forces. We have not presented the results without configuration mixing which can easily be obtained by taking the mixing angle $\theta = 0$. It has been observed that configuration mixing decreases the overall magnitudes of the charge radii in χCQM but the change is very small as compared to the other low-energy properties like spin distribution function, magnetic moments etc. [37–40]. In order to make our calculations for the octet baryon charge radii more responsive, we have also presented the results by neglecting the contributions coming from the three-quark term ($C = 0$) for the $SU(3)$ symmetry breaking case. We find that the inclusion of the three-quark term decreases the value of the octet baryon charge radii which may be due to the spin of the 'quark sea' contributing with an opposite sign. Even though the three-quark contribution is small, it is not negligible. In fact, in some cases the contribution decreases the value of charge radii by a significant amount. This is particularly true for the neutral octet baryons, for example, n, Σ^0, Ξ^0, and Λ.

On comparing our results with the other phenomenological models, we find that for charged octet baryons, our results are in fair agreement in sign and magnitude with the other model predictions. However, for the neutral octet baryons n, Σ^0, Ξ^0, and Λ, different models show opposite sign, for example, if we consider the charge radii for the Λ baryon. Our model prediction (-0.063) is opposite in sign to the predictions of the relativistic constituent quark model (RCQM) [21], covariant constituent quark model (CCQM) [22], $1/N_c$ expansion [24], and PχQM [26]. On the other hand, it is in agreement with the sign of HBχPT [27]. A similar trend has been observed for the charge radii of $\Sigma\Lambda$ transition. The difference in the sign may be due to the chiral fluctuation of a constituent quark leading to the reversal of sign in neutral octet baryons. This can perhaps be substantiated by measuring charge radii of other baryons.

The spin-$\frac{3}{2}^+$ decuplet baryon charge radii, presented in table 6, are in general higher than the octet baryon charge radii which is in line with the trend followed by the octet and decuplet baryons for the other low-energy hadronic matrix elements such as magnetic moments. In this case also, the inclusion of $SU(3)$ symmetry breaking increases the predictions of charge radii. It can be easily shown that $SU(3)$ symmetry results in the following relations for the decuplet baryons:

$$r^2_{\Delta^{++}} = r^2_{\Delta^{+}} = r^2_{\Delta^{-}} = r^2_{\Sigma^{*+}} = r^2_{\Sigma^{*-}} = r^2_{\Xi^{*-}}. \tag{43}$$

These results are affected by the inclusion of $SU(3)$ symmetry breaking and give

$$r^2_{\Xi^{*-}} > r^2_{\Sigma^{*-}} > r^2_{\Delta^{-}} > r^2_{\Delta^{++}} > r^2_{\Delta^{+}} > r^2_{\Sigma^{*+}}. \tag{44}$$

Some relations, derived in $1/N_c$ expansion of QCD [24,25], are found to be independent of $SU(3)$ symmetry breaking parameters in χCQM. Even though the individual charge radii are affected by $SU(3)$ symmetry breaking, the effects cancel exactly for the following relations:

$$\begin{aligned}
&2r^2_{\Delta^{++}} - r^2_{\Delta^{+}} - r^2_{\Delta^{0}} - r^2_{\Delta^{-}} = 0,\\
&2r^2_{\Delta^{++}} - 3r^2_{\Delta^{+}} + 3r^2_{\Delta^{0}} + r^2_{\Delta^{-}} = 0,\\
&r^2_{\Sigma^{*+}} - 2r^2_{\Sigma^{*0}} - r^2_{\Sigma^{*-}} = 0.
\end{aligned} \tag{45}$$

In this case also, $SU(3)$ symmetry breaking is expected to reduce the charge radii with increasing strangeness content. As a consequence, Δ^-, Σ^-, and Ξ^- should have successively decreasing charge radii. However, this suppression disappears in χCQM due to the effect of 'quark sea' and the charge radii of Δ^+, Σ^{*-}, and Ξ^{*0} are of almost the same order as that of Σ^{*+}, Ξ^{*-}, and Σ^{*0}, respectively. Again, the sign and magnitude of the decuplet baryon charge radii in χCQM are in fair agreement with the other phenomenological models with the exception for neutral baryons. One of the important predictions in χCQM is a non-zero Δ^0 charge radii which vanishes in NQM as well as in some other models. This is further endorsed by the predictions of the field theoretical quark model (FTQM) calculations [30]. The contribution of the three-quark term in decuplet baryons is exactly opposite to that for the octet baryons. Unlike the octet baryon case, the inclusion of the three-quark term increases the value of the baryon charge radii.

For the sake of completeness, certain relations between the octet and decuplet baryon charge radii can also be tested for the spacing between the levels. In NQM, we have

$$r^2_{\Sigma^-} - r^2_{\Sigma^{*-}} = r^2_{\Xi^-} - r^2_{\Xi^{*-}} = r^2_{\Sigma^+} - r^2_{\Sigma^{*+}} = r^2_{\Xi^0} - r^2_{\Xi^{*0}} = r^2_{\mathrm{n}}. \tag{46}$$

In χCQM, the inclusion of $SU(3)$ symmetry breaking effects creates a spacing between the octet and decuplet baryon charge radii as

$$\begin{aligned} r^2_{\mathrm{p}} - r^2_{\Delta^+} &= r^2_{\Sigma^+} - r^2_{\Sigma^{*+}} = -0.31, \\ r^2_{\Sigma^-} - r^2_{\Sigma^{*-}} &= r^2_{\Xi^-} - r^2_{\Xi^{*-}} = -0.48, \\ r^2_{\mathrm{n}} - r^2_{\Delta^0} &= r^2_{\Xi^0} - r^2_{\Xi^{*0}} = -0.09. \end{aligned} \tag{47}$$

6. Summary and conclusion

To summarize, χCQM is able to provide a fairly good description of the charge radii of spin-$\frac{1}{2}^+$ octet and spin-$\frac{3}{2}^+$ decuplet baryons using the general parametrization method (GPM). The most significant prediction of the model is the non-zero value pertaining to the charge radii of the neutral octet baryons (n, Σ^0, Ξ^0, Λ) and decuplet baryons (Δ^0, Σ^{*0}, Ξ^{*0}). The effects of $SU(3)$ symmetry breaking have also been investigated and the results show considerable improvement over the $SU(3)$ symmetric case. We have also studied the implications of GPM parameters, particularly the contribution of the three-quark term in the octet and decuplet baryon charge radii. We find that the sign of the three-quark term contribution is opposite for octet and decuplet baryons. The χCQM parameters play an important role in the $SU(3)$ symmetry breaking effects whereas the assumed parametrization plays a dominant role in the valence quark distributions. New experiments aimed at measuring the charge radii of the other baryons are needed for a profound understanding of the hadron structure in the non-perturbative regime of QCD.

In conclusion, we would like to state that at the leading order constituent quarks and the weakly interacting Goldstone bosons constitute the appropriate degrees of freedom in the non-perturbative regime of QCD. The $SU(3)$ symmetry breaking parameters pertaining to the strangeness contribution and the GPM parameters pertaining to the one-, two-, and three-quark contributions are the key parameters in understanding the octet and decuplet baryon charge radii.

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