

Effect of nonthermal ion distribution and dust temperature on nonlinear dust-acoustic solitary waves

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Abstract. Dust-acoustic solitary waves in unmagnetized dusty plasma whose constituents are inertial charged dust grains, Boltzmannian electrons and nonthermal ions have been investigated by taking into account finite dust temperature. The pseudopotential has been used to study solitary solution. The existence of solitary waves having negative potential is reported.

Keywords. Solitons; dusty plasma; dust temperature; Sagdeev pseudopotential.

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1. Introduction

In recent years, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas, which are very common in laboratory and astrophysical environments. It is found that the presence of charged dust grains modifies the existing plasma wave spectra. The dust dynamics may even introduce new eigenmodes in the plasma. Indeed, Rao *et al* [1] were the first to predict theoretically the existence of extremely low-phase velocity dust-acoustic waves in unmagnetized dusty plasmas whose constituents are inertial charged dust grains and Boltzmann distributed ions and electrons. These waves were reported experimentally and their nonlinear features were investigated by Barkan *et al* [2]. Dust modes were investigated by many scientists. Some of them have shown that dusty plasma with inertial dust fluid and Boltzmann distributed ions admit only negative solitary potentials associated with nonlinear dust-acoustic wave [3], whereas, others revealed that the presence of nonthermal ions in dusty plasma leads to the possibility of coexistence of compressive as well as rarefactive solitary waves [4,5]. But, the dust temperature (which may not be negligible) was ignored [6–8]. Motivated by all the previous works, in this paper, we investigate dust-acoustic solitary waves, taking into account dust temperature as well as nonthermal ions and study their effect on the properties of DAWs.

2. Model equations

We consider three-component dusty plasma with extremely massive, micron-sized, negatively charged dust grains, Boltzmannian electrons and nonthermally distributed ions. The quasineutrality at equilibrium is written as $N_{i0} = N_{e0} + Z_d N_{d0}$ where N_{i0} , N_{e0} and N_{d0} are the unperturbed ion, electron and dust densities respectively, and Z_d is the number of elementary charges residing on the dust grain. The electron and nonthermal ion densities are given respectively by,

$$N_e = N_{e0} \exp\left(\frac{e\phi}{T_e}\right), \quad (1)$$

$$N_i = N_{i0} \left(1 + \beta \left(\frac{e\phi}{T_i}\right) + \beta \left(\frac{e\phi}{T_i}\right)^2\right) \exp\left(-\frac{e\phi}{T_i}\right), \quad (2)$$

where $\beta = 4\alpha/(1 + 3\alpha)$, α being a parameter which defines the population of nonthermal ions.

3. Arbitrary amplitude solitary structures

In this section we shall be looking for arbitrary large-amplitude solutions of the nonlinear equation systems. The normalized governing equations of the plasma evolution are given by

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} (n_d u_d) = 0, \quad (3)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + 3\sigma n_d \frac{\partial n_d}{\partial x} = \frac{\partial \Phi}{\partial x}, \quad (4)$$

$$\frac{\partial^2 \Phi}{\partial x^2} = \mu_e n_e - \mu_i n_i + n_d, \quad (5)$$

The electron and ion densities are given respectively by

$$n_e = n_{e0} \exp(\sigma_i \Phi), \quad n_i = n_{i0} (1 + \beta \Phi + \beta \Phi^2) \exp(-\Phi),$$

n_d is the dust particle number density normalized to n_{d0} , u_d is the dust fluid velocity normalized to the dust acoustic speed $C_s = (Z_d T_i / m_d)^{1/2}$, Φ is the electrostatic potential normalized to T_i / e . The time and space variables are in units of $\omega_{pd}^{-1} = (m_d / 4\pi n_{d0} Z_d e^2)^{1/2}$, the dust plasma period and the Debye length $\lambda_D = (T_i / 4\pi n_{d0} Z_d e^2)^{1/2}$ respectively, T_i , T_e and T_d being the ion, electron and dust temperatures, respectively. $\mu = n_{e0} / n_{i0}$, $\mu_i = 1 / (1 - \mu)$ and $\mu_e = \mu / (1 - \mu)$, $\sigma_e = T_e / T_i$, $\sigma = T_d / T_i$.

We confine ourselves to investigate stationary solutions that depend on space and time in the following way: $\xi = x - Mt$, where M is the Mach number. In the stationary frame, we obtain from eqs (3), (5) the density as

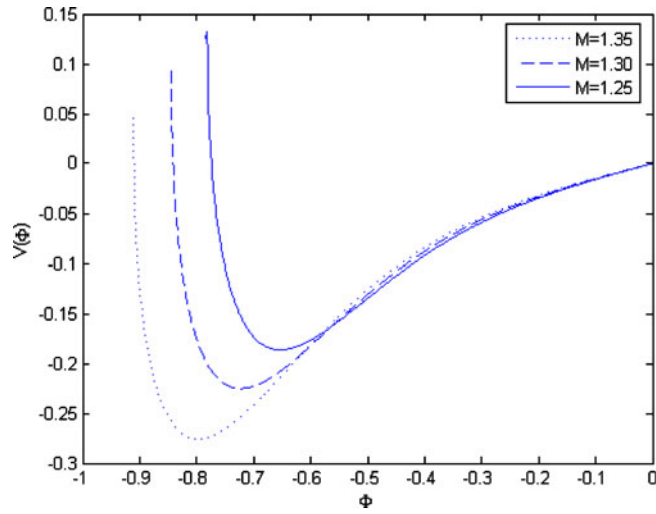


Figure 1. $V(\Phi)$ vs. Φ for $\alpha = 0.20$, $\mu = 0.1$ and $\sigma = 0.02$ shows the existence of solitons.

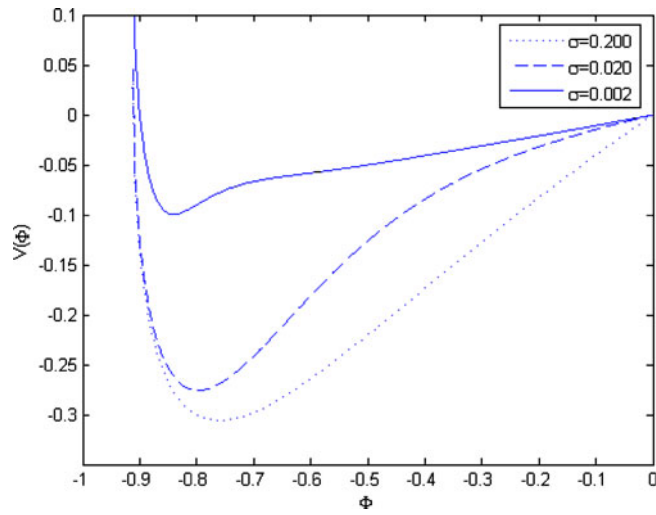


Figure 2. $V(\Phi)$ vs. Φ for $\alpha = 0.20$, $\mu = 0.1$ and $M = 1.35$ shows the existence of solitons.

$$n_d = \frac{\sqrt{2}M}{\sqrt{M^2 + 2\Phi + 3\sigma} + \sqrt{(M^2 + 2\Phi + 3\sigma) - 12M^2\sigma}}, \quad (6)$$

where we have imposed appropriate boundary conditions for the localized disturbances, viz., $u_d \rightarrow 0$, $n_d \rightarrow 1$, $\Phi \rightarrow 0$, at $\xi \rightarrow \infty$. Substituting n_d from eq. (6) in eq. (5) and multiplying both sides of the resulting equation by $d\Phi/d\xi$, integrating once, and taking

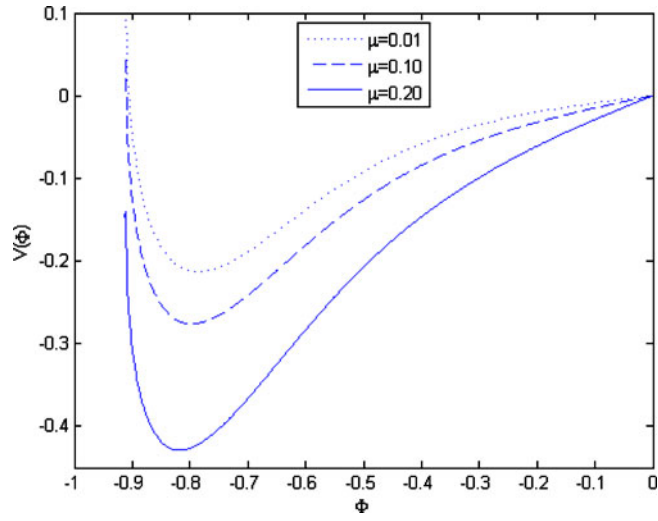


Figure 3. $V(\Phi)$ vs. Φ for $\alpha = 0.20$, $\sigma = 0.02$ and $M = 1.30$.

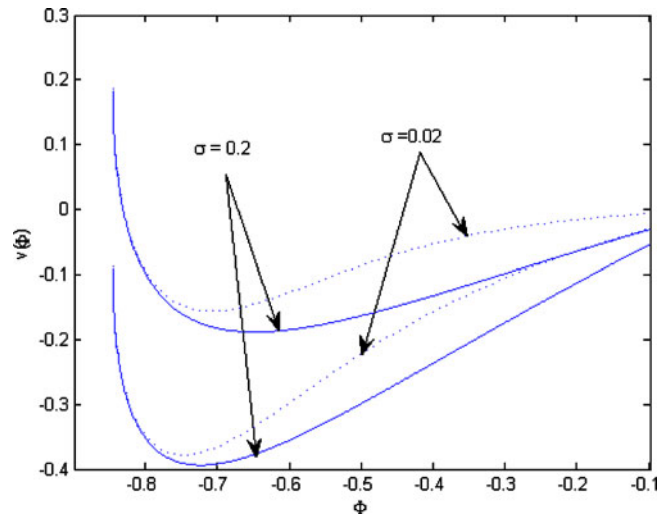


Figure 4. $V(\Phi)$ vs. Φ for $\alpha = 0.20$ and $M = 1.30$. The upper plots are for $\mu = 0.02$ while the lower plots are for $\mu = 0.2$.

into account the appropriate boundary conditions, i.e., $\Phi \rightarrow 0$ and $d\Phi/d\xi \rightarrow 0$ at $\xi \rightarrow \infty$, we obtain the energy integral equation,

$$\frac{1}{2} \left(\frac{d\Phi}{d\xi} \right)^2 + V(\Phi) = 0, \quad (7)$$

where the Sagdeev pseudopotential [9] for our purpose reads as

$$\begin{aligned}
 V(\Phi) = & -\mu_i((1 + 3\beta + 3\beta\Phi + \Phi^2) \exp(-\Phi) - (1 + \beta)) + \frac{\mu_e}{\sigma_e} (1 - \exp(\sigma_e \Phi)) \\
 & + \frac{M(12\sigma M^2)^{1/4}}{\sqrt{2}} \left[\exp\left(\frac{1}{2}\right) \cosh^{-1}(a/\sqrt{b}) \right. \\
 & \quad \left. + \frac{1}{3} \exp\left(-\frac{3}{2}\right) \cosh^{-1}(a/\sqrt{b}) \right] \\
 & - \frac{M(12\sigma M^2)^{1/4}}{\sqrt{2}} \left[\exp\left(\frac{1}{2}\right) \cosh^{-1}(c/\sqrt{b}) \right. \\
 & \quad \left. + \frac{1}{3} \exp\left(-\frac{3}{2}\right) \cosh^{-1}(c/\sqrt{b}) \right]. \quad (8)
 \end{aligned}$$

Here, $a = M^2 + 2\Phi + 3\sigma$, $b = 12\sigma M^2$, $c = M^2 + 3\sigma$.

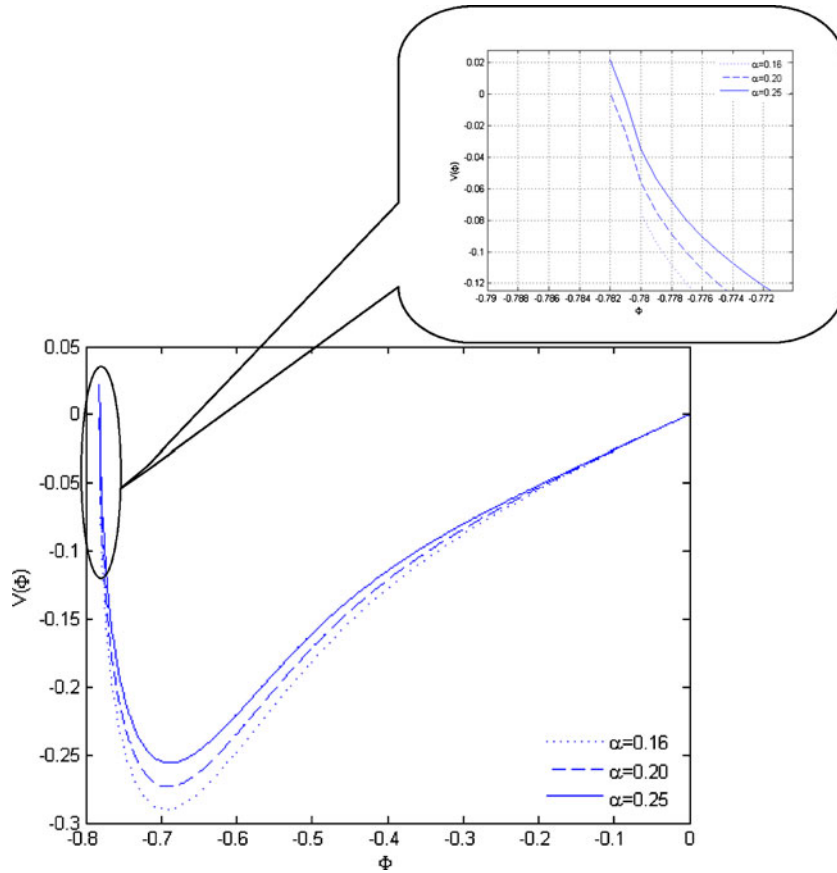


Figure 5. $V(\Phi)$ vs. Φ for $\sigma = 0.01$, $\mu = 0.2$ and $M = 1.25$.

4. Results and discussion

The solitonic solutions of eq. (8) exist when the usual conditions, i.e, $V(\Phi) = (dV(\Phi)/d\Phi) = 0$ at $\Phi = 0$ and $V(\Phi) < 0$ for $0 < |\Phi| < |\Phi_0|$ where $|\Phi_0|$ is the maximum amplitude of the solitons, are satisfied. We plotted the Sagdeev pseudopotential vs. the electrostatic potential Φ for different parameters. It is clearly seen from figure 1, that for $\alpha = 0.20$, $\mu = 0.1$, $\sigma = 0.02$, refractive solitons (solitary waves associated with negative potential) exist for $M \geq 1.25$ and it also shows that the faster the dust is, the deeper is the pseudopotential well. Figure 2 shows that the pseudopotential well is deeper for the hotter dust (which is logical, cooler dust grains have smaller energy). Figure 3 shows that existence of solitons is not possible when μ exceeds 0.10. This means that density has a crucial role on the existence of solitons ($\mu = n_{e0}/n_{i0}$); it is obvious from figure 4 that decrease in dust temperature does not significantly affect the solitons. Figure 5 presents an interesting result; the concentration of nonthermal ions seems to be an important parameter: for $\sigma = 0.01$, $\mu = 0.02$ and $M = 1.25$, solitons exist for $\alpha \geq 0.20$.

5. Conclusion

In this work, we have investigated large-amplitude solitary waves with finite dust temperature incorporating the effect of nonthermal ion distribution in three-component plasma. We used Sagdeev pseudopotential method which takes into account the full nonlinearity of dusty plasma equations. Our results are summarized as follows.

- (i) Only refractive solitons are obtained.
- (ii) Functional dependence of Sagdeev pseudopotential is very sensitive to the variation of μ , α and M .
- (iii) Concentration of nonthermal ions is an important parameter for soliton occurrence.
- (iv) Decrease in dust temperature does not significantly affect the soliton.

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