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Bright and dark soliton solutions of the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili equation and generalized Benjamin equation

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Abstract. In this paper, we obtain the 1-soliton solutions of the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili (gKP) equation and the generalized Benjamin equation. By using two solitary wave ansatz in terms of sech^p and $\tanh^p$ functions, we obtain exact analytical bright and dark soliton solutions for the considered model. These solutions may be useful and desirable for explaining some nonlinear physical phenomena in genuinely nonlinear dynamical systems.

Keywords. Exact solutions; dark soliton; bright soliton; (3 + 1)-dimensional generalized Kadomtsev–Petviashvili equation; the generalized Benjamin equation.

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1. Introduction

The research in the area of nonlinear evolution equation (NEE) has made significant progress in the past decades. There has been a growing interest in finding exact analytical solutions to nonlinear wave equations by using appropriate techniques. Particularly, the existence of soliton solutions for NEEs is of great importance because of their potential application in many physics areas such as chaos, mathematical biology, diffusion process, plasma physics, optical fibres, neural physics, solid state physics etc.

Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are of vital importance in NEEs [1]. In the past decades, many methods such as the tanh–sech method [2,3], extended tanh method [4,5], sine–cosine method [6,7], exp-function method [8,9], homogeneous balance method [10,11], first integral method [12,13], Jacobi elliptic function method [14,15], (G'/G)-expansion method [16,17] and F-expansion method [18,19] were developed for finding exact solutions of NEEs.

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Solitary waves are wave packets or pulses which propagate in nonlinear dispersive media. Due to dynamical balance between the nonlinear and dispersive effects, these waves retain a stable waveform. A soliton is a very special type of solitary wave, which keeps its waveform even after collision with other solitons [20].

Solitons in photonic crystal fibres as well as diffraction Bragg gratings have been studied. In addition, theories of dispersion-managed solitons, quasilinear pulses have also been developed. Dark solitons are also known as topological optical solitons in the context of nonlinear optics media. It is known that dark optical solitons are more stable in the presence of noise and spreads more slowly in the presence of loss, in the optical communication systems, as compared to bright solitons [21–23].

In this paper, one such modern method of integrability will be applied to carry out the integration of a generalized version of the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili and the generalized Benjamin equations. The technique that will be adopted to integrate such equations is the solitary wave ansatz method.

The paper is organized as follows: in §2, we derived the bright and dark soliton solutions of nonlinear (3 + 1)-dimensional gKP equation. In §3, we apply the ansatz method to the generalized Benjamin equation and establish many soliton solutions. In the last section, we briefly make a summary to the results that we have obtained.

2. The (3 + 1)-dimensional gKP equation

The (3 + 1)-dimensional gKP equation, given by [24]

$$(u_t + 6u^n u_x + u_{xxx})_x + 3u_{yy} + 3u_{zz} = 0, \tag{1}$$

describes the dynamics of solitons and nonlinear waves in plasma physics and fluid dynamics [25].

Now, the bright and dark soliton solutions of this equation will be obtained.

2.1 *The bright (non-topological) soliton solution*

The solitary wave ansatz for the 1-soliton solution of (1) is given by the form [26–31]

$$u(x, y, z, t) = \lambda \operatorname{sech}^p \tau, \tag{2}$$

where $\tau = ax + by + cz - vt$ and a, b, c are inverse widths of the soliton. Here λ and v are respectively the amplitude and the velocity of the soliton. The exponent p is unknown at this point and will be determined later. From the ansatz (2), it is possible to obtain

$$u_{tx} = -p^2\lambda a v \operatorname{sech}^p\tau + p(p+1)\lambda a v \operatorname{sech}^{p+2}\tau, \tag{3}$$

$$\begin{aligned}(u^n u_x)_x &= \lambda^{n+1} p a^2 (pn + p)\operatorname{sech}^{pn+p}\tau \\ &\quad -\lambda^{n+1} p a^2 (pn + p + 1)\operatorname{sech}^{pn+p+2}\tau,\end{aligned} \tag{4}$$

$$\begin{aligned}u_{xxxx} &= p^4\lambda a^4 \operatorname{sech}^p\tau - 2p(p+1)(p^2+2p+2)\lambda a^4 \operatorname{sech}^{p+2}\tau \\ &\quad + p(p+1)(p+2)(p+3)\lambda a^4 \operatorname{sech}^{p+4}\tau,\end{aligned} \tag{5}$$

$$u_{yy} = p^2\lambda b^2 \mathrm{sech}^p\tau - p(p+1)\lambda b^2 \mathrm{sech}^{p+2}\tau, \tag{6}$$

$$u_{zz} = p^2\lambda c^2 \mathrm{sech}^p\tau - p(p+1)\lambda c^2 \mathrm{sech}^{p+2}\tau. \tag{7}$$

Substituting eqs (3)–(7) into eq. (1) yields

$$\begin{aligned}
&- p^2\lambda a v \mathrm{sech}^p\tau + p(p+1)\lambda a v \mathrm{sech}^{p+2}\tau \\
&+ 6\lambda^{n+1} p a^2 (pn+p)\mathrm{sech}^{pn+p}\tau - 6\lambda^{n+1} p a^2 (pn+p+1)\mathrm{sech}^{pn+p+2}\tau \\
&+ p^4\lambda a^4 \mathrm{sech}^p\tau - 2p(p+1)(p^2+2p+2)\lambda a^4 \mathrm{sech}^{p+2}\tau \\
&+ p(p+1)(p+2)(p+3)\lambda a^4 \mathrm{sech}^{p+4}\tau \\
&+ 3p^2\lambda b^2 \mathrm{sech}^p\tau - 3p(p+1)\lambda b^2 \mathrm{sech}^{p+2}\tau \\
&+ 3p^2\lambda c^2 \mathrm{sech}^p\tau - 3p(p+1)\lambda c^2 \mathrm{sech}^{p+2}\tau \\
&= 0.
\end{aligned} \tag{8}$$

Now from (8), equating the exponents of $pn + p + 2$ and $p + 4$ leads to

$$pn + p + 2 = p + 4, \tag{9}$$

which gives

$$p = \frac{2}{n}. \tag{10}$$

It needs to be noted that the same result is obtained when the exponents $pn+p$ and $p+2$ are equated to each other. From eq. (8), setting the coefficients of $\mathrm{sech}^{pn+p+2}\tau$ and $\mathrm{sech}^{p+4}\tau$ terms to zero we get

$$-6\lambda^{n+1} p a^2 (pn+p+1) + p(p+1)(p+2)(p+3)\lambda a^4 = 0, \tag{11}$$

so that

$$\lambda = \left[\frac{a^2(p+1)(p+2)(p+3)}{6\,(1+np+p)}\right]^{1/n}. \tag{12}$$

We next set the coefficients of $\mathrm{sech}^p\tau$ terms in eq. (8) to zero to obtain,

$$-p^2\lambda a v + p^4\lambda a^4 + 3p^2\lambda b^2 + 3p^2\lambda c^2 = 0, \tag{13}$$

which gives

$$v = \frac{p^2a^4 + 3b^2 + 3c^2}{a}, \qquad a \neq 0. \tag{14}$$

Finally, we get the bright (non-topological) soliton solution for the $(3+1)$-dimensional gKP equation, when the above expressions of p, λ and v given by eqs (10), (12) and (14) are substituted in eq. (2) as

$$u(x, y, z, t) = \lambda\ \mathrm{sech}^{2/n}(ax + by + cz - vt)\,. \tag{15}$$

In this case, if we take $p = 2$ this yields $n = 1$ and solving eq. (12) we get

$$\lambda = 2a^2. \tag{16}$$

By solving eq. (14) using eq. (13) we get,

$$v = \frac{4a^4 + 3b^2 + 3c^2}{a}, \qquad a \neq 0. \tag{17}$$

Hence the 1-soliton solution to (1) is given by

$$u(x, y, z, t) = 2a^2 \operatorname{sech}^2 \left(ax + by + cz - \left(\frac{4a^4 + 3b^2 + 3c^2}{a} \right) t \right). \tag{18}$$

In this case, if we take $p = 1$ this yields $n = 2$ and solving eq. (12) we get

$$\lambda = \pm a. \tag{19}$$

Solving eq. (14) using eq. (19) we get,

$$v = \frac{a^4 + 3b^2 + 3c^2}{a}, \qquad a \neq 0 \tag{20}$$

Thus, the bright soliton solution to (1) is given by

$$u(x, y, z, t) = \pm a \operatorname{sech} \left(ax + by + cz - \left(\frac{a^4 + 3b^2 + 3c^2}{a} \right) t \right). \tag{21}$$

2.2 *The dark (topological) soliton solution*

In this section, we are interested in finding the dark soliton solution, as defined in [23] for the (3 + 1)-dimensional gKP equation (1).

In order to construct dark soliton solutions for eq. (1), we use an ansatz solution of the form [32,33]

$$u(x, y, z, t) = \lambda \tanh^p \tau, \tag{22}$$

and choose now a suitable solitary wave ansatz with (3 + 1) dependent variables of the form

$$\tau = ax + by + cz - vt, \tag{23}$$

where λ, a, b and c are unknown free parameters and v is the velocity of the soliton, that will be determined. The exponent p is also unknown.

From eq. (22), we have

$$u_{tx} = -p\lambda va \left\{ (p-1) \tanh^{p-2} \tau - 2p \tanh^p \tau + (p+1) \tanh^{p+2} \tau \right\}, \tag{24}$$

$$\begin{aligned} \left(u^n u_x\right)_x = {} & p\lambda^{n+1} a^2 \left\{ (np + p - 1) \tanh^{np+p-2} \tau - 2(np + p) \tanh^{np+p} \tau \right. \\ & \left. + (np + p + 1) \tanh^{np+p+2} \tau \right\}, \end{aligned} \tag{25}$$

$$u_{xxxx} = \lambda p a^4 \left\{ \begin{array}{c} (p-1)(p-2)(p-3) \tanh^{p-4} \tau - 4(p-1)(p^2-2p+2) \tanh^{p-2} \tau \\ +2p(3p^2+5) \tanh^p \tau - 4(p+1)(p^2+2p+2) \tanh^{p+2} \tau \\ +(p+1)(p+2)(p+3) \tanh^{p+4} \tau \end{array} \right\}, \tag{26}$$

$$u_{yy} = \lambda p b^2 \left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{27}$$

$$u_{zz} = \lambda p c^2 \left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\right\}, \tag{28}$$

where $\tau = ax + by + cz - vt$. Substituting eqs (24)–(28) into eq. (1), we obtain

$$\begin{aligned}
&- p\lambda v a \left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\right\} \\
&\quad + 6p\lambda^{n+1}a^2 \left\{(np+p-1)\tanh^{np+p-2}\tau - 2(np+p)\tanh^{np+p}\tau \right.\\
&\qquad\qquad \left. + (np+p+1)\tanh^{np+p+2}\tau\right\} \\
&\quad + \lambda p a^4 \left\{\begin{array}{c} (p-1)(p-2)(p-3)\tanh^{p-4}\tau - 4(p-1)(p^2-2p+2)\tanh^{p-2}\tau \\ + 2p(3p^2+5)\tanh^{p}\tau - 4(p+1)(p^2+2p+2)\tanh^{p+2}\tau \\ + (p+1)(p+2)(p+3)\tanh^{p+4}\tau \end{array}\right\} \\
&\quad + 3\lambda p b^2 \left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\right\} \\
&\quad + 3\lambda p c^2 \left\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\right\} \\
&\quad = 0.
\end{aligned} \tag{29}$$

Thus, from matching the exponents of $\tanh^{np+p+2}\tau$ and $\tanh^{p+4}\tau$ terms in eq. (29), we obtain

$$np + p + 2 = p + 4, \tag{30}$$

which yields

$$p = \frac{2}{n}. \tag{31}$$

By setting the corresponding coefficients of $\tanh^{np+p+2}\tau$ and $\tanh^{p+4}\tau$ terms to zero one gets

$$6p\lambda^{n+1}a^2(np+p+1) + \lambda p a^4(p+1)(p+2)(p+3) = 0, \tag{32}$$

so that

$$\lambda = \left[-\frac{a^2(p+1)(p+2)(p+3)}{6\,(1+np+p)}\right]^{1/n}. \tag{33}$$

We next set the coefficients of $\tanh^{p-2}\tau$ terms to zero to get

$$\begin{aligned}
&-p\lambda v a(p-1) - 4p(p-1)(p^2-2p+2)\lambda a^4 + 3\lambda p b^2(p-1) \\
&\quad + 3\lambda p c^2(p-1) = 0.
\end{aligned} \tag{34}$$

Solving eq. (34) using eq. (33)

$$v = -\frac{4p(p-2)a^4 + 8a^4 - 3b^2 - 3c^2}{a}. \tag{35}$$

Lastly, we can determine the dark (topological) soliton solution for

$$u(x, y, z, t) = \lambda \tanh^{2/n}(ax + by + cz - vt), \tag{36}$$

where the velocity of the solitons v is given in eq. (35) and free parameter λ is given by (33).

In this case, if we take $p = 2$ this yields $n = 1$ and solving eq. (32) we get

$$\lambda = -2a^2. \tag{37}$$

Solving eq. (34) using eq. (37) we get

$$v = -\frac{8a^4 - 3b^2 - 3c^2}{a}, \qquad a \neq 0. \tag{38}$$

Hence the dark soliton solution to (1) is given by

$$u(x, y, z, t) = -2a^2 \tanh^2\left(ax + by + cz + \left(\frac{8a^4 - 3b^2 - 3c^2}{a}\right) t\right). \tag{39}$$

3. Generalized Benjamin equation

We consider nonlinear generalized Benjamin equation which is given by

$$u_{tt} + \alpha(u^n u_x)_x + \beta u_{xxxx} = 0, \tag{40}$$

where α and β are constants. This kind of equation is one of the most important NLPDEs, used in the analysis of long waves in shallow water [34]. Taghizadeh *et al* obtained some soliton solutions and travelling wave solutions using the extended tanh method [24].

3.1 *The bright (non-topological) soliton solution*

In this section the search is going to be for non-topological 1-soliton solution to the generalized Benjamin equation given by (40). To begin with, let us assume the following solitary wave ansatz:

$$u(x, t) = \lambda\, \mathrm{sech}^p \tau \tag{41}$$

and

$$\tau = B(x - vt). \tag{42}$$

Here λ is the soliton amplitude, v is the soliton velocity and B is the inverse width of the soliton. The unknown p will be determined during the derivation of the solutions of eq. (40).

Therefore, from (41), it is possible to get

$$u_{tt} = p^2 \lambda v^2 B^2 \mathrm{sech}^p \tau - p(p+1)\lambda v^2 B^2 \mathrm{sech}^{p+2} \tau, \tag{43}$$

$$\begin{aligned}(u^n u_x)_x = {} & \lambda^{n+1} p B^2 (pn + p)\, \mathrm{sech}^{pn+p} \tau \\ & -\lambda^{n+1} p B^2 (pn + p + 1)\, \mathrm{sech}^{pn+p+2} \tau,\end{aligned} \tag{44}$$

$$u_{xxxx} = p^4\lambda B^4 \text{sech}^p\tau - 2p(p+1)(p^2+2p+2)\lambda B^4 \text{sech}^{p+2}\tau$$
$$+p(p+1)(p+2)(p+3)\lambda B^4 \text{sech}^{p+4}\tau, \quad (45)$$

where

$$\tau = B(x - vt).$$

Thus, substituting the ansatz (43)–(45) into (40), yields the relation

$$\begin{aligned}
&p^2\lambda v^2 B^2 \text{sech}^p\tau - p(p+1)\lambda v^2 B^2 \text{sech}^{p+2}\tau \\
&\quad + \alpha\lambda^{n+1} pB^2 (pn+p) \,\text{sech}^{pn+p}\tau \\
&\quad -\alpha\lambda^{n+1} pB^2 (pn+p+1) \,\text{sech}^{pn+p+2}\tau + p^4\lambda\beta B^4 \text{sech}^p\tau \\
&\quad - 2p(p+1)(p^2+2p+2)\lambda\beta B^4 \text{sech}^{p+2}\tau \\
&\quad + p(p+1)(p+2)(p+3)\lambda\beta B^4 \text{sech}^{p+4}\tau \\
&\quad = 0.
\end{aligned} \quad (46)$$

Now, from (46), equating the exponents $pn + p + 2$ and $p + 4$ leads to

$$pn + p + 2 = p + 4, \quad (47)$$

so that

$$p = \frac{2}{n}. \quad (48)$$

Setting the cofficients of $\text{sech}^{pn+p+2}\tau$ and $\text{sech}^{p+4}\tau$ terms to zero in eq. (46) we get

$$-\alpha\lambda^{n+1} pB^2 (pn+p+1) + p(p+1)(p+2)(p+3)\lambda\beta B^4 = 0, \quad (49)$$

which after some calculations gives,

$$\lambda = \left[\frac{\beta B^2(p^3+6p^2+11p+6)}{\alpha\,(1+np+p)}\right]^{1/n}. \quad (50)$$

Equation (50) shows that solitons will exist for $\alpha \cdot \beta > 0$, if n is an even integer.
We next set the coefficients of $\text{sech}^p\tau$ terms to zero in eq. (46) to obtain

$$p^2\lambda v^2 B^2 + p^4\lambda\beta B^4 = 0, \quad (51)$$

which after some calculations gives

$$v = pB\sqrt{-\beta}. \quad (52)$$

Thus, from (52) it is possible to conclude that the solitons will exist for $\beta < 0$.

Finally, we get the bright (non-topological) soliton solution for the generalized Benjamin equation when the expressions of p, λ and v given by eqs (48), (50) and (52) are substituted in (41) as

$$u(x,t) = \lambda\ \text{sech}^{2/n}(B(x - vt)). \quad (53)$$

Case I: $p = 2$.

This yields

$$n = 1 \tag{54}$$

so that the generalized Benjamin equation given by eq. (40) modifies to

$$u_{tt} + \alpha\,(uu_x)_x + \beta u_{xxxx} = 0. \tag{55}$$

Further substitution of $p = 2$ into eqs (49) and (51) gives, respectively,

$$\lambda = \frac{12\beta B^2}{\alpha}, \qquad \alpha \neq 0, \tag{56}$$

$$v = 2B\sqrt{-\beta}, \qquad \beta < 0. \tag{57}$$

Thus, in this case, the bright soliton solution is given by

$$u(x,t) = \frac{12\beta B^2}{\alpha}\,\mathrm{sech}^2(B(x - 2B\sqrt{-\beta}t)). \tag{58}$$

Case II: $p = 1$.

This yields

$$n = 2 \tag{59}$$

so that the generalized Benjamin equation given by eq. (40) modifies to

$$u_{tt} + \alpha(u^2u_x)_x + \beta u_{xxxx} = 0. \tag{60}$$

Further substitution of $p = 1$ into eqs (50) and (52) gives, respectively,

$$\lambda = B\sqrt{\frac{6\beta}{\alpha}}, \qquad \alpha \neq 0, \tag{61}$$

From eq. (61) we clearly see that the solitons will exist for $\alpha \cdot \beta > 0$.

$$v = B\sqrt{-\beta}, \qquad \beta < 0. \tag{62}$$

Thus, in this case, the bright soliton solution is given by

$$u(x,t) = B\sqrt{\frac{6\beta}{\alpha}}\mathrm{sech}(B(x - B\sqrt{-\beta}t)). \tag{63}$$

3.2 *The dark (topological) soliton solution*

In order to start off with the solution hypothesis, the following solitary wave ansatz is assumed:

$$u(x,t) = \lambda \tanh^p \tau \tag{64}$$

and

$$\tau = B(x - vt), \tag{65}$$

where λ and B are the free parameters and v is the velocity of the soliton. The exponent p is also unknown. These will be determined

From (64) it is possible to obtain

$$u_{tt} = pv^2\lambda B^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\} \tag{66}$$

$$(u^n u_x)_x = p\lambda^{n+1}B^2\left\{(np+p-1)\tanh^{np+p-2}\tau - 2(np+p)\tanh^{np+p}\tau \right.$$
$$\left. + (np+p+1)\tanh^{np+p+2}\tau\right\}, \tag{67}$$

$$u_{xxxx} = p\lambda B^4\begin{Bmatrix} (p-1)(p-2)(p-3)\tanh^{p-4}\tau \\ -4(p-1)(p^2-2p+2)\tanh^{p-2}\tau \\ +2p(p^2+5)\ \tanh^{p}\tau \\ -4(p+1)(p^2+2p+2)\ \tanh^{p+2}\tau \\ +(p+1)(p+2)(p+3)\ \tanh^{p+4}\tau \end{Bmatrix}, \tag{68}$$

where

$$\tau = B(x - vt).$$

Substituting eqs (66)–(68) into eq. (40), gives

$$\begin{aligned} & pv^2\lambda B^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^{p}\tau + (p+1)\tanh^{p+2}\tau\} \\ & \quad + p\alpha\lambda^{n+1}B^2\left\{(np+p-1)\tanh^{np+p-2}\tau - 2\alpha(np+p)\tanh^{np+p}\tau\right. \\ & \qquad\qquad \left. + \alpha(np+p+1)\tanh^{np+p+2}\tau\right\} \\ & \quad + p\beta\lambda B^4(p-1)(p-2)(p-3)\tanh^{p-4}\tau \\ & \quad - 4p\beta\lambda B^4(p-1)(p^2-2p+2)\tanh^{p-2}\tau \\ & \quad + 2\beta p^2\lambda B^4(p^2+5)\tanh^{p}\tau - 4p\beta\lambda B^4(p+1)(p^2+2p+2)\tanh^{p+2}\tau \\ & \quad + p\lambda\beta B^4(p+1)(p+2)(p+3)\tanh^{p+4}\tau \\ & \quad = 0. \end{aligned} \tag{69}$$

Now, from (69) equating the exponents of $\tanh^{np+p+2}\tau$ and $\tanh^{p+4}\tau$ gives,

$$np + p + 2 = p + 4 \tag{70}$$

which yields

$$p = \frac{2}{n}. \tag{71}$$

Setting the coefficients of $\tanh^{np+p+2}\tau$ and $\tanh^{p+4}\tau$ terms in eq. (69) to zero, we have

$$\alpha p\lambda^{n+1}B^2(np+p+1) + p\lambda\beta B^4(p+1)(p+2)(p+3) = 0 \tag{72}$$

which after some calculations gives,

$$\lambda = \left[-\frac{\beta B^2(p+1)(p+2)(p+3)}{\alpha\,(1+np+p)}\right]^{1/n}. \tag{73}$$

From eq. (73) it is important to note that $\alpha \cdot \beta < 0$, if n is an even integer.

By equating the coefficients of $\tanh^{p-2}\tau$ terms in eq. (69) to zero we obtain

$$pv^2\lambda B^2(p-1) - 4p\beta\lambda B^4(p-1)(p^2-2p+2) = 0. \tag{74}$$

Solving eq. (74) using (73),

$$v = 2B\sqrt{\beta p^2 - 2p\beta + 2\beta} \tag{75}$$

which shows that solitons will exist for $\left\{\beta p^2 - 2p\beta + 2\beta\right\} > 0$.

Thus, we can determine the dark (topological) soliton solution for

$$u(x,t) = \lambda \tanh^{2/n}(B(x-vt)), \tag{76}$$

where the velocity of the solitons v is given by eq. (75) and free parameter λ is given by (73).

Case I: $p = 2$.

This yields

$$n = 1 \tag{77}$$

so that the generalized Benjamin equation given by eq. (40) modifies to

$$u_{tt} + \alpha\,(uu_x)_x + \beta u_{xxxx} = 0. \tag{78}$$

Further substitution of $p = 2$ into eqs (73) and (75) gives, respectively,

$$\lambda = -\frac{12\beta B^2}{\alpha}, \qquad \alpha \neq 0, \tag{79}$$

$$v = 2B\sqrt{2\beta}, \qquad \beta > 0. \tag{80}$$

Thus, in this case, the dark soliton solution is given by

$$u(x,t) = -\frac{12\beta B^2}{\alpha}\tanh^2(B(x - 2B\sqrt{2\beta}t)). \tag{81}$$

Case II: $p = 1$.

This yields

$$n = 2 \tag{82}$$

so that the generalized Benjamin equation given by eq. (40) modifies to

$$u_{tt} + \alpha(u^2u_x)_x + \beta u_{xxxx} = 0. \tag{83}$$

Further substitution of $p = 1$ into eqs (73) and (75) gives, respectively,

$$\lambda = B\sqrt{-\frac{6\beta}{\alpha}}, \qquad \alpha \neq 0, \qquad \alpha \cdot \beta < 0 \tag{84}$$

$$v = \pm B\sqrt{2\beta}, \qquad \beta > 0. \tag{85}$$

Thus, in this case, the dark soliton solution is given by

$$u(x,t) = B\sqrt{-\frac{6\beta}{\alpha}} \tanh(B(x \pm B\sqrt{2\beta}t)). \tag{86}$$

Remark 1. As a result, we find dark and bright soliton solutions of eqs (1) and (40) different from the solutions which are found in [24,25,34].

Remark 2. With the aid of Maple, we have verified all solutions we obtained in §3 and 4, by putting them back into the original eqs (1) and (40).

Remark 3. Comparing other methods with the solitary wave ansatz method shows that the latter gives an abundant variety of solutions compared to the other methods. This can be easily obtained by selecting a variety of arbitrary values for the parameters p and n, provided that $np = 2$.

4. Conclusion

In this paper, we obtained the exact bright and dark soliton solutions of some nonlinear evolution equations. In view of the analysis, we see that the examined equation is an interesting model for soliton-type (dark, bright, kink, shock, etc.) solutions. In addition, we note that the solitary wave ansatz method is an efficient method for constructing exact soliton solutions for such nonlinear evolution equation. To our knowledge, these new solutions have not been reported earlier. They may be of significant importance for explaining some special physical phenomena. We hope that the present solutions may be useful in further numerical analysis and these results are going to be very useful in future research.

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