PRAMANA
— journal of
physics

Vol. 78, No. 5
May 2012
pp. 767–778

Gravitational Jaynes–Cummings model beyond the rotating wave approximation

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MS received 17 September 2011; revised 14 November 2011; accepted 8 December 2011

Abstract. In this paper, the quantum properties of a two-level atom and the cavity-field in the Jaynes–Cummings model with the gravity beyond the rotating wave approximation are investigated. For this purpose, by solving the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the counter-rotating terms on the dynamical behaviour of atomic population inversion and the probability distribution of the cavity-field as quantum properties is explored. The results in the atom–field system beyond the rotating wave approximation with the gravity show that the quantum properties are not completely suppressed under certain conditions.

Keywords. Jaynes–Cummings model; atomic motion; gravity; quantum properties; beyond the rotating wave approximation.

PACS Nos 42.50.Vk; 42.50.Dv

1. Introduction

A fundamental task in physics is the description of the matter–light interaction using the Jaynes–Cummings model (JCM) [1]. The JCM is analytically solvable by rotating wave approximation (RWA) when the counter-rotating terms (CRTs) are neglected [2]. This approximation is valid when coupling regime is weak and the detuning is small [3,4]. Yet, with the advent of circuit QED it has become feasible experimentally to explore regimes of the model where the dynamics is not well described within the RWA [5–7]. There have already been many investigations exploring analytically and numerically the local dynamics of the model beyond the RWA [8,9]. Some of the developed techniques deal with nearly resonant but strong couplings [10–12] and some deal with highly detuned and strong coupling scenarios [13–15].

On the other hand, with development of technologies of laser cooling and atom trapping, the interaction between a moving atom and the field has attracted much attention [16,17]. Experimentally, atomic beams with very low velocities are generated in laser

DOI: 10.1007/s12043-012-0267-8; ***e*Publication: 13 April 2012**

cooling and atomic interferometry [18]. It is obvious that for atoms moving with a velocity of a few millimeters or centimeters per second for a time period of several milliseconds or more, the influence of Earth's acceleration becomes important and cannot be neglected [19].

A semiclassical description of a two-level atom interacting with a classical running laser wave in a gravitational field is given in [20,21]. However, the semiclassical treatment does not permit us to study the pure quantum effects occurring in the course of atom–radiation interaction. Recently, within a quantum treatment of the internal and external dynamics of the atom, we have presented our research to investigate the influence of classical homogeneous gravitational field on the atom–field properties in the JCM with the RWA [22,23].

In this paper, the quantum properties of two-level atom and the cavity-field are investigated, when the two-level atom interacts with the single-mode travelling wave field in optical ring cavity (cavity-field) with the gravity and beyond the RWA. By solving the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the CRTs on the dynamical behaviour of atomic population inversion and the probability distribution of the cavity-field is explored. In §2, we present a quantum treatment of the internal and external dynamics of the atom. In the interaction picture, we obtain an effective Hamiltonian describing the interaction of two-level atom with the single-mode cavity-field with the gravity and with respect to CRTs. In §3, we investigate the dynamical evolution of the system and show how the CRTs may affect the dynamical properties of the JCM with the gravity. In §4, we study the influence of RCTs on atomic population inversion and the probability distribution of the cavity-field as quantum properties under certain conditions. Finally, we summarize our conclusion in §5.

2. The effective Hamiltonian for the JCM with the gravity beyond the RWA

The total Hamiltonian for the atom–field system with the gravity and in the absence of RWA with the atomic motion along the position vector $\hat{\vec{x}}$ is given by [24]

$$\hat{H} = \hat{H}_{\text{free}} + \hat{H}_{\text{RWA}} + \hat{H}_{\text{CRT}}, \tag{1}$$

where

$$\hat{H}_{\text{free}} = \frac{\hat{p}^2}{2M} - M\vec{g}\cdot\hat{\vec{x}} + \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{1}{2}\hbar\omega_{\text{eg}}\hat{\sigma}_z, \tag{2}$$

$$\hat{H}_{\text{RWA}} = \hbar\lambda[\exp(-i\vec{q}\cdot\hat{\vec{x}})\hat{a}^\dagger\hat{\sigma}_- + \exp(i\vec{q}\cdot\hat{\vec{x}})\hat{\sigma}_+\hat{a}], \tag{3}$$

$$\hat{H}_{\text{CRT}} = \hbar\lambda[\exp(i\vec{q}\cdot\hat{\vec{x}})\hat{a}^\dagger\hat{\sigma}_+ + \exp(-i\vec{q}\cdot\hat{\vec{x}})\hat{\sigma}_-\hat{a}], \tag{4}$$

where $\hat{a}$ and $\hat{a}^\dagger$ denote, respectively, the annihilation and creation operators of a single-mode travelling wave with frequency ω, $\vec{q}$ is the wave vector of the running wave and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels

$|e\rangle$, $|g\rangle$ and Bohr transition frequency ω_{eg}. The atom–field coupling is given by the parameter λ and $\hat{\vec{p}}$, $\hat{\vec{x}}$ denote, respectively, the momentum and position operators of the atomic centre of mass motion and g is the Earth's gravitational acceleration. The Schrödinger equation is given by

$$i\hbar\frac{\partial|\varphi(t)\rangle}{\partial t} = \hat{H}|\varphi(t)\rangle. \tag{5}$$

It is convenient to consider the evolution of the state vector $|\varphi(t)\rangle$ of the total system in an interaction picture induced by the unitary operator

$$\hat{T}(t) = \exp\left(\frac{-it\hat{H}_0}{\hbar}\right)\exp\left(\frac{-i\int\hat{H}_p(t')\mathrm{d}t'}{\hbar}\right), \tag{6}$$

where

$$\hat{H}_0 = \frac{\hat{p}^2}{2M} + M\vec{g}\cdot\vec{x} - \hbar\left[\frac{q^2\hbar}{8M} - (\hat{a}^\dagger\hat{a} + |e\rangle\langle e|)\omega\right] - \hbar\hat{\sigma}_z\hat{\delta}, \tag{7}$$

$$\hat{\delta} = \left[2\omega - \frac{3}{2}\omega_{\mathrm{eg}} - \vec{q}\cdot\left(\hat{\vec{x}} + \frac{\hat{\vec{p}}}{M}\right)\right], \tag{8}$$

with

$$\hat{H}_p(t) = \left(\hat{\vec{p}} + \vec{q}\right)\cdot\vec{g}t + \frac{1}{2}Mg^2t^2. \tag{9}$$

In this picture, the evolution of the transformed state vector $|\varphi_{1e}(t)\rangle = \hat{T}(t)|\varphi(t)\rangle$ is governed by the Hamiltonian

$$\begin{aligned}\hat{H}_{1e} =&\ \hat{T}^\dagger\hat{H}\hat{T} - i\hbar\hat{T}^\dagger\dot{\hat{T}}\\ =&\ \hbar\lambda(\exp[-it\hat{\Delta}_-(\hat{\vec{p}},\vec{g},t)]\hat{a}^\dagger\hat{\sigma}_- + \exp[it\hat{\Delta}_-(\hat{\vec{p}},\vec{g},t)]\sigma_+\hat{a})\\ &+ \hbar\lambda(\exp[it\hat{\Delta}_+(\hat{\vec{p}},\vec{g},t)]\hat{a}^\dagger\hat{\sigma}_+ + \exp[-it\hat{\Delta}_+(\hat{\vec{p}},\vec{g},t)]\sigma_-\hat{a}),\end{aligned} \tag{10}$$

where

$$\hat{\Delta}_\pm(\hat{\vec{p}},\vec{g},t) = \omega \pm \omega_{\mathrm{eg}} - \left(\frac{\vec{q}\cdot\hat{\vec{p}}}{M} + \vec{q}\cdot\vec{g}t + 3\frac{\hbar q^2}{2M}\right), \tag{11}$$

has been introduced as the Doppler shift detuning at time t. The Schrödinger equation governing the JCM beyond the RWA is

$$i\hbar\frac{\partial|\varphi_{1e}(t)\rangle}{\partial t} = \hat{H}_{1e}|\varphi_{1e}(t)\rangle. \tag{12}$$

It is not easy to solve eq. (12) because of the presence of the CRTs in (10). Thus, the unitary squeezing operator is defined as

$$\hat{S}(\eta) = \exp\left(\frac{\eta^*}{2}\hat{a}^2 - \frac{\eta}{2}\hat{a}^{\dagger 2}\right). \tag{13}$$

The Hamiltonian corresponding to the state vector $|\phi_{2e}(t)\rangle = \hat{S}^\dagger(\eta)|\varphi_{1e}(t)\rangle$ is obtained as

$$\begin{aligned}\hat{H}_{2e} = & \ \hat{S}^\dagger \hat{H}_{1e}\hat{S} - i\hbar\hat{S}^\dagger\dot{\hat{S}} \\ = & \ \hbar\lambda[A(t)\hat{a}^2 + A^*(t)\hat{a}^{\dagger 2} + \hat{\zeta}(t)\hat{a}^\dagger\hat{\sigma}_- \\ & + \hat{\zeta}^*(t)\hat{\sigma}_+\hat{a} + \hat{\xi}^*(t)\hat{a}^\dagger\hat{\sigma}_+ + \hat{\xi}(t)\hat{\sigma}_-\hat{a} + C(t)],\end{aligned} \tag{14}$$

where

$$A(t) = \frac{\eta^*\sinh\sqrt{4\eta^*\eta}}{\sqrt{4\eta^*\eta}}\left[\frac{i(\dot{\eta}\eta^* - \dot{\eta}^*\eta)}{4\eta^*\eta} - \omega\right] - \frac{i(\dot{\eta}\eta^* - \dot{\eta}^*\eta)}{4\eta^*\eta} - \frac{i\dot{\eta}^*}{2}, \tag{15}$$

$$\hat{\zeta}(t) = \hat{\gamma}_-(t)\cosh\sqrt{\eta^*\eta} - \hat{\gamma}_+(t)\frac{\sinh\sqrt{\eta^*\eta}}{\sqrt{\eta^*\eta}}, \tag{16}$$

$$\hat{\xi}(t) = \hat{\gamma}_+(t)\cosh\sqrt{\eta^*\eta} - \hat{\gamma}_-(t)\eta^*\frac{\sinh\sqrt{\eta^*\eta}}{\sqrt{\eta^*\eta}}, \tag{17}$$

$$C(t) = \left[\frac{\omega}{2} - \frac{i(\dot{\eta}\eta^* - \dot{\eta}^*\eta)}{8\eta^*\eta}\right](\cosh\sqrt{4\eta^*\eta} - 1), \tag{18}$$

with

$$\hat{\gamma}_\pm(t) = \exp[-it\hat{\Delta}_\pm(\hat{\vec{p}}, \vec{g}, t)]. \tag{19}$$

By considering the two constraints $A = 0$ and $\xi = 0$, the time-dependent function $\eta(t)$ is obtained as

$$\eta(t) = \frac{1}{2}\exp(i\chi(t))\ln\left[\frac{2\omega_{\mathrm{eg}}\exp(i\chi(t)/2) + i\omega f_1(t)\exp(-i\chi(t)/2)}{2\omega_{\mathrm{eg}}\exp(i\chi(t)/2) - i\omega f_1(t)\exp(-i\chi(t)/2)}\right], \tag{20}$$

with

$$\chi(t) = -i\ln[\omega f(t)], \tag{21}$$

where

$$f(t) = \omega f_1(t) - \frac{i}{2\omega}\ln f_2(t), \tag{22}$$

$$f_2(t) = \frac{i\omega f_1(t)}{2\omega_{\mathrm{eg}}}[2 + if_1^*(t) + 2\sqrt{1 + if_1^*(t)}], \tag{23}$$

$$f_1(t) = \frac{-2i\omega_{\mathrm{eg}}}{\omega}\exp(-2i\omega_{\mathrm{eg}}t). \tag{24}$$

By using (20), the Hamiltonian (14) is rewritten as

$$\hat{H}_{2e} = \hbar\lambda[\hat{\zeta}(t)\hat{a}^\dagger\hat{\sigma}_- + \hat{\zeta}^*(t)\hat{\sigma}_+\hat{a} + C(t)]. \tag{25}$$

The Schrödinger equation with regard to (25) is

$$i\hbar\frac{\partial|\varphi_{2e}(t)\rangle}{\partial t} = \hat{H}_{2e}|\varphi_{2e}(t)\rangle. \tag{26}$$

Finally, by using $T_e = \exp(-i\int_0^t C(t')\mathrm{d}t')$, the effective Hamiltonian is given by

$$\begin{aligned}\hat{H}_{\text{eff}} = {} & T_e^\dagger \hat{H}_{2e} T_e - i\hbar T_e^\dagger \dot{T}_e = \hat{H}_{2e} - C(t)\\ = {} & \hbar\lambda[\hat{\zeta}(t)\hat{a}^\dagger\hat{\sigma}_- + \hat{\zeta}^*(t)\hat{\sigma}_+\hat{a}],\end{aligned} \tag{27}$$

where we have defined $\hat{\zeta}(t)$ and $C(t)$ in (16) and (18), respectively.

3. Dynamical evolution beyond the RWA

In §2, we obtained an effective Hamiltonian for the atom–field system with the gravity and in the absence of RWA. In this section, the dynamical evolution of the system is investigated. We shall show how the CRTs may affect the quantum dynamics of the JCM with the gravity. For this purpose, we solve the Schrödinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \hat{H}_{\text{eff}}|\psi(t)\rangle, \tag{28}$$

for the state vector $|\psi(t)\rangle = T_e(t)|\varphi_{2e}(t)\rangle$ with the effective Hamiltonian (27). Since the Hamiltonian couples only the states $|g, n+1\rangle \otimes |\vec{p}\rangle$ and $|e, n\rangle \otimes |\vec{p}\rangle$, the state vector is introduced by the following form:

$$\begin{aligned}|\psi(t)\rangle = {} & \int \mathrm{d}^3 p \sum_{n=0} (\psi_{e,n}(\vec{p}, \vec{g}, t)|e, n\rangle \otimes |\vec{p}\rangle\\ & +\psi_{g,n+1}(\vec{p}, \vec{g}, t)|g, n+1\rangle \otimes |\vec{p}\rangle).\end{aligned} \tag{29}$$

The equations of motion for the time-dependent probability amplitudes $\psi_{e,n}(\vec{p}, \vec{g}, t) \equiv \psi_1$, $\psi_{g,n+1}(\vec{p}, \vec{g}, t) \equiv \psi_2$ by substituting (27) and (29) into (28) are found as

$$\dot{\psi}_1 = -i\lambda\sqrt{(n+1)}\zeta^*(t)\psi_2, \tag{30}$$

and

$$\dot{\psi}_2 = -i\lambda\sqrt{(n+1)}\zeta(t)\psi_1. \tag{31}$$

At time $t = 0$ the atom is uncorrelated with the single-mode cavity-field and the state vector of the system can be written as a direct product

$$\begin{aligned}|\psi(t=0)\rangle = {} & |\psi_{\text{c.m.}}(0)\rangle \otimes |\psi_{\text{atom}}(0)\rangle \otimes |\psi_{\text{field}}(0)\rangle\\ = {} & \left(\int \mathrm{d}^3 p\phi(\vec{p})|\vec{p}\rangle\right) \otimes (c_e|e\rangle + c_g|g\rangle)\\ & \otimes \left(\sum_{n=0} w_n|n\rangle\right),\end{aligned} \tag{32}$$

where we have assumed that initially the field is in a single-mode coherent superposition of Fock states, the atom is in a coherent superposition of its excited and ground states, and the state vector for the centre-of-mass degree of freedom is $|\psi_{\text{c.m.}}(0)\rangle = \int \mathrm{d}^3 p \phi(\vec{p})|\vec{p}\rangle$. The initial state (32) reads as

$$|\psi(t=0)\rangle = \int \mathrm{d}^3 p \sum_{n=0} (w_n c_e \phi(\vec{p})|e, n\rangle \otimes |\vec{p}\rangle + w_{n+1} c_g \phi(\vec{p})|g, n+1\rangle \otimes |\vec{p}\rangle). \tag{33}$$

When we compare eq. (33) with eq. (29) we find the following initial conditions:

$$\psi_1(t=0) = w_n c_e \phi(\vec{p}), \quad \psi_2(t=0) = w_{n+1} c_g \phi(\vec{p}). \tag{34}$$

We can solve two coupled first-order differential eqs (30) and (31) in a straightforward way. We have

$$\frac{\partial^2 \psi_1}{\partial t^2} - a^*(t)\frac{\partial \psi_1}{\partial t} + b_n(t)\psi_1 = 0, \tag{35}$$

and

$$\frac{\partial^2 \psi_2}{\partial t^2} - a(t)\frac{\partial \psi_2}{\partial t} + b_n(t)\psi_2 = 0, \tag{36}$$

where

$$a(t) = \frac{\dot{\zeta}(t)}{\zeta(t)}, \quad b_n(t) = \lambda^2(n+1)\zeta^*(t)\zeta(t), \tag{37}$$

are time-dependent. The exact solutions of eqs (35) and (36) read as, respectively,

$$\psi_1(t) = \exp\left(\frac{t b_n(t)}{a(t)}\right)[C_n(1)D_{1n}(t) + C_n(2)E_{1n}(t)], \tag{38}$$

and

$$\psi_2(t) = \exp\left(\frac{t b_n(t)}{a(t)}\right)[C_n(1)D_{2n}(t) + C_n(2)E_{2n}(t)], \tag{39}$$

where

$$D_{1n}(t) = H\left[\frac{b_n^2(t)}{a^{*3}(t)}, \frac{-\sqrt{2}b_n(t)}{a^{*3/2}(t)} + \frac{t\sqrt{a^*(t)}}{\sqrt{2}}\right], \tag{40}$$

$$E_{1n}(t) = {}_1F_1\left[\frac{-b_n^2(t)}{2a^{*3}(t)}, 1/2; \left(\frac{-\sqrt{2}b_n(t)}{a^{*3/2}(t)} + \frac{t\sqrt{a^*(t)}}{\sqrt{2}}\right)^2\right], \tag{41}$$

$$D_{2n}(t) = H\left[\frac{b_n^2(t)}{a^3(t)}, \frac{-\sqrt{2}b_n(t)}{a^{3/2}(t)} + \frac{t\sqrt{a(t)}}{\sqrt{2}}\right], \tag{42}$$

$$E_{2n}(t) = {}_1F_1\left[\frac{-b_n^2(t)}{2a^3(t)}, 1/2; \left(\frac{-\sqrt{2}b_n(t)}{a^{3/2}(t)} + \frac{t\sqrt{a(t)}}{\sqrt{2}}\right)^2\right], \tag{43}$$

with

$$C_n(1) = \frac{E_{2n}(0)\psi_1(0) - E_{1n}(0)\psi_2(0)}{E_{2n}(0)D1n(0) - E_{1n}(0)D_{2n}(0)}, \tag{44}$$

$$C_n(2) = \frac{D_{2n}(0)\psi_1(0) - D_{1n}(0)\psi_2(0)}{E_{1n}(0)D2n(0) - E_{2n}(0)D_{1n}(0)}, \tag{45}$$

and H, ${}_1F_1$ denote, respectively the Hermite and the confluent hypergeometric functions.

4. Dynamical properties of the model beyond the RWA

In this section, influence of the CRTs on the quantum properties of the two-level atom and the quantized radiation field with the gravity is studied.

4.1 *Atomic inversion*

The atomic population inversion is expressed as

$$W(t) = \langle\psi(t)|\sigma_z|\psi(t)\rangle. \tag{46}$$

By using the atom–field state $|\psi(t)\rangle$ given by eq. (29), we obtain

$$W(t) = \int \mathrm{d}^3 p \sum_{n=0} [|\psi_1|^2 - |\psi_2|^2]. \tag{47}$$

Therefore, by substituting (38) and (39) into (47) we have

$$\begin{aligned} W(t) = {} & \int \mathrm{d}^3 p \sum_{n=0}^{\infty} \exp\left(t\frac{a^*(t)b_n(t) + a(t)b_n^*(t)}{|a(t)|^2}\right) \\ & \times\{|C_n(1)|^2(|D_{1n}(t)|^2 - |D_{2n}(t)|^2) \\ & + |C_n(2)|^2(|E_{1n}(t)|^2 - |E_{2n}(t)|^2) \\ & + C_n(1)C_n^*(2)(D_{1n}(t)E_{1n}^*(t) - D_{2n}(t)E_{2n}^*(t)) \\ & + C_n(2)C_n^*(1)(E_{1n}(t)D_{1n}^*(t) - E_{2n}(t)D_{2n}^*(t))\}, \end{aligned} \tag{48}$$

where according to eqs (16) and (19), $\zeta(t)$ and $\gamma_\pm(t)$ in $a(t)$ and $b_n(t)$ are functions of $\vec{p}$. We have shown in [23] that because of strong gravity $\vec{q}\cdot\vec{g} = 1.5\times 10^7\,\mathrm{s}^{-2}$ in atom–field system with the RWA, the quantum properties such as atomic population inversion $W(t)$ and the probability distribution of cavity field $P(n,t)$ are suppressed. Furthermore, investigations of quantum properties such as $W(t)$ and $P(n,t)$ for various physical systems are extremely important due to experimental realizations [25,26]. Therefore, we are interested in establishing conditions in which not only both the gravity and the CRTs are considered but also the quantum properties of the system are not suppressed. For this purpose, we consider an atom–field system beyond the RWA with the weak gravity $\vec{q} \cdot \vec{g} = 0.01 \times 10^7\,\mathrm{s}^{-2}$, because based on [23], the quantum properties of the system are not completely suppressed when considering the weak gravity with proper value for atom–field detuning $\Delta = \omega - \omega_{\mathrm{eg}}$. Then, by considering different values for coupling parameter λ, the other condition for our purpose is found. Influence of CRTs on the evolution of atomic population inversion for three values of coupling parameter λ with the weak gravity $\vec{q} \cdot \vec{g} = 0.01 \times 10^7\,\mathrm{s}^{-2}$ and the atom–field detuning $\Delta = 0.18 \times 10^7$ rad s^{-1} [23] is shown in figure 1. We assume that at $t = 0$, the two-level atom is in a coherent superposition of the excited state and the ground state with $c_g(0) = 1/\sqrt{2}$, $c_e(0) = 1/\sqrt{2}$ and the cavity-field is prepared in a Glauber coherent state $w_n(0) = \exp(-\frac{|\alpha|^2}{2})\alpha^n/\sqrt{n!}$. In figures 1 and 2 we set $q = 10^7$ m^{-1}, $M = 10^{-26}$ kg, $g = 9.8$ m s^{-2}, $\omega_{\mathrm{rec}} = \hbar q^2/2M = 0.5 \times 10^6$ rad s^{-1} [20–24], $\omega_{\mathrm{eg}} = 8.82 \times 10^7$ rad s^{-1}, $\omega = 9 \times 10^7$ rad s^{-1}, $\alpha = 2$ and $\phi(\vec{p}) = \frac{1}{\sqrt{2\pi\sigma_0}} \exp(-p^2/\sigma_0^2)$ with $\sigma_0 = 1$ [23,27]. Here, it is necessary to point out that the relevant time-scale introduced by the gravitational influence is $\tau_a = 1/\sqrt{\vec{q} \cdot \vec{g}}$ [22]. Therefore, for an optical field with $|\vec{q}| = 10^7$ m^{-1}, τ_a is about 10^{-4} s. In figure 1a, one can see the Rabi-like oscillations in the atomic population inversion $W(t)$ in weak coupling regime with $\lambda = 0.01\omega$ [28,29] which is agreeable with [30]. In the atom–field system beyond the RWA, not only the weak gravity is considered but also the atom–field detuning $\Delta = 0.18 \times 10^7$ rad s^{-1} [23] is selected. By comparing figures 1b and 1c for strong coupling $\lambda = 1.11\omega$ [31] and ultra-strong coupling $\lambda = 11.1\omega$ [32] respectively, the Rabi-like oscillations in the atomic population inversion $W(t)$ is completely suppressed which is agreeable with [33,34]. Moreover, with increasing value of coupling parameter λ, the amplitude of $W(t)$ increases.

4.2 *The probability distribution of the cavity-field*

The probability distribution function $P(n,t)$ that there are n photons in the cavity-field at time t is given by

$$P(n,t) = |\langle n|\psi(t)\rangle|^2. \tag{49}$$

By using the expressions (27), (36) and (37) we have

$$P(n,t) = \int \mathrm{d}^3 p[|\psi_1(t)|^2 + |\psi_2(t)|^2]. \tag{50}$$

Therefore, we obtain

$$\begin{aligned} P(n,t) = &\int \mathrm{d}^3 p \exp\left(t\frac{a^*(t)b_n(t) + a(t)b_n^*(t)}{|a(t)|^2}\right) \\ &\times \{|C_n(1)|^2(|D_{1n}(t)|^2 + |D_{2n}(t)|^2) \\ &+ |C_n(2)|^2(|E_{1n}(t)|^2 + |E_{2n}(t)|^2) \\ &+ C_n(1)C_n^*(2)(D_{1n}(t)E_{1n}^*(t) + D_{2n}(t)E_{2n}^*(t)) \\ &+ C_n(2)C_n^*(1)(E_{1n}(t)D_{1n}^*(t) + E_{2n}(t)D_{2n}^*(t))\}. \end{aligned} \tag{51}$$

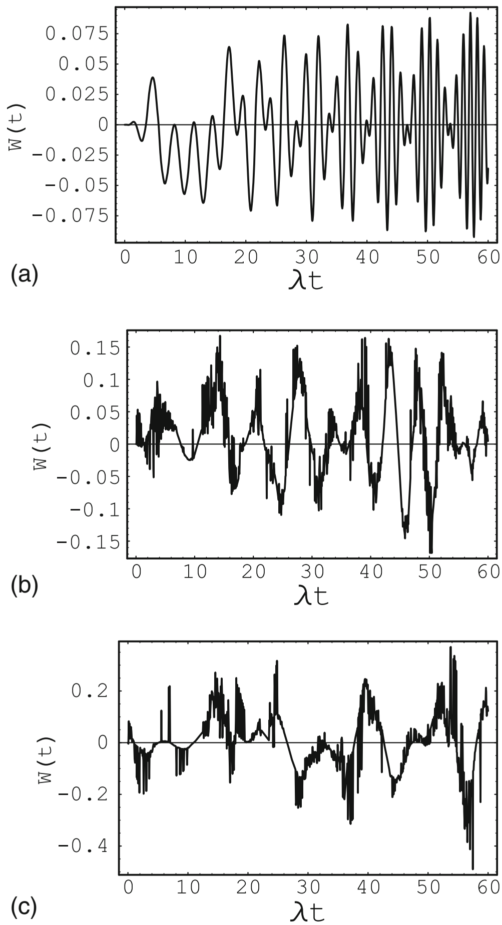

Figure 1. Time evolution of the atomic population inversion vs. the scaled time λt. Here we have set $q = 10^7\ \mathrm{m}^{-1}$, $M = 10^{-26}$ kg, $g = 9.8\ \mathrm{m\ s}^{-2}$, $\omega_{\mathrm{rec}} = 0.5 \times 10^6\ \mathrm{rad\ s}^{-1}$, $\omega_{\mathrm{eg}} = 8.82 \times 10^7\ \mathrm{rad\ s}^{-1}$, $\omega = 9 \times 10^7\ \mathrm{rad\ s}^{-1}$, $\alpha = 2$, $\vec{q} \cdot \vec{g} = 0.01 \times 10^7\ \mathrm{s}^{-2}$, $\Delta = 0.18 \times 10^7\ \mathrm{rad\ s}^{-1}$ and $c_e = c_g = 1/\sqrt{2}$ with coherent state for initial cavity-field: (**a**) For $\lambda = 0.01\omega$, (**b**) for $\lambda = 1.11\omega$ and (**c**) for $\lambda = 10.1\omega$.

The three-peak structure of probability distribution of the cavity-field $P(n, t)$ in [23] has decreased with increase in gravity, when the RWA is considered. Influence of CRTs on the three-dimensional plot of the $P(n, t)$ with the weak gravity $\vec{q} \cdot \vec{g} = 0.01 \times 10^7\,\mathrm{s}^{-2}$ and the atom–field detuning $\Delta = 0.18 \times 10^7$ rad s^{-1} [23] is shown in figure 2. By comparing figures 2a–2c for weak coupling $\lambda = 0.01\omega$ [28,29], strong coupling $\lambda = 1.11\omega$ [31] and ultra-strong coupling $\lambda = 11.1\omega$ [32] respectively, one can see that the number of peaks in the $P(n, t)$ increase because of CRTs (compare with [23]). Moreover, the multipeak structure of $P(n, t)$ is an evidence for the nonclassical behaviour of the cavity-field which means, the nonclassical behaviour of cavity-field is suppressed with respect to CRTs by

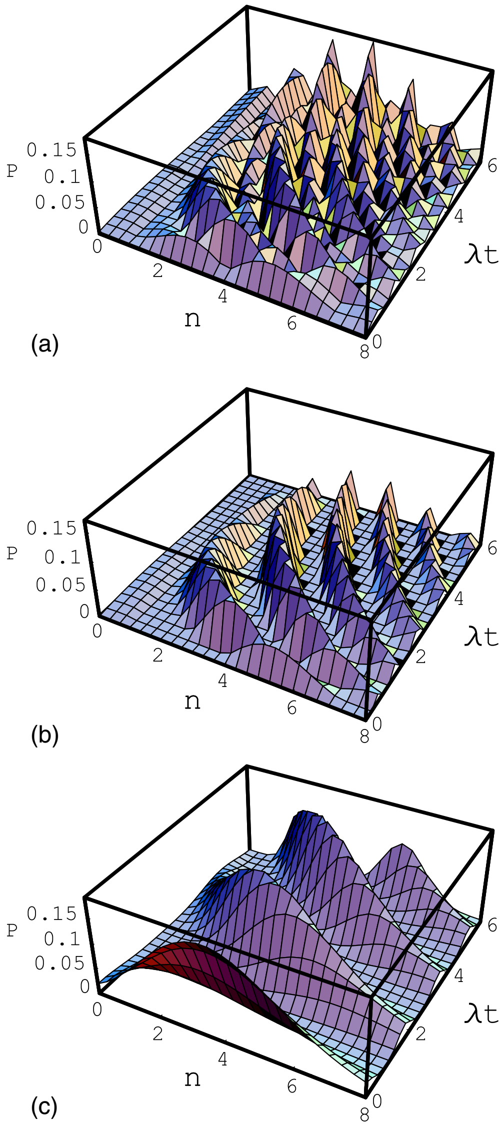

Figure 2. The three-dimensional plot of the probability distribution function $P(n, t)$ vs. the scaled time λt and n with the same corresponding data used in figure 1: (**a**) For $\lambda = 0.01\omega$, (**b**) for $\lambda = 1.11\omega$ and (**c**) for $\lambda = 10.1\omega$.

increasing the coupling parameter. By comparing figures 1 and 2, it is found that the nonclassical behaviour of quantum properties for the atom and the field in our system without the RWA is not suppressed by considering three conditions: (I) the weak gravity $\vec{q} \cdot \vec{g} = 0.01 \times 10^7\,\mathrm{s}^{-2}$, (II) the weak coupling $\lambda = 0.01\omega$ and (III) the atom–field detuning $\Delta = 0.18 \times 10^7$ rad s^{-1}.

5. Summary and conclusions

In this paper, the quantum properties of a two-level atom and the cavity-field in the JCM with the gravity beyond the RWA are investigated. For this purpose, by solving the Schrödinger equation in the interaction picture, the evolving state of the system is found by which the influence of the CRTs on the dynamical behaviour of atomic population inversion of two-level atom and the probability distribution of the cavity-field is explored. The results are summarized as follows: with increase in coupling parameter in our system without the RWA (1) the Rabi-like oscillations in the atomic population inversion are disappeared, (2) the nonclassical behaviour of cavity-field is suppressed and (3) the nonclassical properties of the atom–field system beyond the RWA with the gravity are not completely suppressed by considering three conditions: (I) the weak gravity, (II) the weak coupling and (III) the proper detuning.

Acknowledgements

The author wishes to thank the Office of Research of Shahreza Branch, the Islamic Azad University, for their support.

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