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A transformed rational function method for (3+1)-dimensional potential Yu–Toda–Sasa–Fukuyama equation

SHENG ZHANG[1,2,*] and HONG-QING ZHANG[2]
[1]Department of Mathematics, Bohai University, Jinzhou 121013, People's Republic of China
[2]School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, People's Republic of China
*Corresponding author. E-mail: zhshaeng@yahoo.com.cn

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Abstract. A direct method, called the transformed rational function method, is used to construct more types of exact solutions of nonlinear partial differential equations by introducing new and more general rational functions. To illustrate the validity and advantages of the introduced general rational functions, the (3+1)-dimensional potential Yu–Toda–Sasa–Fukuyama (YTSF) equation is considered and new travelling wave solutions are obtained in a uniform way. Some of the obtained solutions, namely exponential function solutions, hyperbolic function solutions, trigonometric function solutions, Jacobi elliptic function solutions and rational solutions, contain an explicit linear function of the independent variables involved in the potential YTSF equation. It is shown that the transformed rational function method provides more powerful mathematical tool for solving nonlinear partial differential equations.

Keywords. Nonlinear partial differential equations; transformed rational function method; Jacobi elliptic function solutions; hyperbolic function solutions; trigonometric function solutions.

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1. Introduction

Nonlinear complex physical phenomena are related to nonlinear partial differential equations (PDEs) which are involved in many fields from physics to biology, chemistry, mechanics, etc. Searching for exact solutions of nonlinear PDEs plays an important role in the study of these physical phenomena and gradually becomes one of the most important and significant tasks. In recent years, many direct methods for obtaining exact solutions of nonlinear PDEs have been presented, such as the tanh-function method [1–3], the homogeneous balance method [4], the sine-cosine method [5], the Jacobi elliptic function expansion method [6–8], the F-expansion method [9–11], the auxiliary equation method [12–14], and the exp-function method [15–17].

Based on the observation that it has been a successful idea to generate exact solutions of nonlinear wave equations by reducing PDEs into ordinary differential equations (ODEs), more recently Ma and Lee [18] proposed a new direct method called the transformed rational function method for constructing exact solutions of nonlinear equations by using rational function transformations. The new method which unifies the tanh-function methods, the homogeneous balance method, the exp-function method, the mapping method and the F-expansion methods provides a more systematical and convenient handling of the solution process of nonlinear PDEs. The interesting idea of using a fractional transformation to connect the rational solutions of nonlinear equations with the elliptic functions has been used earlier for solving nonlinear Schrödinger equation (NLSE) with a source by Raju *et al* [19] and higher-order NLSE with a source by Vyas *et al* [20].

The present paper is motivated by the desire to extend the transformed rational function method to construct more types of exact solutions of nonlinear PDEs by introducing new and more general rational functions. To illustrate the validity and advantages of the general rational functions, we shall consider the (3+1)-dimensional potential YTSF equation [21]:

$$-u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z + 3u_{yy} = 0, \tag{1}$$

which can be derived from the (3+1)-dimensional YTSF equation

$$[-4v_t + \Phi(v)v_z]_x + 3v_{yy} = 0, \quad \Phi(v) = \partial_x^2 + 4v + 2v_x\partial_x^{-1}, \tag{2}$$

by using the potential $v = u_x$. It was Yu *et al* [22] who extended the (2+1)-dimensional Bogoyavlenskii–Schif (BS) equation [23]

$$v_t + \Phi(v)v_z = 0, \quad \Phi(v) = \partial_x^2 + 4v + 2v_x\partial_x^{-1} \tag{3}$$

to the (3+1)-dimensional nonlinear evolution equation in the form of eq. (2). The extension schemes in which the (1+1)-dimensional KdV equation $v_t+\Phi(v)v_x = 0$, $\Phi(v) = \partial_x^2+4v+2v_x\partial_x^{-1}$ is extended to the (2+1)-dimensional KP equation $[-4v_t + \Phi(v)v_x]_x + 3v_{yy} = 0$, $\Phi(v) = \partial_x^2 + 4v + 2v_x\partial_x^{-1}$ and BS equation (3), and then the (3+1)-dimensional YTSF equation (2) are schematically written in the following form [22]:

$$\begin{array}{ccc} \text{KdV equation} & \Longrightarrow & \text{BS equation} \\ \Downarrow & & \Downarrow \\ \text{KP equation} & \Longrightarrow & \text{YTSF equation} \end{array}.$$

The rest of this paper is organized as follows. In §2, we describe the transformed rational function method combining new and more general rational functions. In §3, we take the (3+1)-dimensional potential YTSF equation (1) as an example to illustrate the validity and advantages of the generalized work. In §4, conclusions are given.

2. Description of the transformed rational function method combining new and more general rational functions

In this section, let us recall the basic idea of the transformed rational function method [18] and outline its generalization. For a given nonlinear PDE, say, in four variables x, y, z and t

$$P(x, y, z, t, u, u_x, u_y, u_z, u_t, \ldots) = 0, \tag{4}$$

we first use the following transformation:

$$u(x, y, z, t) = u(\xi), \quad \xi = \xi(x, y, z, t), \tag{5}$$

and usually set $\xi = ax + by + cz - \omega t$ in the constant-coefficient case, while $\xi = a(t)x + b(t)y + c(t)z - \omega(t)$ in the t-dependent-coefficient case, where a, b, c and ω are constants; $a(t)$, $b(t)$, $c(t)$ and $\omega(t)$ are functions of t. Then eq. (4) is reduced to an ODE:

$$Q(x, y, z, t, u^{(r)}, u^{(r+1)}, \ldots) = 0, \tag{6}$$

where $u^{(i)} = \mathrm{d}^i u/\mathrm{d}\xi^i$, $i \geq r$, and r is the least order of derivatives in the equation. If Q is a total ξ-derivative of another function, integrating eq. (6) with respect to ξ, we further reduce the transformed equation.

Secondly, we search for exact solutions determined by

$$u^{(r)}(\xi) = v(\eta) = \frac{\sum_{j=0}^{m} p_{1j}\eta^j + \sum_{j=1}^{m} p_{2j}\eta^{j-1}\eta'}{\sum_{k=0}^{m} q_{1k}\eta^k + \sum_{k=1}^{m} q_{2k}\eta^{k-1}\eta'}, \quad \eta = \eta(\xi), \tag{7}$$

where m and n are two natural numbers, p_j and q_k are normally constants or functions of the independent variables and η satisfies a solvable ODE, for example, a first-order differential equation

$$\eta' = T = T(\xi, \eta). \tag{8}$$

Some special solvable cases of T were used in open literature, such as $T = T(\eta) = \eta$ in the exp-function method, $T = T(\eta) = \eta^2 + \alpha$ (α is a constant) in the extended tanh-function method and $T^2 = T^2(\eta) = f\eta^4 + g\eta^2 + h$ (f, g and h are constants) in the extended F-expansion method. We, therefore, have

$$\begin{aligned} u^{(r+1)}(\xi) &= Tv'(\eta), \\ u^{(r+2)}(\xi) &= T\partial_\eta u^{(r+1)}(\xi) = T^2 v''(\eta) + TT'v'(\eta), \ldots, \end{aligned} \tag{9}$$

which is based on $\partial_\xi = T\partial_\eta$, prime denotes the derivative with respect to the involved variable, for instance, $u'(\xi) = \mathrm{d}u(\xi)/\mathrm{d}\xi$, $v'(\eta) = \mathrm{d}v(\eta)/\mathrm{d}\eta$, $T' = \mathrm{d}T/\mathrm{d}\eta$.

Thirdly, we get a system of algebraic equations for all variables a, b, c, ω, p_j and q_k by substituting eqs (7) and (9) into eq. (6) and setting the numerator of the resulting rational function to be zero. Solving this system of algebraic equations, we determine $p(\eta)$, $q(\eta)$ and ξ.

Finally, integrating $v(\eta)$ with respect to ξ, r times, we obtain a class of exact solutions

$$\begin{aligned} u(x, y, z, t) &= u(\xi) = \underbrace{\int\int\cdots\int}_{r} \frac{p(\eta(\xi))}{q(\eta(\xi))}\mathrm{d}\xi\cdots\mathrm{d}\xi \\ &= \int^{\xi}\int^{\xi_r}\cdots\int^{\xi_2} \frac{p(\eta(\xi_1))}{q(\eta(\xi_1))}\mathrm{d}\xi_1\cdots\mathrm{d}\xi_{r-1}\mathrm{d}\xi_r + \sum_{k=1}^{r} d_k \xi^{r-k}, \end{aligned} \tag{10}$$

where d_k are arbitrary constants. If $r = 1$, there is only the last defined integral over $[0, \xi]$ in eq. (10). The obtained solutions will definitely contain a polynomial part in ξ when $r > 1$.

Remark 1. It is easy to see that when $p_{1j} = p_j$, $q_{1k} = q_k$, $p_{2j} = q_{2k} = 0$, from eq. (7) we have

$$v(\eta) = \frac{\sum_{j=0}^{m} p_j \eta^j}{\sum_{k=0}^{m} q_k \eta^k}, \tag{11}$$

which is the rational functions (2.9) used in [18]. Thus, we may get new and more types of exact solutions of eq. (4) by using the proposed rational functions in eq. (7) due to the terms $p_{2j}\eta^{j-1}\eta'$ and $q_{2k}\eta^{k-1}\eta'$ included there.

3. Application to the (3+1)-dimensional potential YTSF equation

In this section we apply the generalized method developed in §2 to the (3+1)-dimensional potential YTSF eq. (1) and consider three special cases of T in eq. (8).

Using the transformation

$$u(x, y, z, t) = u(\xi), \quad \xi = ax + by + cz - \omega t, \tag{12}$$

we reduce eq. (1) into an ODE of the form

$$a^3 c u^{(4)} + 6a^2 c u' u'' + (3b^2 + 4a\omega) u'' = 0, \tag{13}$$

where the prime denotes the derivative with respect to ξ. Then integrating it once with respect to ξ yields

$$a^3 c u^{(3)} + 3a^2 c (u')^2 + (3b^2 + 4a\omega) u' = 0, \tag{14}$$

further setting $r = 1$ and $u' = v$, we have the transformed potential YTSF equation

$$a^3 c T^2 v'' + a^2 c T T' v' + 3a^2 c v^2 + (3b^2 + 4a\omega) v = 0, \tag{15}$$

where the prime denotes the derivative with respect to η.

3.1 *The case of $\eta' = \eta$ in the exp-function method*

In this case, eq. (15) becomes

$$a^3 c \eta^2 v'' + a^2 c \eta v' + 3a^2 c v^2 + (3b^2 + 4a\omega) v = 0. \tag{16}$$

If a rational solution v is constructed in the form

$$v(\eta) = \frac{p_{12}\eta^2 + p_{11}\eta + p_{10}}{q_{12}\eta^2 + q_{11}\eta + q_{10}} \tag{17}$$

with the help of *Mathematica* we have only two choices for non-constant v and $abc \neq 0$

$$v(\eta) = \frac{4aq_{11}q_{12}\eta}{(q_{11} + 2q_{12}\eta)^2}, \quad \omega = -\frac{a^3 c + 3b^2}{4a}, \tag{18}$$

$$v(\eta)=\frac{p_{10}(16p_{10}^2+16p_{10}p_{11}\eta+p_{11}^2\eta^2)}{q_{10}(4p_{10}-p_{11}\eta)^2},\quad a=-\frac{3p_{10}}{q_{10}},$$
$$\omega=\frac{9cp_{10}^3+b^2q_{10}^3}{4p_{10}q_{10}^2}. \tag{19}$$

According to eqs (18) and (19) and the solution e^{ξ} of $\eta'=\eta$, we obtain exponential function solutions of eq. (1)

$$u(x,y,z,t)=-\frac{2aq_{11}}{q_{11}+2q_{12}e^{\xi}}+d,\quad \xi=ax+by+cz+\frac{a^3c+3b^2}{4a}t, \tag{20}$$

$$u(x,y,z,t)=\frac{24p_{10}^2}{q_{10}(4p_{10}-p_{11}e^{\xi})}+\frac{p_{10}}{q_{10}}\xi+d,$$
$$\xi=-\frac{3p_{10}}{q_{10}}x+by+cz-\frac{9cp_{10}^3+b^2q_{10}^3}{4p_{10}q_{10}^2}t, \tag{21}$$

where the involved constants are all arbitrary. We note that solution (21) contains an explicit linear function of ξ, and that all solutions reported in [24] are equivalent to solution (20).

3.2 *The case of $\eta'=\eta^2+\alpha$ in the extended tanh-function method*

In this case, eq. (15) changes into

$$a^3c\eta^4v''+2a^3c\eta^3v'+2a^3c\alpha\eta^2v''+a^3c\alpha^2v''+2a^3c\alpha\eta v'$$
$$+3a^2cv^2+(3b^2+4a\omega)v=0. \tag{22}$$

If a rational solution v is constructed in the form

$$v(\eta)=\frac{p_{14}\eta^4+p_{13}\eta^3+p_{12}\eta^2+p_{11}\eta+p_{10}}{q_{12}\eta^2+q_{11}\eta+q_{10}} \tag{23}$$

with the help of *Mathematica* we have four different choices for non-constant v and $abc\neq 0$

$$v(\eta)=-2a\alpha-2a\eta^2,\quad \omega=\frac{4a^3c\alpha-3b^2}{4a}, \tag{24}$$

$$v(\eta)=-\frac{2}{3}a\alpha-2a\eta^2,\quad \omega=-\frac{4a^3c\alpha+3b^2}{4a}, \tag{25}$$

$$v(\eta)=-4a\alpha-2a\eta^2-2a\alpha^2\frac{1}{\eta^2},\quad \omega=\frac{16a^3c\alpha-3b^2}{4a}, \tag{26}$$

$$v(\eta)=\frac{4}{3}a\alpha-2a\eta^2-2a\alpha^2\frac{1}{\eta^2},\quad \omega=-\frac{16a^3c\alpha+3b^2}{4a}. \tag{27}$$

According to eqs (24)–(27) and the well-known special solutions [12] of $\eta'=\eta^2+\alpha$, when $\alpha<0$ we obtain hyperbolic function solutions of eq. (1)

$$u(x,y,z,t)=2a\sqrt{-\alpha}\tanh(\sqrt{-\alpha}\xi)+d,$$
$$\xi=ax+by+cz-\frac{4a^3c\alpha-3b^2}{4a}t, \tag{28}$$

$$u(x, y, z, t) = 2a\sqrt{-\alpha}\coth(\sqrt{-\alpha}\xi) + d,$$
$$\xi = ax + by + cz - \frac{4a^3c\alpha - 3b^2}{4a}t, \tag{29}$$

$$u(x, y, z, t) = 2a\sqrt{-\alpha}\tanh(\sqrt{-\alpha}\xi) + 2a\sqrt{-\alpha}\coth(\sqrt{-\alpha}\xi) + d,$$
$$\xi = ax + by + cz - \frac{16a^3c\alpha - 3b^2}{4a}t. \tag{30}$$

$$u(x, y, z, t) = 2a\sqrt{-\alpha}\tanh(\sqrt{-\alpha}\xi) + \frac{4a\alpha}{3}\xi + d,$$
$$\xi = ax + by + cz + \frac{4a^3c\alpha + 3b^2}{4a}t, \tag{31}$$

$$u(x, y, z, t) = 2a\sqrt{-\alpha}\coth(\sqrt{-\alpha}\xi) + \frac{4a\alpha}{3}\xi + d,$$
$$\xi = ax + by + cz + \frac{4a^3c\alpha + 3b^2}{4a}t, \tag{32}$$

$$u(x, y, z, t) = 2a\sqrt{-\alpha}\tanh(\sqrt{-\alpha}\xi) + 2a\sqrt{-\alpha}\coth(\sqrt{-\alpha}\xi) + \frac{16a\alpha}{3}\xi + d,$$
$$\xi = ax + by + cz + \frac{16a^3c\alpha + 3b^2}{4a}t. \tag{33}$$

When $\alpha > 0$, we obtain trigonometric function solutions of eq. (1)

$$u(x, y, z, t) = -2a\sqrt{\alpha}\tan(\sqrt{\alpha}\xi) + d,$$
$$\xi = ax + by + cz - \frac{4a^3c\alpha - 3b^2}{4a}t, \tag{34}$$

$$u(x, y, z, t) = 2a\sqrt{\alpha}\cot(\sqrt{\alpha}\xi) + d,$$
$$\xi = ax + by + cz - \frac{4a^3c\alpha - 3b^2}{4a}t, \tag{35}$$

$$u(x, y, z, t) = -2a\sqrt{\alpha}\tan(\sqrt{\alpha}\xi) + 2a\sqrt{\alpha}\cot(\sqrt{\alpha}\xi) + d,$$
$$\xi = ax + by + cz - \frac{16a^3c\alpha - 3b^2}{4a}t, \tag{36}$$

$$u(x, y, z, t) = -2a\sqrt{\alpha}\tan(\sqrt{\alpha}\xi) + \frac{4a\alpha}{3}\xi + d,$$
$$\xi = ax + by + cz + \frac{4a^3c\alpha + 3b^2}{4a}t, \tag{37}$$

$$u(x, y, z, t) = 2a\sqrt{\alpha}\cot(\sqrt{\alpha}\xi) + \frac{4a\alpha}{3}\xi + d,$$

$$\xi = ax + by + cz + \frac{4a^3c\alpha + 3b^2}{4a}t, \tag{38}$$

$$u(x, y, z, t) = -2a\sqrt{\alpha}\tan(\sqrt{\alpha}\xi) + 2a\sqrt{\alpha}\cot(\sqrt{\alpha}\xi) + \frac{16a\alpha}{3}\xi + d,$$

$$\xi = ax + by + cz + \frac{16a^3c\alpha + 3b^2}{4a}t. \tag{39}$$

When $\alpha = 0$, we obtain rational solutions of eq. (1)

$$u(x, y, z, t) = \frac{8a^2}{4a^2x + 4aby + 4acz + 3b^2t} + d. \tag{40}$$

We note that solutions (31)–(33) and (37)–(39) contain an explicit linear function of ξ, and that all solutions reported in [21] are special cases of solutions (28), (29), (31), (32), (34), (35), (37), (38), respectively.

3.3 *The case of $\eta'^2 = f\eta^4 + g\eta^2 + h$ in the extended F-expansion method*

In this case, eq. (15) turns into

$$a^3cf\eta^4v'' + 2a^3cf\eta^3v' + a^3cg\eta^2v'' + a^3cg\eta v' + a^3chv'' + 3a^2cv^2 + (3b^2 + 4a\omega)v = 0. \tag{41}$$

If we try to construct a rational solution v in the form

$$v(\eta) = \frac{p_{14}\eta^4 + p_{13}\eta^3 + p_{12}\eta^2 + p_{11}\eta + p_{10} + p_{23}\eta^2\eta' + p_{22}\eta\eta' + p_{21}\eta'}{q_{12}\eta^2 + q_{11}\eta + q_{10} + q_{22}\eta\eta' + q_{21}\eta'} \tag{42}$$

with the help of *Mathematica* we have four different choices for non-constant v and $abc \neq 0$

$$v(\eta) = -\frac{2}{3}a(g \pm \sqrt{g^2 - 3fh}) - 2af\eta^2,$$

$$\omega = \frac{\pm 4a^3c\sqrt{g^2 - 3fh} - 3b^2}{4a}, \tag{43}$$

$$v(\eta) = -\frac{2}{3}a(g \pm \sqrt{g^2 + 12fh}) - 2af\eta^2 - 2ah\frac{1}{\eta^2},$$

$$\omega = \frac{\pm 4a^3c\sqrt{g^2 + 12fh} - 3b^2}{4a}, \tag{44}$$

$$v(\eta) = -\frac{1}{6}a(g \pm \sqrt{g^2 + 12fh}) - af\eta^2 \pm a\sqrt{f}\eta',$$

$$\omega = \frac{\pm 4a^3c\sqrt{g^2 + 12fh} - 3b^2}{4a}, \tag{45}$$

$$v(\eta) = -\frac{1}{6}a\left(g \mp 6\sqrt{fh} \pm \sqrt{g^2 \pm 60g\sqrt{fh} + 132fh}\right) - af\eta^2$$
$$- ah\frac{1}{\eta^2} \pm a\sqrt{f}\eta' \pm a\sqrt{h}\frac{1}{\eta^2}\eta',$$
$$\omega = \frac{\pm a^3c\sqrt{g^2 \pm 60g\sqrt{fh} + 132fh} - 3b^2}{4a}. \tag{46}$$

Using eqs (43)–(46) and selecting one appropriate special solution [11] of $\eta'^2 = f\eta^4 + g\eta^2 + h$, for example, $\eta = \mathrm{sn}(\xi, m)$ along with $f = m^2$, $g = -(1+m^2)$ and $h = 1$, we obtain Jacobi elliptic function solutions of eq. (1)

$$u(x,y,z,t) = -2am^2\int^{\xi} \mathrm{sn}^2(\xi_1, m)\mathrm{d}\xi_1$$
$$- \frac{2}{3}a(-1 - m^2 \pm \sqrt{m^4 - m^2 + 1})\xi + d,$$
$$\xi = ax + by + cz - \frac{\pm 4a^3c\sqrt{m^4 - m^2 + 1} - 3b^2}{4a}t \tag{47}$$

$$u(x,y,z,t) = -2a\int^{\xi} [m^2\mathrm{sn}^2(\xi_1, m) + \mathrm{ns}^2(\xi_1, m)]\mathrm{d}\xi_1$$
$$- \frac{2}{3}a(-1 - m^2 \pm \sqrt{m^4 + 14m^2 + 1})\xi + d,$$
$$\xi = ax + by + cz - \frac{\pm 4a^3c\sqrt{m^4 + 14m^2 + 1} - 3b^2}{4a}t, \tag{48}$$

$$u(x,y,z,t) = -am\int^{\xi} [m\,\mathrm{sn}^2(\xi_1, m) \mp \mathrm{cn}(\xi_1, m)\mathrm{dn}(\xi_1, m)]\mathrm{d}\xi_1$$
$$- \frac{1}{6}a(-1 - m^2 \pm \sqrt{m^4 + 14m^2 + 1})\xi + d,$$
$$\xi = ax + by + cz - \frac{\pm 4a^3c\sqrt{m^4 + 14m^2 + 1} - 3b^2}{4a}t, \tag{49}$$

$$u(x,y,z,t) = -a\int^{\xi} [m^2\mathrm{sn}^2(\xi_1, m) + \mathrm{ns}^2(\xi_1, m)$$
$$\mp m\,\mathrm{cn}(\xi_1, m)\mathrm{dn}(\xi_1, m) \mp \mathrm{cs}(\xi_1, m)\mathrm{ds}(\xi_1, m)]\mathrm{d}\xi_1$$
$$- \frac{1}{6}a[-1 - m^2 \mp 6m \pm \sqrt{m^4 \mp 60m(1+m^2) + 134m^2 + 1}]\xi + d,$$
$$\xi = ax + by + cz + \frac{\pm a^3c\sqrt{m^4 \mp 60m(1+m^2) + 134m^2 + 1} - 3b^2}{4a}t. \tag{50}$$

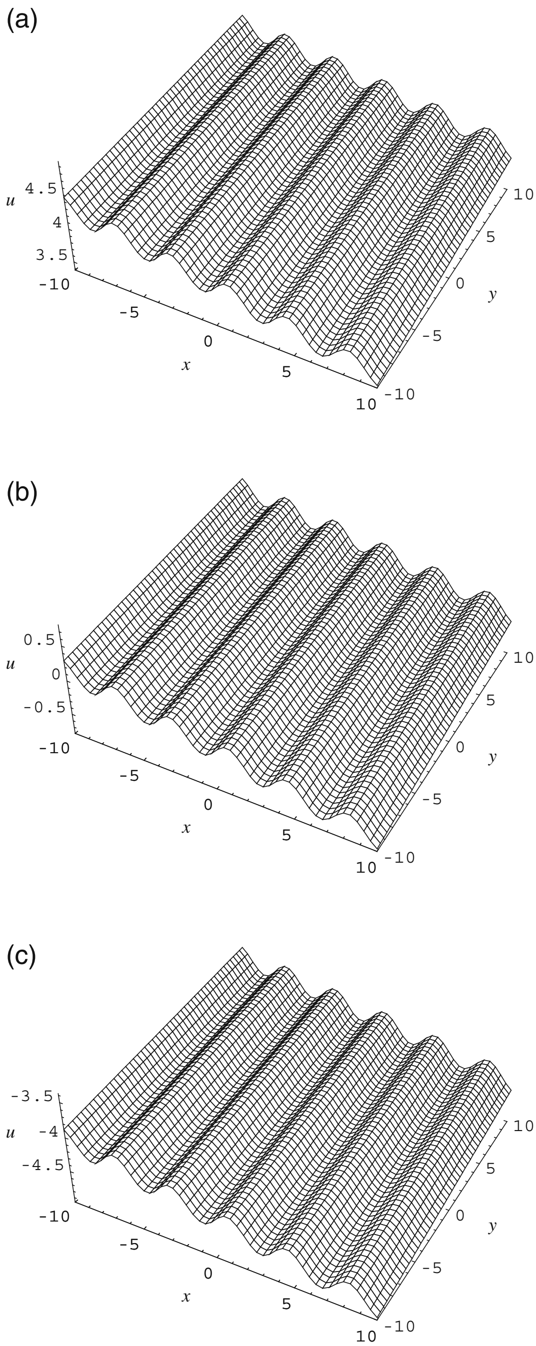

Figure 1. The asymptotical property of Jacobi doubly periodic solution (47) with (+) branch for parameters $a = 1$, $b = 0.1$, $c = 2$, $d = 0$, $m = 0.5$, $z = 0$ at different times: (**a**) $t = -5$, (**b**) $t = 0$, (**c**) $t = 5$.

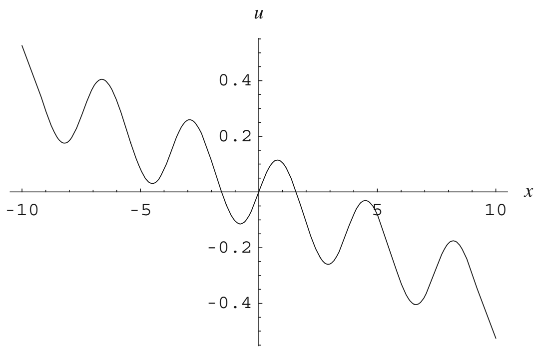

Figure 2. The asymptotical property of Jacobi doubly periodic solution (47) with (+) branch for parameters $a = 1$, $b = 0.1$, $c = 2$, $d = 0$, $m = 0.5$, $y = 0$, $z = 0$ at time $t = 0$.

We note that solutions (47)–(50) with an explicit linear function of ξ have not been obtained in [19,24–31]. To the best of our knowledge, solutions (47)–(50) are new. If we continue to select other appropriate special solutions [11] of $\eta'^2 = f\eta^4 + g\eta^2 + h$, we can also obtain other new Jacobi elliptic function solutions of eq. (1). However, we omit them here for simplicity.

It is well known that solitary wave solutions and Jacobi doubly periodic solutions are interesting and physically relevant. Here we take solution (47) as an example to further show its asymptotic properties by figures 1 and 2.

Remark 2. All solutions obtained above have been checked with *Mathematica* by putting them back into the original eq. (1).

4. Conclusion

In this paper, the transformed rational function method has been successfully extended to construct more types of exact solutions of the (3+1)-dimensional potential YTSF equation owing to the proposed new and more general rational functions. Some of the obtained exponential function solutions, hyperbolic function solutions, trigonometric function solutions, Jacobi elliptic function solutions and rational solutions, contain an explicit linear function of the independent variables involved in the potential YTSF equation. It may be important to explain some physical phenomena. The paper shows that the transformed rational function method provides more powerful mathematical tool for solving nonlinear partial differential equations.

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