PRAMANA
— journal of
physics

Vol. 80, No. 2
February 2013
pp. 355–360

Particle–particle Tamm–Dancoff approximation and particle–particle random phase approximation calculations for ^{18}O and ^{18}F nuclei

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MS received 17 April 2012; revised 5 June 2012; accepted 24 July 2012

Abstract. The nuclear structures of ^{18}O and ^{18}F nuclei are studied using particle–particle Tamm–Dancoff approximation (pp TDA) and particle–particle random phase approximation (pp RPA). All possible single-particle states of the allowed angular momenta are considered in the 0p and 1s–0d shells. The Hamiltonian is diagonalized in the presence of Warburton and Brown interactions. The results containing energy-level schemes and transition strength $B(E2)$ are compared with the available experimental data.

Keywords. Nuclear structure; collective excitations and transition strength.

PACS Nos 21.10.Re; 21.10.–k; 21.60.–n

1. Introduction

The basic assumption of the shell model considers that each nucleon moves independently in an average field and in its simple calculations, the spectra of two valence nucleons are formed by the addition of two nucleons beyond an inert core. But, there are many correlations that cannot reproduce within the framework of simple shell model calculations. The simple correlations beyond Hartree–Fock (HF) can only be taken into account by breaking the HF core and raising a nucleon from below to above Fermi level [1]. According to the collective models, the excited states of $A+2$ nuclei can be described as a linear combination of particle–particle pairs. Such an approximation is called particle–particle Tamm–Dancoff approximation (pp TDA) [2]. A system of states more general than the one considered in TDA appears when treating the particle–particle pairs of ground and excited states. Such an approximation is called the particle–particle random phase approximation (pp RPA) (see figure 1).

In this study, the structures of ^{18}O and ^{18}F nuclei are studied in the framework of pp TDA and pp RPA. The basis of single-particle states is considered to include $0p_{3/2}$, $0p_{1/2}$,

DOI: 10.1007/s12043-012-0468-1; ***e*Publication: 17 January 2013**

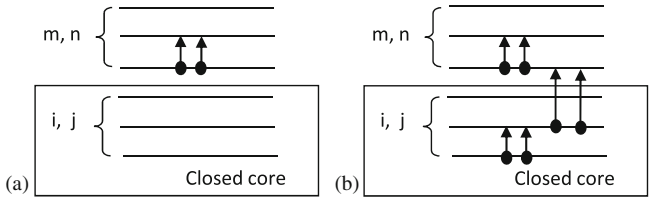

Figure 1. Collective excited model space, (**a**) pp TDA and (**b**) pp RPA.

$0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$. The Hamiltonian is diagonalized in the presence of Warburton and Brown interactions (WBP). The results having energy-level schemes and transition strengths are compared with the available experimental data.

2. Theory

The RPA is a generalization of TDA. This method was originally introduced by Bohm and Pines for studying the plasma oscillations of the electron gas. The collective excited states of the $A+2$ system of multipolarity J and isospin T are generated by operating pp TDA operator on HF vacuum $|0\rangle$ of A nucleons system.

$$Q^{\dagger}_{JT}\,|0\rangle = |A+2, JT\rangle = \sum_{mn} X^{JT}_{mn} a^{\dagger}_{m} a^{\dagger}_{n}\,|A,0\rangle\,.$$

This can be generated either by creating or by destroying a particle–particle pair from the ground state. This is illustrated in figure 2. Thus, the pp RPA operators have the general form:

$$Q^{\dagger}_{JT}\,|0\rangle = |A+2, JT\rangle = \left(\sum_{mn} X^{JT}_{mn} a^{\dagger}_{m} a^{\dagger}_{n} - \sum_{ij} Y^{JT}_{ij} a^{\dagger}_{i} a^{\dagger}_{j}\right)|A,0\rangle\,.$$

The pp RPA eigenvalue equation is given by ref. [2]:

$$\begin{pmatrix} A & B \\ B^{\dagger} & C \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = E\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \tag{1}$$

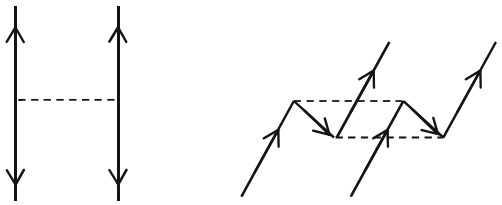

Figure 2. Excitation in pp TDA and pp RPA.

with

$$A^{JT}_{mnm'n'} = -(\epsilon_m + \epsilon_n)\,\delta_{mm'}\delta_{nn'} + V^{JT}_{mnm'n'}$$

$$C^{JT}_{iji'j'} = (\epsilon_i + \epsilon_j)\delta_{ii'}\delta_{jj'} + V^{JT}_{iji'j'}$$

$$B^{JT}_{ijmn} = -V^{JT}_{ijmn}.$$

The indices mn and ij represent the quantum numbers of orbits above and below the Fermi sea, respectively. ϵ is the single-particle energy. V is the two-particle matrix element of the effective interaction. E and X and Y are eigenvalues and eigenvectors to be obtained from the diagonalization of eq. (1). If submatrices C and $B = 0$, then the pp RPA equation reduces to pp TDA equation [2].

The reduced electric transition strength is given by ref. [3]:

$$B(EJ) = \frac{Z^2}{4\pi}\left[\frac{(2J+1)!!}{k^J}\right]^2 \left|F^L_J(k)\right|^2, \tag{2}$$

where $k = E_x/\hbar c$ and F^L_J is the longitudinal inelastic scattering form factor. More details can be obtained in refs [4–6].

3. Results

The structures of ^{18}O, $T = 1$ (isoscalar) and ^{18}F, $T = 0$ (isoscalar) are studied in the framework of pp TDA and pp RPA. Equation (1) is diagonalized in the presence of

Table 1. Eigenvalues and eigenvectors for the low-lying isovector states of ^{18}O ($0p_{3/2} \equiv 2, 0p_{1/2} \equiv 3, 0d_{5/2} \equiv 4, 1s_{1/2} \equiv 5, 0d_{3/2} \equiv 6$).

	E (MeV)			X ($\times 10^{-3}$)						Y ($\times 10^{-3}$)		
	Exp.	Calc.		4 4	5 4	6 4	5 5	6 5	6 6	2 2	2 3	3 3
0^+	−12.18	TDA	−12.17	888			387		244			
	0		0									
		RPA	−12.17	888			387		244	3.06		1.89
			0									
2^+	1.98	TDA	2.17	778	570	92.3		211	125			
		RPA	2.17	778	570	92.3		211	125	−1.6	2.19	
4^+	3.55	TDA	3.78	966		257						
		RPA	3.78	966		257						
0^+	3.63	TDA	4.32	−393			919		−29			
		RPA	4.32	−393			919		−29	−0.39		−0.25
2^+	3.92	TDA	4.43	−610	765	51.3		191	−50.5			
		RPA	4.43	−610	765	51.3		191	−50.5	1.45	2.62	
3^+	5.37	TDA	5.72		990	140						
		RPA	5.72		990	140				0		

Table 2. Eigenvalues and eigenvectors for the low-lying isoscalar states of ^{18}F ($0p_{3/2} \equiv 2$, $0p_{1/2} \equiv 3$, $0d_{5/2} \equiv 4$, $1s_{1/2} \equiv 5$, $0d_{3/2} \equiv 6$).

	E (MeV)			X ($\times 10^{-3}$)						Y ($\times 10^{-3}$)		
	Exp.	Calc.		4 4	5 4	6 4	5 5	6 5	6 6	2 2	2 3	3 3
1^+	−13.23	TDA	−13.365	534		−614	559	−153	−0.44			
	0		0									
		RPA	−13.433	525		−611	570	−157	−0.55	−19.2	−29.9	14.4
			0									
3^+	0.937	TDA	1.153	566	792	−222			−60.9			
		RPA	1.221	566	792	−222			−60.9	0		
5^+	1.121	TDA	1.243	1000								
		RPA	1.311	1000								
1^+	3.724	TDA	4.108	−728		−58	507	−453	65.1			
		RPA	4.118	732		36.2	−510	445	−62.7	25.8	17.8	−17.3
2^+	3.839	TDA	4.087		945	−0.5		328				
		RPA	4.154		945	−0.5		328		0		0
3^+	4.116	TDA	4.288	780	−602	−124			−117			
		RPA	4.356	780	−602	−124			−117	0		

WBP interaction [7] within the single-particle model space $mn \equiv 0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ and single hole states $ij \equiv 0p_{3/2}$, $0p_{1/2}$ using the subroutine NROOT [8], which computes eigenvalues and eigenvectors of a real non-symmetric matrix of the form B-inverse

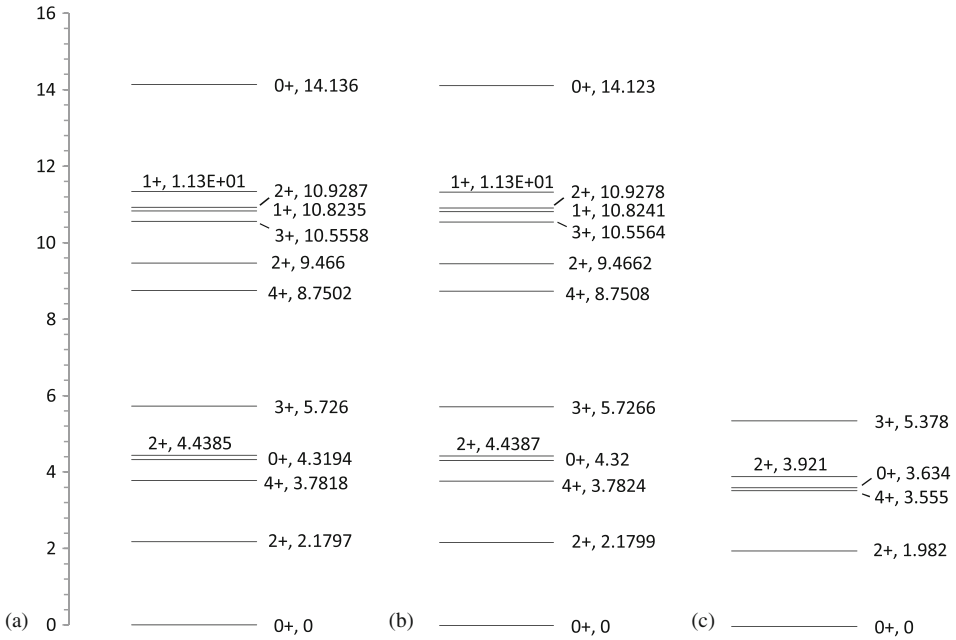

Figure 3. Energy-level schemes of ^{18}O, $T = 1$. (**a**) Results of pp TDA calculations, (**b**) results of pp RPA calculations and (**c**) experimental data.

times A. Eigenvalues and eigenvectors for the low-lying isovector states of ^{18}O and isoscalar states of ^{18}F are given in tables 1 and 2, respectively.

The low-lying positive parity, $T = 1$ spectra of ^{18}O are presented in figure 3. pp TDA results plotted in first column and pp RPA results are plotted in second column and compared with experimental spectrum. The calculated results of pp RPA are obtained to be similar to that of pp TDA. Both calculations agree quite well with the experiment.

For ^{18}F, we get the ground state and low-lying excited states very nicely for both calculations. The results of pp TDA predict that the first 2^+ is lower than second 1^+. This discrepancy is removed in pp RPA calculation as shown in figure 4.

Table 3 gives the calculated values of the reduced electric transition strengths $B(E2)$ for ^{18}O with the oscillator parameter $b = 1.82$ fm and effective charges $e_p = 1.35e$ and $e_n = 0.35e$, which has been found suitable for sd shell [9]. Good agreements are obtained

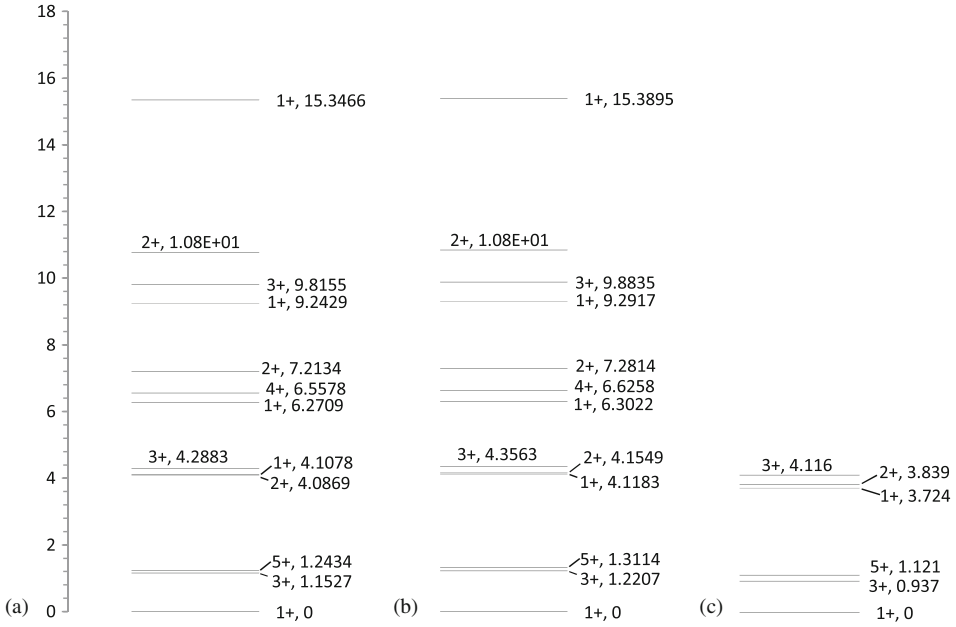

Figure 4. Energy-level schemes of ^{18}F, $T = 0$. (**a**) Results of pp TDA calculations, (**b**) results of pp RPA calculation and (**c**) experimental data.

Table 3. Reduced transition strengths $B(E2)$ of ^{18}O.

E_x (MeV)	Calculated $B(E2)$ e^2 fm^4	Experimental $B(E2)$ e^2 fm^4
1.98	6.23	44.8
3.92	23.72	22.2
5.26	10.96	28.3
8.21	8.6	7.7

for second 2^+ (3.92 MeV) and fourth 2^+ (8.21 MeV), while the results of first 2^+ (1.98 MeV) and third 2^+ (5.26 MeV) under-predict the measured values as shown in table 3. By including higher-allowed orbits up to $10\hbar\omega$ excitations as in ref. [3], results will be in better agreement with experimental data.

4. Conclusion

When the Hamiltonian is diagonalized in the presence of WBP, the calculated results of pp RPA are obtained to be similar to that of pp TDA for ^{18}O. Both calculations agree quite well with the experiment. But for ^{18}F, the results of pp RPA removed the discrepancy of predicting the position of first 2^+ and second 1^+. The calculated electric transition strength $B(E2)$ for ^{18}O is well-obtained for second and fourth 2^+ and under-estimated for first and third 2^+.

Acknowledgements

This work was accomplished during the author's visit to the Physics Department of Florida State University. The author thanks Prof. Alexander Volya for motivating discussions and comments.

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