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Spherical Kadomtsev–Petviashvili equation for dust acoustic waves with dust size distribution and two-charges-ions

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Abstract. The nonlinear dust acoustic waves in dusty plasmas with negative as well as positive ions and the combined effects of bounded spherical geometry and the transverse perturbation and the size distribution of dust grains are studied. Using the perturbation method, a spherical Kadomtsev–Petviashvili (SKP) equation that describes the dust acoustic waves is deduced.

Keywords. Dusty plasma; spherical Kadomtsev–Petviashvili equation; size distribution.

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1. Introduction

Nowadays, there is a great deal of interest in understanding different types of collective processes in dusty plasmas, which are very common in laboratory and astrophysical environments [1–7]. It is found that the presence of charged dust grains modifies the existing plasma wave spectra, and the dust dynamics may even introduce new eigenmodes in the plasma [8–17]. Rao *et al* [18] first predicted theoretically the existence of extremely low-phase velocity dust acoustic waves in unmagnetized dusty plasmas whose constituents were inertial charged dust grains and Boltzmann-distributed ions and electrons. These waves were reported experimentally and their nonlinear features investigated by Barkan *et al* [19]. Due to their importance, the solitary waves in unmagnetized plasma without geometry distortion and the dissipation effects have been extensively investigated and it was found that the solitary waves could be described by the Korteweg–de Vries (KdV) equation or Kadomtsev–Petviashvili (KP) equation [20–22]. However, recent theoretical studies indicate that the properties of solitary waves in bounded nonplanar spherical geometry differ from that in unbounded planar geometry [23]. In addition, it is well known that the transverse perturbation (which always exist in the higher-dimensional system) introduces anisotropy into the system and also modifies the structure and stability. The combined effects of both nonplanar geometry and the transverse perturbation on the dust acoustic waves (DAWs) have been considered by several authors, but most of them have paid attention to wave propagation in monosized dust grains because it is comparatively easy to study monosized grains [24–27]. But, the dust grains have different sizes in both space and laboratory plasmas [28]. Special complex plasmas containing electrons, both negative and positive ions, as well as dust grains appear in most space and laboratory plasmas. In Earth's mesosphere plasmas, the negative ions are often present [29]. Wang *et al* [30] studied the effect of negative ions on dust acoustic waves and found that in such a case only solitons with negative potential could exist.

Motivated by all the previous works, we tried to study a more realistic dusty plasma. In this paper, DAWs are investigated by including four effects, namely, (i) two-charges-ions, (ii) dust size distribution (by considering different dust size distributions, viz., power-law distribution), (iii) nonplanar spherical geometry and (iv) the transverse perturbation. A spherical Kadomtsev–Petviashvili equation is obtained analytically using the well-known reductive perturbation method [31].

2. Formulation

We consider dusty plasma with extremely massive, highly negatively charged dust grains, Boltzmann distributed electrons, positive and negative ions. We assume that there are N different-sized dust grains with masses m_j ($j = 1,2,...,N$). Charge neutrality at equilibrium requires $n_{i0+} = n_{i0-} + n_{e0} + \sum_{j=1} Z_{d0j} n_{d0}$, where $n_{i0+},\ n_{i0-},\ n_{e0}$ and n_{d0j} are the unperturbed positive ions, negative ions, electrons and jth dust grains number densities, respectively. Z_{d0j} is the unperturbed number of charges residing on the jth dust grain measured in units of electron charge. So, the usual dusty fluid equations governing the dust acoustic waves in spherical geometry are

$$\begin{aligned}
&\frac{\partial n_{dj}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 n_{dj} u_{dj}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(n_{dj} v_{dj}\right) + \frac{n_{dj} v_{dj}}{r}\cot g\theta = 0,\\
&\frac{\partial u_{dj}}{\partial t} + u_{dj}\frac{\partial u_{dj}}{\partial r} + \frac{v_d}{r}\frac{\partial u_{dj}}{\partial \theta} - \frac{v_{dj}^2}{r} = \frac{Z_{dj}}{m_{dj}}\frac{\partial \phi}{\partial r},\\
&\frac{\partial u_{dj}}{\partial t} + u_{dj}\frac{\partial v_{dj}}{\partial r} + \frac{v_d}{r}\frac{\partial v_{dj}}{\partial \theta} - \frac{u_{dj} v_{dj}}{r} = \frac{1}{r}\frac{Z_{dj}}{m_{dj}}\frac{\partial \phi}{\partial r},\\
&\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial \phi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\cot g\theta}{r^2}\frac{\partial \phi}{\partial \theta}\\
&\quad = \sum_{j=1}^{N} Z_{dj} n_{dj} + \mu_e \exp(\sigma_e \phi) - \mu_{i+}\exp(-\phi) + \mu_{i-}\exp(\sigma_- \phi)\,, \qquad (1)
\end{aligned}$$

where r, θ are the radial and angle coordinates and $u_{dj,} v_{dj}$ are the dust fluid velocity in r and θ directions, respectively. n_d, ϕ represent the dust density and the electrostatic potential. The variables t, r, n_d, u_d, v_d and ϕ are normalized to the dust plasma frequency $\omega_p^{-1} = \sqrt{4\pi n_{d0} Z_d^2 e^2/m_d}$, Debye radius $\lambda_{\mathrm{D}} = \sqrt{k_{\mathrm{B}} T_i/4\pi n_{d0} Z_d^2 e^2}$, unperturbed equilibrium dust density n_d, effective dust acoustic velocity $C_D = \sqrt{Z_d k_B T_i / m_d}$, and $k_B T/e$, respectively. Here we have denoted

$$\mu_e = n_{e0}/Z_d n_{d0} = 1/(\mu_+ - \mu_- - 1),$$
$$\mu_{i+} = n_{+0}/Z_d n_{d0} = \mu_+/(\mu_+ - \mu_- - 1),$$
$$\mu_{i-} = n_{-0}/Z_d n_{d0} = \mu_-/(\mu_+ - \mu_- - 1),$$
$$\mu_- = \frac{n_{i0-}}{n_{e0}}, \quad \mu_+ = \frac{n_{i0+}}{n_{e0}}, \quad \sigma_e = \frac{T_{i+}}{T_e}, \quad \sigma_- = \frac{T_{i+}}{T_{i-}}, \quad \mu = \frac{\mu_{i+}}{\mu_{i-}}.$$

To study the dynamics of small-amplitude dust-acoustic solitary waves, we use the so-called reductive perturbation method [30]. We can then expand the variables n_d, u_d and ϕ about the unperturbed states in power series of ε (ε is a small parameter). So let,

$$\begin{aligned} n_{dj} &= 1 + \varepsilon n_{dj1} + \varepsilon^2 n_{dj2} + \varepsilon^3 n_{dj3} + \cdots, \\ u_{dj} &= \varepsilon u_{dj1} + \varepsilon^2 u_{dj2} + \varepsilon^3 u_{dj3} + \cdots, \\ \phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \cdots. \end{aligned} \tag{2}$$

We can rewrite eqs (1) taking into account eqs (2) and the stretched coordinates $\xi = \varepsilon^{1/2}(r - v_0 t)$, $\tau = \varepsilon^{3/2} t$, $\eta = \varepsilon^{-1/2}\theta$, where v_0 is the wave velocity, to get the following system of equations to the lowest order in ε:

$$\begin{aligned} n_{d1j} &= -\frac{Z_{dj}}{v_0^2 m_{dj}} \phi_1, \\ u_{d1j} &= -\frac{Z_{dj}}{v_0 m_{dj}} \phi_1, \\ v_0^2 &= (1/(\mu_e \sigma_e + \mu_{i-}\sigma_{i-} + \mu_{i+})) \quad \left(\sum_{j=1}^{N} Z_{dj}^2 / m_{dj} \right), \\ \frac{\partial v_{dj}}{\partial \xi} &= -\frac{1}{v_0^2 \tau} \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi_1}{\partial \eta}. \end{aligned} \tag{3}$$

For the next order in ε we get the following set of equations:

$$\begin{aligned} &\frac{\partial n_1}{\partial \tau} - v_0 \frac{\partial n_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \frac{\partial \left(n_{dj} u_{dj} \right)}{\partial \xi} + \frac{1}{v_0 \tau} \frac{\partial v_1}{\partial \eta} + \frac{1}{v_0 \tau} \left(2v_1 + \frac{1}{\eta} v_1 \right) = 0, \\ &\frac{\partial u_1}{\partial \tau} - v_0 \frac{\partial u_2}{\partial \xi} + u_1 \frac{\partial u_1}{\partial \xi} - \sum_{j=1}^{N} \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi_2}{\partial \xi} = 0, \\ &\frac{\partial^2 \phi_1}{\partial \xi^2} - \frac{1}{2}(\mu_e \sigma_e^2 + \mu_{i-}\sigma_{i-}^2 - \mu_{i+}) \phi_1^2 - (\mu_e \sigma_e + \mu_{i-}\sigma_{i-} + \mu_{i+}) \phi_2 \\ &\qquad - \sum_{j=1}^{N} Z_{dj} n_{d2} = 0. \end{aligned} \tag{4}$$

Using eqs (2) and (3) and eliminating n_2, μ_2 and ϕ from (4), we obtain the SKP equation

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + \frac{1}{\tau}\phi_1\right] + C\left[\frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial \phi_1}{\partial \eta}\right] = 0,$$

$$A = -\frac{1}{2}\left[\frac{3}{v_0} + v_0^3\left(\mu_e \sigma_e^2 + \mu_{i-}\sigma_{i-}^2 - \mu_{i+}\right)\right],\ B = \frac{v_0^3}{2},\ C = \frac{1}{2v_0\tau^2}. \tag{5}$$

It is important to point out that, if the wave propagates without the transverse perturbation, the last term in the left-hand side of eq. (5) disappears and the SKP equation (5) reduces to the ordinary spherical KdV equation. We can find an exact solitary wave solution for the SKP equation (5) by using a suitable variable transformation. In eq. (5), the two terms with variable coefficient can be cancelled if we assume $\zeta = \xi - \frac{v_0}{2}\eta^2\tau$ and $\phi_1 = \Phi(\zeta, \tau)$.

Then the SKP equation (6) is reduced to the standard KdV equation

$$\frac{\partial \Phi}{\partial \tau} + A\Phi\frac{\partial \Phi}{\partial \xi} + B\frac{\partial^3 \Phi}{\partial \xi^3} = 0,\ \Phi(\zeta) = \frac{3U_0}{A}\mathrm{sech}^2\left[\sqrt{\frac{U_0}{4B}}(\zeta - U_0\tau)\right], \tag{6}$$

U_0 is a constant representing wave velocity. Thus we get an exact solitary wave solution of the SKP equation (6)

$$\Phi(\zeta) = \frac{3U_0}{A}\mathrm{sech}^2\left[\sqrt{\frac{U_0}{4B}}\left(\xi - \left(U_0 + \frac{v_0}{2}\right)\tau\right)\right]. \tag{7}$$

On the one hand it is clear that the amplitude and wave velocity of our solitary wave described by SKP equation (5) are exclusively determined by the parameters of the system and only depending on the initial conditions. Equation (7) indicates that the phase velocity of the solitary wave is angle-dependent in the phase. This means that the spherical wave described by eq. (5) will slightly deform as time goes on. The solution of eq. (7) gives

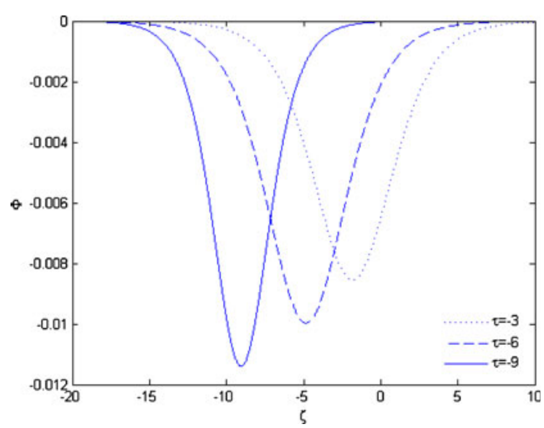

Figure 1. Electrostatic potential Φ vs. ξ for $\sigma_e = 0.2$ and $\mu = 1.50$.

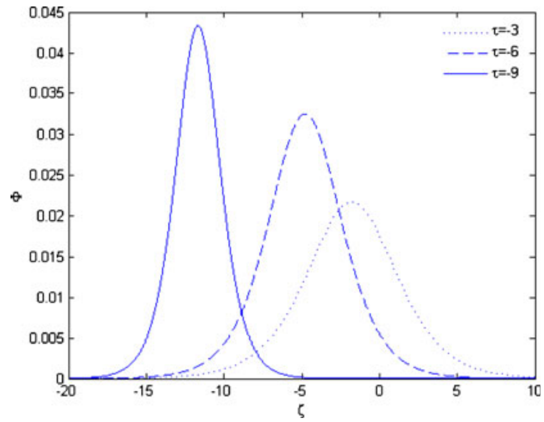

Figure 2. Electrostatic potential Φ vs. ξ for $\sigma_e = 0.1$ and $\mu = 1.50$.

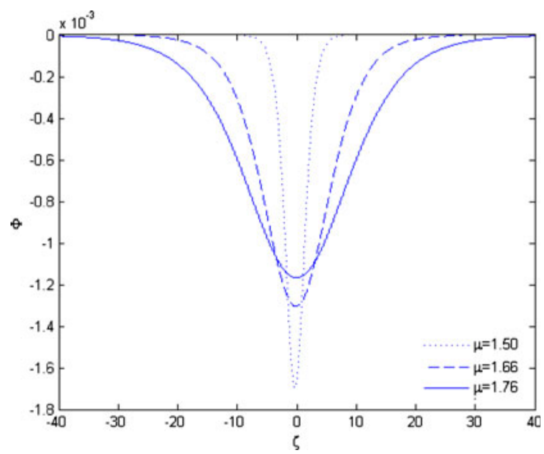

Figure 3. Electrostatic potential Φ vs. ξ for $\sigma_e = 0.35$.

us an additional information: a spherical soliton with constant amplitude can exist if the transverse perturbation is considered. Indeed, as indicated in figures 1–3, the amplitude increases with decreasing values of τ, but, the width is proportional to τ. On the other hand, it is clear that if a solitary wave exists in this system, the velocities of different dust particles will be different. They are inversely proportional to the square of the dust size. This result is the same as in ref. [32]. This suggests that different dust particles will have different velocities and different propagation distances in dusty plasmas. Furthermore, it must be noted that both types of solitons are admissible depending on the particle's temperature. Figure 1 shows rarefactive solitons (peak amplitude is negative) and figure 2 shows a compressive soliton corresponding to the positive peak amplitude. It is clear from figure 3 that as μ decreases the width too decreases, while the amplitude increases. This is an indication of negative ions which means that when negative ions are more (high density), the amplitude of the solitary wave will be greater.

We point out that with monosize grain the SKP equation exhibits only one type of solitons [29]. But solitary waves in two dimensions in dusty plasma with dust grain sizes distributed and negative ions leads to both types of solitons depending on various parameters.

3. Conclusion

In conclusion, let us recall, that in this note we have investigated the effect of different dust size distributions for solitary wave in dusty plasma with negative as well as positive ions, taking into account the combined effects of bounded nonplanar spherical geometry and the transverse perturbation. We use reductive perturbation technique and derive KP equation. A spherical Kadomtsev–Petviashvili (SKP) equation that describes the dust acoustic waves is deduced.

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