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A higher-dimensional Bianchi type-I inflationary Universe in general relativity

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Abstract. A five-dimensional Bianchi type-I inflationary Universe is investigated in the presence of massless scalar field with a flat potential. To get an inflationary Universe, a flat region in which the potential V is constant is considered. Some physical and kinematical properties of the Universe are also discussed.

Keywords. Higher-dimensional; inflationary Universe; general relativity.

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1. Introduction

The study of higher-dimensional cosmology space-time because of the underlying that the cosmos at its early stage of evolution of the Universe might have had a higher-dimensional era. The dimensionality of the world has long been a subject of discussion because our sense perceived only four dimensions, but there is nothing in the equation of relativity which restricts us to four dimensions. Witten [1], Appelquist *et al* [2], Chodos and Detweiler [3] and Marciano [4] are some of the authors who have initiated the discussion of higher-dimensional cosmological models. Many authors [5–8] have studied physics of the Universe in higher-dimensional space-time. These models are believed to have physical relevance possibly at the early times before the Universe has undergone compactification transitions.

In recent years there has been lot of interest in inflationary models of the Universe in general relativity. Inflationary models play an important role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the Universe. Inflation, the stage of accelerated expansion of the Universe, first proposed by Gliner [9], nowadays receives a great deal of attention. Gliner proposed $p = -\rho$

as equation of state for vacuum with non-zero energy density, because the corresponding stress-energy tensor $T_{\alpha\beta} = \rho g_{\alpha\beta}$ has an infinite set of co-moving reference frames and no distinguished one. Gliner suggested that at super-high density, matter undergoes a transition into a superdense vacuum. In 1970, he also suggested that the physical state $p = -\rho$ and the de-Sitter geometry it generates, can be the initial state for the expanding Universe with negative pressure as the intrinsic reason for expansion. Guth [10] proposed inflationary model in the context of grand unified theory (GUT), which has been accepted soon as the model of the early Universe. Barrow and Turner [11] showed that the large anisotropy prevents transition into an inflationary era contrary to Guth's original inflationary scenario. Scalar fields are the simplest classical fields and there exist extensive literature containing numerous solutions of the Einstein equation where the scalar field is minimally coupled to the gravitational field. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology.

Burd and Barrow [12], Wald [13], Barrow [14], Ellis and Madsen [15] and Heusler [16] studied several aspects of scalar field in the evolution of the Universe and FRW models. Using the concept of Higgs field ϕ with potential $V(\phi)$ has a flat region and the ϕ field evolves slowly but the Universe expands in an exponential way due to vacuum field energy. The role of self-interacting scalar fields in inflationary cosmology in four-dimensional space-time has been investigated by Bhattacharjee and Baruah [17], Bali and Jain [18], Rahaman *et al* [19], Singh *et al* [20], Reddy *et al* [21], Reddy and Naidu [22]. In recent years, Katore *et al* [24], and Reddy and Naidu [23] have studied the cosmological models with constant deceleration parameter of the Universe in the context of different aspects of different space-time. Reddy [25] has discussed Bianchi type-V inflationary Universe in general relativity. Very recently, Katore *et al* [26,27] have discussed inflationary Universe in general relativity.

In this paper, we have investigated a five-dimensional inflationary cosmological model in the presence of massless scalar field with a flat potential in general relativity and this is an extension of work by Raj Bali and Jain [18]. To get a determinate solution, we have considered a flat region in which the potential is constant.

2. Metric and field equations

We consider a homogeneous LRS Bianchi type-I metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2(\mathrm{d}y^2 + \mathrm{d}z^2) + C^2\mathrm{d}w^2, \tag{1}$$

where A, B and C are functions of t only.

In this case of gravity minimally coupled to a scalar field $V(\phi)$, the Lagrangian

$$L = \int \left[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\right]\sqrt{-g}\,\mathrm{d}^4x, \tag{2}$$

which, when L is varied with respect to the dynamical fields, changes to Einstein field equations

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij}, \tag{3}$$

with

$$T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right] g_{ij}. \tag{4}$$

The conservation relation is of the form

$$\frac{1}{\sqrt{-g}}\partial_u\left(\sqrt{-g}\,\partial^u\phi\right) = -\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi},$$

i.e.

$$\phi^{i}_{;i} = -\frac{\mathrm{d}V}{\mathrm{d}\phi}, \tag{5}$$

where comma and semicolon indicate ordinary and covariant differentiation respectively.

Other symbols have their usual meaning and units are taken so that

$$8\pi G = C = 1.$$

Now the field equations (3) for the metric (1) are given by

$$2\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} = -\frac{\phi_5^2}{2} + V(\phi)\,, \tag{6}$$

$$\frac{A_{55}}{A} + \frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{A_5}{A}\frac{B_5}{B} + \frac{A_5}{A}\frac{C_5}{C} + \frac{B_5}{B}\frac{C_5}{C} = -\frac{\phi_5^2}{2} + V(\phi)\,, \tag{7}$$

$$\frac{A_{55}}{A} + 2\frac{B_{55}}{B} + \frac{B_5^2}{B^2} + 2\frac{A_5}{A}\frac{C_5}{C} = -\frac{\phi_5^2}{2} + V(\phi)\,, \tag{8}$$

$$2\frac{A_5}{A}\frac{B_5}{B} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} + \frac{A_5}{A}\frac{C_5}{C} = \frac{\phi_5^2}{2} + V(\phi)\,, \tag{9}$$

and eq. (5), for the scalar field, takes the form

$$\left(\frac{A_5}{A} + 2\frac{B_5}{B} + \frac{C_5}{C}\right)\phi_5 + \phi_{55} = \frac{\mathrm{d}V}{\mathrm{d}\phi}, \tag{10}$$

where the subscript 5 denotes ordinary differentiation with respect to t.

3. Solutions of the field equations and the model

We are interested, here, in inflationary solutions of the field equations (6)–(10).

Stein-Schabas [28] has shown that Higgs field ϕ with potential $V(\phi)$ has a flat region and the field evolves slowly but the Universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the Universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. Thus, we are interested here, in inflationary solutions of the field equations.

The flat region is considered where the potential is constant, i.e.

$$V(\phi) = \text{constant} = v_0. \tag{11}$$

For complete determinacy of the system, one extra condition is needed. For this purpose, we assume $A = BC$.

Using the above condition, the set of field equations (6)–(10) reduces to

$$2\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{B_5^2}{B^2} + 2\frac{B_5}{B}\frac{C_5}{C} + \frac{\phi_5^2}{2} - V(\phi) = 0, \tag{12}$$

$$2\frac{B_{55}}{B} + 2\frac{C_{55}}{C} + \frac{B_5^2}{B^2} + \frac{C_5^2}{C^2} + 5\frac{B_5}{B}\frac{C_5}{C} + \frac{\phi_5^2}{2} - V(\phi) = 0, \tag{13}$$

$$3\frac{B_{55}}{B} + \frac{C_{55}}{C} + 3\frac{B_5^2}{B^2} + 4\frac{B_5}{B}\frac{C_5}{C} + \frac{\phi_5^2}{2} - V(\phi) = 0, \tag{14}$$

$$3\frac{B_5^2}{B^2} + \frac{C_5^2}{C^2} + 5\frac{B_5}{B}\frac{C_5}{C} - \frac{\phi_5^2}{2} - V(\phi) = 0, \tag{15}$$

$$\left(3\frac{B_5}{B} + 2\frac{C_5}{C}\right)\phi_5 + \phi_{55} = \frac{\mathrm{d}V}{\mathrm{d}\phi}. \tag{16}$$

We solve the above set of field equations (12)–(16) with the transformation, $B = \mathrm{e}^{\alpha}, C = \mathrm{e}^{\beta}, \mathrm{d}t = ABC\mathrm{d}T = B^2C^2\mathrm{d}T$ and using the relation (11), the above set of equations reduces to

$$2\alpha'' + \beta'' - \alpha'^2 - \beta'^2 - 4\alpha'\beta' + \frac{\phi'^2}{2} = v_0\mathrm{e}^{(4\alpha+4\beta)} \tag{17}$$

$$2\alpha'' + 2\beta'' - \alpha'^2 - \beta'^2 - 3\alpha'\beta' + \frac{\phi'^2}{2} = v_0\mathrm{e}^{(4\alpha+4\beta)} \tag{18}$$

$$3\alpha'' + \beta'' - \beta'^2 - 4\alpha'\beta' + \frac{\phi'^2}{2} = v_0\mathrm{e}^{(4\alpha+4\beta)} \tag{19}$$

$$3\alpha'^2 + \beta'^2 + 5\alpha'\beta' - \frac{\phi'^2}{2} = v_0\mathrm{e}^{(4\alpha+4\beta)} \tag{20}$$

$$\phi'' + \alpha'\phi' = 0, \tag{21}$$

where the prime indicates differentiation with respect to T.

The set of equations (17)–(21) admit an exact solution

$$\begin{aligned} A &= d\,(aT+b)^{1+c/a}\,,\\ B &= (aT+b)\,,\\ C &= d\,(aT+b)^{c/a}\,,\\ \phi &= f\log(aT+b)^{e/a}\,, \end{aligned} \tag{22}$$

where *a, b, c, d, e, f* are constants of integration.

The five-dimensional inflationary cosmological model corresponding to the solution can be written as

$$\begin{aligned} \mathrm{d}s^2 = {} & -\mathrm{d}t^2 + \mathrm{d}^2\,(aT+b)^{2(1+(c/a))}\,\mathrm{d}x^2 \\ & + (aT+b)^2\left(\mathrm{d}y^2+\mathrm{d}z^2\right) + \mathrm{d}^2\,(aT+b)^{2c/a}\,\mathrm{d}w^2. \end{aligned} \tag{23}$$

For a positive value of a, the model is free from singularities. For a negative value of a, the model has a singularity at $T = -b/a$.

4. Some physical and kinematical properties

The model (23) represents a five-dimensional inflationary cosmological model in general relativity in the presence massless scalar field with flat potential. The model has no initial singularities at $T = 0$.

For the model (23), the physical and kinematical variables which are important, in cosmology, are

Spatial volume:

$$V=\sqrt{-g} = d^2T^{\eta}, \quad \text{where } \eta = 3 + 2(c/a). \tag{24}$$

Expansion scalar:

$$\theta = U^i_{;i} = \frac{\eta}{T}, \quad \text{where } \eta = 1 + 2a + (2c/a). \tag{25}$$

Shear scalar:

$$\sigma^2 = \frac{4\eta^2}{9T^2}. \tag{26}$$

Hubble parameter:

$$H = \frac{R_4}{R} = \frac{1}{3d^2T^{3+2c}}. \tag{27}$$

The spatial volume increases with T and it becomes infinite for large values of T. Thus inflation is possible for large T. Also volume becomes zero at $T = 0$ and hence there is a Big-Bang at $T = 0$. $\rho \to 0$ as $T \to \infty$. Thus the model gives essentially an empty Universe as $T \to \infty$. Collins and Wainwright [29] have pointed out that the shear scalar plays an important role in general relativistic cosmology. The shear tensor arises in the decomposition of four-vector velocity of the fluid. The shear scalar is non-zero for $T > 0$ and becomes infinitely large as $T \to \infty$. Thus the Universe is not shear-free at infinite time.

It can be observed that for large T, the parameters θ, σ and H vanish and diverge when $T \rightarrow 0$.

Also for large value of T, the ratio $(\sigma^2/\theta^2) \neq 0$ and hence the model (23) is anisotropic and does not approach isotropy. At the time of evolution, the anisotropy of the Universe is constant. Thus the Universe remains anisotropic throughout the evolution.

Self-interacting scalar fields play a central role in the study of inflationary cosmology. During the inflationary era the dominant contributions to the cosmic energy density came from a vacuum and a scalar field. Also for $T = 0$, the scalar field ϕ diverges.

5. Conclusion

In this paper, we have obtained a five-dimensional inflationary Universe in the presence of massless scalar field with flat potential in general relativity. It can be observed that for large T, the parameters θ, σ, H vanish and diverge when $T \rightarrow 0$. The model is expanding and does not approach isotropy at late times. Our investigations are similar to the investigations of Bali and Jain [18]. The inflationary model obtained here has considerable astrophysical significance. For example, classical scalar fields are essential in the study of present day cosmological models. There is an increasing interest, in recent years, in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary Universe and they help us to describe the early stages of evolution of the Universe.

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References

[1] E Witten, *Phys. Lett.* **B144**, 351 (1984)
[2] T Appelquist, A Chodos and P G O Freund, *Modern Kaluza–Klein theories* (Addison–Wesley, Reading, 1987)
[3] A Chodos and S Detweiler, *Phys. Rev.* **D21**, 2167 (1980)
[4] W J Marciano, *Phys. Rev. Lett.* **52**, 498 (1984)
[5] D Sahdev, *Phys. Rev.* **D30**, 2495 (1984)
[6] V M Emelyanov, Yu P Nikitin, J L Rozental and A V Berkov, *Phys. Rep.* **143**, 1 (1986)
[7] S Chatterjee and B Bhui, *Int. J. Theor. Phys.* **32**, 671 (1993)
[8] B C Paul and S Mukherjee, *Phys. Rev.* **D42**, 2595 (1990)
[9] E B Gliner, *Zh. Eksp. Tear. Fiz.* **49**, 542 (1965) [*Sov. Phys. JETP* **22**, 378 1966]
[10] A H Guth, *Phys. Rev.* **D23**, 347 (1981)
[11] D Barrow and M S Turner, *Nature (London)* **35**, 292 (1981)
[12] A B Burd and J D Barrow, *Nucl. Phys.* **B308**, 923 (1988)
[13] R Wald, *Phys. Rev.* **D28**, 2118 (1983)
[14] J D Barrow, *Phys. Lett.* **B187**, 12 (1987)
[15] G F R Ellis and M S Madsen, *Class. Quant. Grav.* **8**, 667 (1991)
[16] M Heusler, *Phys. Lett.* **B253**, 33 (1991)

[17] R Bhattacharjee and K K Baruah, *Indian J. Pure Appl. Math.* **32**, 47 (2001)
[18] R Bali and V C Jain, *Pramana – J. Phys.* **59**, 1 (2002)
[19] F Rahaman, G Bag, B C Bhui and S Das, *Fizika* **B12**, 193 (2003)
[20] C P Singh and S Kumar, *Pramana – J. Phys.* **68(5)**, 707 (2007)
[21] D R K Reddy, R L Naidu and A S Rao, *Int. J. Theor. Phys.* **47**, 1016 (2008)
[22] D R K Reddy and R L Naidu, *Int. J. Theor. Phys.* **47**, 2339 (2008)
[23] D R K Reddy, R L Naidu and A S Rao, *Astrophys. Space Sci.* **319**, 89 (2009)
[24] S D Katore, R S Rane and V B Kurkure, *Astrophys. Space Sci.* **315**, 347 (2008)
[25] D R K Reddy, *Int. J. Theor. Phys.* **48**, 2036 (2009), DOI:10.1007/s10773-009-9979-z
[26] D R K Reddy, K S Adhav, S D Katore and K S Wankhade, *Int. J. Theor. Phys.* **48**, 2884 (2009)
[27] S D Katore, R S Rane, K S Wankhade and N K Sarkate, *Pramana – J. Phys.* **74(4)**, 669 (2010)
[28] J A Stein-Schabas, *Phys. Rev.* **D35**, 2345 (1987)
[29] C B Collins and J Wainwright, *Phys. Rev.* **D27**, 1209 (1983)