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Gravitational collapse with decaying vacuum energy

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Abstract. The effect of dark energy on the end state of spherical radiation collapse is considered within the context of the cosmic censorship hypothesis. It is found that it is possible to have both black holes as well as naked singularities.

Keywords. Gravitational collapse; cosmic censorship hypothesis; vacuum energy.

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1. Introduction

Recent results, e.g., coming from the analysis of high redshift type-Ia supernovae [1,2], indicate that the Universe is presently accelerating. To explain this acceleration, it is necessary to have some exotic field such as dark energy with large negative pressure. The Universe consists of approximately 72% dark energy, 21% dark matter, 4.5% visible matter and 0.5% radiation. Whilst numerous candidates have been proposed for dark energy and dark matter, their nature is still unknown. Following de Campos [3] and Bandyopadhyay *et al* [4], we consider the effect of dark energy in the form of decaying vacuum energy on the gravitational collapse of a typical star.

One of the most important unsolved theoretical problems in classical general relativity is the cosmic censorship hypothesis [5], i.e., will gravitational collapse of a typical massive star always lead to a black hole as opposed to a naked singularity? It was Chandrasekhar [6] who first proposed that a star more massive than about 1.4 solar masses cannot end up as a white dwarf. Unfortunately he did not pursue other possibilities further, and the studies of gravitational collapse had to wait for about three decades before further progress was made. It is now known that a star slightly more massive than 1.4 solar masses can also end up as a neutron star. However, a star more massive than about 10 solar masses must end up in a singularity.

Penrose [5] first proposed that general relativity does not admit naked singularities. There are two versions of the cosmic censorship hypothesis. In its weak form, the hypothesis states that a reasonable spacetime (matter satisfying the energy conditions, and with regular density and velocity profiles) does not have a naked singularity. In its strong form, the hypothesis states that even if a naked singularity forms, it must be hidden behind an event horizon, and therefore not be visible to a distant observer. Despite many attempts, there is still no proof of the hypothesis, and thus one resorts to studying examples to see whether naked singularities or black holes, or both, form.

It is now known that there are a number of solutions of Einstein's equations which admit naked singularities (see, e.g., [7]). This includes the effect of different types of matter (dust, radiation, scalar fields, etc). Spherically symmetric radiation collapse (the Vaidya solution) is known to give rise to both black holes and naked singularities [8]. An interesting question that arises is whether the presence of dark energy, which makes up the bulk of the matter content of the Universe, will lead to black holes or naked singularities. In this paper, we study what effect the dark energy (in the form of decaying vacuum energy) would have on Vaidya collapse.

2. Generalized Vaidya solution

Einstein's field equations are

$$G_{ab} = T_{ab}. \tag{1}$$

We consider the energy–momentum tensor of the star to be given by

$$T_{ab} = \frac{2}{r^2}\dot{m}k_a k_b - \Phi g_{ab}, \tag{2}$$

where m represents the mass of the star, the null vector k_a satisfies $k_a = -\delta_a^v$ and $k^a k_a = 0$, and Φ represents the contribution due to dark energy in the form of decaying vacuum energy. Then we have the generalized Vaidya solution, which in (v, r, θ, ϕ) coordinates reads as:

$$\mathrm{d}s^2 = -\left[1 - \frac{2m(v)}{r} - \frac{\Phi(v)r^2}{3}\right]\mathrm{d}v^2 + 2\mathrm{d}v\,\mathrm{d}r + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{3}$$

where $r \in [0, \infty)$, $v \in (-\infty, \infty)$ represents advanced Eddington time, in which r is decreasing towards the future along a ray $v = \text{const}$.

We consider the mass function m to be given by $2m(v) = \alpha v$ and the vacuum energy Φ to decay as $\Phi(v) = 3\beta/v^2$ where α and β are constants. Then there exists a curvature singularity at $r = 0$, $v = 0$ as can be seen from the divergence of the Kretschmann scalar. We now investigate if the singularity that occurs is a black hole or a naked singularity by checking if radial null geodesics emerge from the singularity. From the metric (3), radial null geodesics are given by

$$\frac{\mathrm{d}r}{\mathrm{d}v} = \frac{1}{2}\left[1 - \alpha y - \frac{\beta}{y^2}\right], \tag{4}$$

where $y = v/r$. In order to determine the nature of the limiting value of y as we approach $r = 0$, $v = 0$, we now let $y_0 = \lim_{r\to 0\ v\to 0} y$ and obtain from (4)

$$\alpha y_0^3 - y_0^2 + 2y_0 + \beta = 0. \tag{5}$$

This algebraic equation tells us if geodesics emerge from the singularity. There exist values for α and β for which eq. (5) admits positive real roots, and values for which it does not.

Table 1. Roots for eq. (5).

α	β	Root 1	Root 2	Root 3
0.05	33.64	12.40	12.14	−4.488
0.05	33.65	−4.489	Imaginary	Imaginary
0.075	9.651	7.758	7.723	−2.148
0.075	9.652	−2.148	Imaginary	Imaginary
0.1	2.614	5.474	5.409	−0.8829
0.1	2.615	−0.8831	Imaginary	Imaginary
0.125	−1	5.236	0.7639	2.000
0.125	0	4	4	–
0.125	1	−0.4111	Imaginary	Imaginary

The existence of positive real roots corresponds to an outgoing node, and is associated with geodesics emerging from the singularity, and hence naked singularities, whereas the non-existence is associated with black holes. So we can have both black holes as well as naked singularities depending upon the values of α and β.

In table 1, we show some typical roots of the cubic eq. (5) for different values of α and β, showing that both naked singularities as well as black holes can form.

The case $\alpha = 0.125$, $\beta = 0$ yields a quadratic equation which corresponds to Vaidya collapse [8]. In general, for each value of α in the cubic, there corresponds a range of values of β which yield both positive and negative roots. For example, for $\alpha = 0.1$, if $\beta \in (0, 2.614]$ we have some positive roots and hence naked singularities, whereas for $\beta \geq 2.615$ we have no positive roots, and hence black holes.

3. Conclusion

De Campos [3] studied the gravitational collapse of a spherically symmetric star made up of a dust cloud of dark matter by considering the decay of dark energy into dark particles, and found that black holes occur. Bandyopadhyay *et al* [4] investigated a similar problem within the brane-world scenario and found that both black holes and naked singularities can form. In this paper, we have looked at what effect the dark energy (in the form of decaying vacuum energy) would have on Vaidya collapse. We found that both black holes and naked singularities can form depending upon certain parameters. Whilst this may not be an actual physical situation, it does illustrate that dark energy may not be able to solve the cosmic censorship problem.

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