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Covariant anomalies and Hawking radiation from Kaluza–Klein AdS black holes

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Abstract. In this paper, Hawking radiation is studied from four-dimensional (4D) Kaluza–Klein (KK) AdS black holes via the method of anomaly cancellation. The KK-AdS black hole considered is a non-extremal charged rotating solution in the theory of 4D gauged supergravity. Its Hawking fluxes of electric charge, angular momentum and energy momentum tensor are derived here. Our results support the common view that Hawking radiation is the quantum effect arising at the event horizon.

Keywords. Hawking radiation; quantum anomaly; Kaluza–Klein black hole.

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1. Introduction

Hawking radiation is a universal quantum phenomenon existing in any geometry background with event horizon. Since Hawking discovered the thermal radiation effect of Schwarzschild black hole more than 30 years ago, various methods have been developed to derive Hawking radiation. A recent interesting method is the anomaly cancellation method proposed by Wilczek and his collaborators [1–3]. This method associates Hawking radiation with the gravitational and gauge anomalies at the event horizon. To be more precise, the Hawking fluxes are seen as compensating fluxes to cancel the gauge and gravitational anomalies at the event horizon and preserve the invariance of fundamental physics under gauge and general coordinate transformations. Until now, the anomaly cancellation method has been developed and applied to various cases [4–25].

In the context of the original anomaly cancellation method [1–3], the gauge and gravitational anomalies take consistent forms. However, the boundary condition needed to fix the fluxes is that the covariant current vanishes at the event horizon. In ref. [17], the authors generalized the anomaly cancellation approach in refs [1–3] to derive Hawking fluxes totally in terms of covariant expressions, namely, all the anomalies and boundary

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conditions are covariant. The generalization in ref. [17] makes the anomaly cancellation method more economical and conceptually cleaner. Extensions of ref. [17] may be found in refs [18–21].

In this paper, we calculate the Hawking fluxes of the newly found 4D Kaluza–Klein AdS (KK-AdS) black hole in ref. [27] by cancelling covariant gravitational and gauge anomalies. The 4D KK-AdS black hole is asymptotically AdS_4. It is a non-extremal charged rotating black hole solution in 4D gauged supergravity with single non-zero charge. Our result further supports that Hawking radiation is a quantum effect taking place at the event horizon. In §2, a brief review of the 4D KK-AdS black hole is provided. In §3, the Hawking fluxes of electric charge, angular momentum and energy–momentum tensor via covariant gauge and gravitational anomlies are computed. The conclusion are given in the last section.

2. A brief review of the 4D KK-AdS black hole

In this section, a brief review of the 4D KK-AdS black hole solution in ref. [27] is given.

The 4D KK-AdS black hole in ref. [27] is an exact charged and rotating black hole solution in 4D gauged supergravity. The Lagrangian for this black hole solution contains a gravitational field, a dilaton field Φ and a non-zero $U(1)$ gauge field A_μ. It is read as

$$L = \sqrt{-g}\left[R - \frac{3}{2}(\partial\Phi)^2 - \frac{1}{4}\mathrm{e}^{-3\Phi}F^2 + 3\hat{g}^2(\mathrm{e}^{\Phi} + \mathrm{e}^{-\Phi})\right], \tag{1}$$

where $F = dA$ and $\hat{g}$ is a gauge-coupling constant. Let the three parameters m, a, δ denote the mass, the angular momentum and the charge, respectively. The 4D KK-AdS black hole, satisfying the field equations derived from the Lagrangian (1), takes the form

$$\begin{aligned} \mathrm{d}s^2 = {} & \frac{1}{\sqrt{H}\rho^2\Xi^2}[-\Delta_\theta(\Delta_\theta\Delta_r - V^2(\hat{g})a^2\sin^2\theta)\mathrm{d}t^2 \\ & - 4macr\Delta_\theta\sqrt{\chi}\sin^2\theta\mathrm{d}t\,\mathrm{d}\phi \\ & + (\Delta_\theta a^4 V^2(a^{-1}) - \Delta_r a^2\sin^2\theta)\sin^2\theta\,\mathrm{d}\phi^2] \\ & + \rho^2\sqrt{H}\left(\frac{\mathrm{d}r^2}{\Delta_r} + \frac{\mathrm{d}\theta^2}{\Delta_\theta}\right), \end{aligned} \tag{2}$$

$$A = \frac{2msr}{\Xi H\rho^2}(c\Delta_\theta\,\mathrm{d}t - a\sqrt{\chi}\sin^2\theta\,\mathrm{d}\phi), \tag{3}$$

$$\Phi = -\frac{1}{2}\log H, \tag{4}$$

where the functions Δ_r, Δ_θ, ρ, $V(z)$ and H are defined by

$$\begin{aligned}
\Delta_r &= r^2 - 2mr + a^2 + \hat{g}^2 r^2 (r^2 + 2ms^2 r + a^2),\\
\Delta_\theta &= 1 - a^2 \hat{g}^2 \cos^2\theta, \qquad \rho^2 = r^2 + a^2 \cos^2\theta,\\
V^2(z) &= (1 + z^2 r^2)(1 + z^2 r^2 + 2ms^2 z^2 r),\\
H &= 1 + \frac{2mrs^2}{\rho^2},
\end{aligned} \tag{5}$$

respectively, and the four constants

$$\Xi = 1 - \hat{g}^2 a^2, \quad \chi = 1 + s^2 \hat{g}^2 a^2, \quad c = \cosh\delta, \quad s = \sinh\delta. \tag{6}$$

Setting $m = 0$ and performing suitable coordinate transformation to the coordinates (r, θ), one can find that the 4D KK-AdS black hole (2) is asymptotically AdS_4. In the absence of charge, i.e. $\delta = 0$, the solution (2) reduces to the well-known 4D Kerr-AdS black hole metric. Its Hawking radiation via quantum anomalies was studied in ref. [6]. Very recently, the 4D KK-AdS black hole solution was generalized to the one in arbitrary dimensions [28].

After obtaining a black hole solution, it is very important to compute its thermodynamical quantities, such as the mass, the charge, the angular momentum, the entropy and the temperature. All these quantities are presented in ref. [27]. For our purpose, we only calculate the Hawking temperature. Let r_+ denote the outside event horizon, which is the largest root of the event horizon equation $\Delta_r = 0$. The Hawking temperature of the 4D KK-AdS black hole via surface gravity formula is given as

$$T_{\mathrm{H}} = \frac{1}{4\pi} \frac{\partial_r \Delta_r}{a^2 V(a^{-1})}\bigg|_{r_+} = \frac{\Delta_r'(r_+)\sqrt{1 - s^2 g^2 r_+^2}}{4\pi c (r_+^2 + a^2)}, \tag{7}$$

where the prime $'$ denotes the derivative to the coordinate r. The central aim of this paper is to rederive the Hawking temperature (7) by the method of covariant anomaly cancellation.

3. Hawking radiation via covariant anomalies

In this section, the Hawking radiation of the 4D KK-AdS black hole (2) is investigated by using the anomaly cancellation method proposed in refs [1–3]. But unlike the original anomaly cancellation method, we adopted the modified one in ref. [17], where both the gravitational and gauge anomalies are covariant. The main idea of the anomaly cancellation method is as follows: Performing dimension reduction by considering a massless scalar field, the original higher-dimensional space-time can be reduced to a (1+1)-dimensional one near the horizon. If we omit the classically irrelevant ingoing modes inside the horizon of the reduced (1+1)-dimensional metric, we shall get a 2D chiral effective theory, which exhibits gauge and gravitational anomalies. In other words, the gauge current and energy–momentum tensor are non-conserved. However, the fundamental physical theory is anomaly-free. Thus, to cancel these anomalies and restore the invariance under gauge and diffeomorphism transformations, one gets compensating fluxes that are equal to the Hawking fluxes for charge and energy–momentum tensor.

As a beginning of our analysis, we now perform dimension reduction to the 4D KK-AdS black hole (2) by taking into account the action for the massless scalar field

$$
\begin{aligned}
S[\varphi] &= \frac{1}{2}\int \mathrm{d}^4x\varphi^* D_\mu\left(\sqrt{-g}g^{\mu\nu}D_\nu\varphi\right)\\
&= \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi\frac{\sin\theta}{\Xi}\varphi^*\\
&\quad\times\left\{-\frac{a^4V^2(a^{-1})}{\Delta_r}\left[D_t+\left(\Omega(r)-\frac{\Delta_r\sqrt{\chi}}{ca^3V^2(a^{-1})}\right)D_\phi\right]^2\right.\\
&\quad+\partial_r(\Delta_r\partial_r)+\frac{\hat{g}^2\Delta_r-V^2(a^{-1})+\Xi^2}{\hat{g}^2a^2V^2(a^{-1})}D_\phi^2\\
&\quad\left.+\frac{a^2\sin^2\theta}{\Delta_\theta}D_t^2+\frac{\Xi}{\sin^2\theta}D_\phi^2+\frac{1}{\sin\theta}\partial_\theta(\sin\theta\,\Delta_\theta\partial_\theta)\right\}\varphi,
\end{aligned}
\tag{8}
$$

where $D_\mu = \partial_\mu + ieA_\mu$, and the function $\Omega(r)$ reads as

$$
\Omega(r) = \frac{a(1+\hat{g}^2r^2)\sqrt{\chi}}{c(r^2+a^2)}\,. \tag{9}
$$

It is worth noting that the angular velocity of the horizon is evaluated as $\Omega_\mathrm{H} = \Omega(r_+)$ with the help of the expression of $\Omega(r)$.

In the near-horizon region, performing a partial wave decomposition

$$
\varphi = \sum_{ln}\varphi_{ln}(t,r)\mathrm{e}^{in\phi}\Theta_{ln}(\theta)
$$

and only restoring the dominant terms, eq. (8) is written as

$$
\begin{aligned}
S[\varphi] \simeq{}& \frac{1}{2a^2\Xi V(a^{-1})}\sum_{ln}\int \mathrm{d}t\,\mathrm{d}r\,\varphi_{ln}^*\left\{-\frac{1}{f(r)}[\partial_t+ie(A_t+\Omega(r)A_\phi)\right.\\
&\left.+in\Omega(r)]^2+\partial_r[f(r)\partial_r]\right\}\varphi_{ln},
\end{aligned}
\tag{10}
$$

where

$$
f(r) = \frac{\Delta_r}{a^2V(a^{-1})}\,. \tag{11}
$$

Equation (10) near the event horizon shows that the massless scalar field in the background of the original 4D KK-AdS black hole can be effectively described by an infinite set of massless scalar fields in the (1+1)-dimensional space-time. The (1+1)-dimensional effective metric and $U(1)$ gauge field are

$$
\mathrm{d}s^2 = -f(r)\mathrm{d}t^2+\frac{\mathrm{d}r^2}{f(r)}, \tag{12}
$$

$$
\begin{aligned}
\tilde{A}_t &= e\tilde{A}_t^\mathrm{G}+n\tilde{A}_t^\mathrm{R} = -\frac{es(1+\hat{g}^2r^2)}{c}-n\Omega(r),\\
\tilde{A}_r &= 0,
\end{aligned}
\tag{13}
$$

where the gauge field $\tilde{A}_t^\mathrm{G}$ originates from the gauge field (3) in the 4D space-time and $\tilde{A}_t^\mathrm{R}$ comes from the contribution of the $U(1)$ rotating symmetry.

In the remainder of this section, our task is to derive the currents of gauge fields and the Hawking fluxes of the energy–momentum tensor via covariant gauge and gravitational anomalies. We first compute the currents corresponding to the gauge field $\tilde{A}_t^{\mathrm{G}}$. Near the horizon ($r \in [r_+, r_+ + \varepsilon]$), omitting the classically irrelevant ingoing modes leads to the breakdown of the classical gauge symmetry. Thus the gauge current $J_{(\mathrm{H})}^{(\mathrm{G})\mu}$ obeys the anomaly Ward identity [26]

$$\nabla_\mu \frac{1}{e} J_{(\mathrm{H})}^{(\mathrm{G})\mu} = \frac{-1}{4\pi\sqrt{-g}} \epsilon^{\alpha\beta} \tilde{F}_{\alpha\beta}, \tag{14}$$

where $\tilde{F} = d\tilde{A}$ and the antisymmetry tensor density $\epsilon^{\alpha\beta}$ is defined as $\epsilon^{tr} = -\epsilon_{tr} = 1$. Solving eq. (14), we have

$$J_{(\mathrm{H})}^{(\mathrm{G})r} = c_{\mathrm{H}}^{(\mathrm{G})} + \frac{e}{2\pi} [\tilde{A}_t(r) - \tilde{A}_t(r_+)], \tag{15}$$

where the integration constant $c_{\mathrm{H}}^{(\mathrm{G})}$ is the gauge current at the event horizon. In the region outside the event horizon ($r \in [r_+ + \varepsilon, +\infty)$), the anomaly-free current $J_{(\mathrm{O})}^{(\mathrm{G})\mu}$ takes the conserved form

$$\nabla_\mu J_{(\mathrm{O})}^{(\mathrm{G})\mu} = 0. \tag{16}$$

That is,

$$J_{(\mathrm{O})}^{(\mathrm{G})r} = c_{\mathrm{O}}^{(\mathrm{G})}, \tag{17}$$

where the integration constant $c_{\mathrm{O}}^{(\mathrm{G})}$ is the gauge current at infinity. With the help of two step functions $\Theta(r) = \Theta(r - r_+ - \varepsilon)$ and $H(r) = 1 - \Theta(r)$, we express the total gauge current $J^{(\mathrm{G})\mu}$ as

$$J^{(\mathrm{G})\mu} = J_{(\mathrm{O})}^{(\mathrm{G})\mu} \Theta(r) + J_{(\mathrm{H})}^{(\mathrm{G})\mu} H(r). \tag{18}$$

Then the Ward identity becomes

$$\partial_r J^{(\mathrm{G})r} = \partial_r \Big(\frac{e}{2\pi} \tilde{A}_t H \Big) + \Big(J_{(\mathrm{O})}^{(\mathrm{G})r} - J_{(\mathrm{H})}^{(\mathrm{G})r} + \frac{e}{2\pi} \tilde{A}_t \Big) \delta(r - r_+ - \varepsilon). \tag{19}$$

To restore the gauge symmetry, the second term should vanish at the horizon, which leads to

$$c_{\mathrm{O}}^{(\mathrm{G})} = c_{\mathrm{H}}^{(\mathrm{G})} - \frac{e}{2\pi} \tilde{A}_t(r_+). \tag{20}$$

Imposing the boundary condition $c_{\mathrm{H}}^{(\mathrm{G})} = 0$, which means that the covariant current vanishes at the horizon, we obtain the gauge current

$$c_{\mathrm{O}}^{(\mathrm{G})} = -\frac{e}{2\pi} \tilde{A}_t(r_+) = \frac{e}{2\pi} \Big(\frac{es(1 + \hat{g}^2 r_+^2)}{c} + n\Omega(r_+) \Big). \tag{21}$$

By a parallel analysis, we get the gauge currents $c_{\mathrm{O}}^{(\mathrm{R})}$ and c_{O} corresponding to $\tilde{A}_t^{\mathrm{R}}$ and $\tilde{A}_\mu$, respectively. They read as

$$c_{\mathrm{O}}^{(\mathrm{R})} = -\frac{n}{2\pi} \tilde{A}_t(r_+) = \frac{n}{2\pi} \Big(\frac{es(1 + \hat{g}^2 r_+^2)}{c} + n\Omega(r_+) \Big), \tag{22}$$

$$c_{\mathrm{O}} = -\frac{1}{2\pi} \tilde{A}_t(r_+) = \frac{1}{2\pi} \Big(\frac{es(1 + \hat{g}^2 r_+^2)}{c} + n\Omega(r_+) \Big). \tag{23}$$

Next, we calculate the flux of energy–momentum tensor via covariant gravitational anomaly. In the region near the horizon ($r \in [r_+, r_+ + \varepsilon]$), the invariance under general coordinate transformation will break down if the quantum effect of the ingoing modes is omitted, which leads the two-dimensional (2D) effective field theory to exhibit a gravitational anomaly. For the right-handed fields, the 2D covariant gravitational anomaly obeys the anomalous Ward identity [26]

$$\nabla_\mu T^\mu_{\ \nu} = \frac{1}{96\pi\sqrt{-g}}\epsilon_{\nu\mu}\partial^\mu R = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_{\ \nu}. \tag{24}$$

Adding the contribution from the gauge field, the anomalous Ward identity near the horizon becomes

$$\nabla_\mu T^\mu_{(\mathrm{H})\nu} = \tilde{F}_{\mu\nu}\tilde{J}^\mu_{(\mathrm{H})} + \frac{1}{96\pi\sqrt{-g}}\epsilon_{\nu\mu}\partial^\mu R, \tag{25}$$

where

$$\tilde{J}^\mu_{(\mathrm{H})} = \frac{1}{e}J^{(G)\mu}_{(\mathrm{H})}.$$

Solving $\nu = t$ component of eq. (25), we get

$$T^r_{(\mathrm{H})t} = a_\mathrm{H} + \left[c_\mathrm{O}\tilde{A}_t(r) + \frac{1}{4\pi}\tilde{A}_t^2(r) + N^r_{\ t}\right]\Bigg|^r_{r_+}, \tag{26}$$

where the integration constant a_H is the flux at horizon and

$$N^r_{\ t} = \frac{1}{192\pi}\left(2ff'' - f'^2\right). \tag{27}$$

In the region far away from the horizon, the energy–momentum tensor satisfies the Lorentz force law

$$\nabla_\mu T^\mu_{(\mathrm{O})\nu} = \tilde{F}_{\mu\nu}\tilde{J}^\mu_{(\mathrm{O})}. \tag{28}$$

The $\nu = t$ component of the above equation is solved as

$$T^r_{(\mathrm{O})t} = a_\mathrm{O} + c_\mathrm{O}\tilde{A}_t(r), \tag{29}$$

where the constant a_O is the flux of the energy–momentum tensor at infinity.

As before, we use the two step functions $\Theta(r)$ and $H(r)$ to express the total energy–momentum tensor $T^\mu_{\ \nu}$ as the combination of $T^\mu_{(\mathrm{O})\nu}$ and $T^\mu_{(\mathrm{H})\nu}$, that is,

$$T^\mu_{\ \nu} = T^\mu_{(\mathrm{O})\nu}\Theta(r) + T^\mu_{(\mathrm{H})\nu}H(r). \tag{30}$$

Then, we have

$$\begin{aligned}\nabla_\mu T^\mu_{\ t} =\ & c_\mathrm{O}\partial_r\tilde{A}_t + \partial_r\left[\left(\frac{1}{4\pi}\tilde{A}_t^2 + N^r_{\ t}\right)H\right]\\ & + \left(T^r_{(\mathrm{O})t} - T^r_{(\mathrm{H})t} + \frac{1}{4\pi}\tilde{A}_t^2 + N^r_{\ t}\right)\delta(r - r_+ - \varepsilon)\,.\end{aligned} \tag{31}$$

In eq. (31), we have used eqs (26) and (29). To preserve the general invariance under coordinate transformation, the third term in eq. (31) must vanish at the event horizon, which leads to

$$a_\mathrm{O} = a_\mathrm{H} + \frac{1}{4\pi}\tilde{A}_t^2(r_+) + \frac{\kappa^2}{48\pi}, \tag{32}$$

where the surface gravity at horizon κ is given by $\kappa^2 = -48\pi N'^r_t(r_+) = f'^2/4$. We once again impose the covariant boundary condition that the flux for the energy–momentum tensor vanishes at the horizon, namely, $a_{\mathrm{H}} = 0$. Then, the flux at infinity is evaluated as

$$a_{\mathrm{O}} = \frac{1}{4\pi}\tilde{A}_t^2(r_+) + \frac{\kappa^2}{48\pi}\,. \tag{33}$$

By cancelling the covariant gauge and gravitational anomalies near the horizon, we have obtained the gauge currents $c_{\mathrm{O}}^{(\mathrm{G})}$ and $c_{\mathrm{O}}^{(\mathrm{R})}$ and the flux of the energy–momentum tensor a_{O}. In fact, they are equal to the Hawking fluxes of electric charge, angular momentum and energy–momentum tensor, respectively [3]. This means that Hawking temperature of the 4D KK-AdS black hole can be derived via covariant anomalies.

4. Summary

In this paper, we have extended the anomaly cancellation method to compute the Hawking fluxes of 4D KK-AdS black holes. Considering a scalar quantum field to perform dimension reduction to the 4D KK-AdS black hole (2), we found that the quantum field in the original 4D KK-AdS black hole space-time can be effectively described by an infinity collection of fields in the (1+1)-dimensional space-time with the metric (12) and the $U(1)$ gauge field (13) near the event horizon. In terms of both the 2D metric and the gauge field, we derived the gauge currents $c_{\mathrm{O}}^{(\mathrm{G})}$ and $c_{\mathrm{O}}^{(\mathrm{R})}$ and the flux of the energy–momentum tensor a_{O}, by cancelling the covariant gauge and gravitational anomalies, which correspond to the Hawking thermal fluxes of electric charge, angular momentum and energy–momentum tensor, respectively. During our calculations, the quantum anomalies and the boundary conditions localize at the event horizon. It implies that our results support the universal view that Hawking radiation is a quantum phenomenon arising at the event horizon. By using the effective action approach in ref. [22] to compute the Hawking fluxes of the 4D KK-AdS black hole on the basis of the 2D effective metric (12) and gauge field (13), one can obtain the same results.

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