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Collisional effect on the Weibel instability in the limit of high plasma temperature

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Abstract. The Weibel instability (WI) of relativistic electron beam (REB) penetrating an infinite collisional plasma was studied in the following models: (i) REB model, where the total equilibrium distribution function $f_0(\vec{p})$ is approximated by nonrelativistic background electron and REB distribution functions and (ii) relativistic monoenergetic beam (RMB) model, where $f_0(\vec{p})$ is approximated by nonrelativistic background electrons and RMB distribution functions.

The dispersion equation including the effect of collision for a purely transverse mode describing each model was derived and solved analytically to obtain growth rates and conditions of excitation of the WI in the limit of high plasma temperature.

The purpose of this paper is to determine the effect of collision within the plasma on the growth rate of the WI for the two models. It was proved that the plasma collision frequency reduces the growth rate of WI at high plasma temperature. That is to say, collisions are inversely proportional to the growth rate. This leads to the important result: WI can be stabilized by increasing the plasma temperature.

Comparing the growth rate of WI in the two models (RMB and REB models), we came to the conclusion that growth rate of WI is more in the second case (REB case).

Keywords. Weibel instability (WI); relativistic electron beam (REB); relativistic monoenergetic beam (RMB); Krook collision term.

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1. Introduction

The Boltzmann equation with the relaxation model for the collision term has proved useful, particularly, for the weakly ionized plasma [1]. The electrostatic instability including the effect of collisions was considered previously by Vranjas *et al* [2].

The Weibel instability (WI) is most remarkable at the peak of the superintense femtosecond laser pulse [3]. Yang *et al* [4] showed how the WI stoped when the particles in the plasma started to gyrate in the magnetic fields (they generated magnetic trapping).

Nishikawa [5] calculated the emission from nonthermal electrons in the turbulent magnetic fields generated by the WI. This instability played an important role in generating the magnetic field in the laser-produced plasmas. It is purely growing electromagnetic instability [6]. Intense self-generated magnetic fields are produced by the mechanism of WI in undense plasmas. If this instability is excited, strong (gigagauss) magnetic fields can be generated from the magnetic field due to the electron thermal motion [6].

Lee and Lampe [7] reported the result of a nonlinear study with respect to the WI of an REB propagating in a plasma. Shokri and Ghorbanalilu [8] showed that the growth rate in the relativistic case was higher than that obtained for the non-relativistic case by a factor depending on the electric field strength and the plasma frequency.

Morse and Nielson [9] described and applied numerical simulation of the WI in one and two dimensions. Krall and Trivelpiece [10] calculated the dispersion relation for electromagnetic waves in a bi-Maxwellian collisionless plasma without drifts and in the absence of beams. They have solved it to obtain growth rate and condition of excitation of WI. We consider the same problem but with a bi-Maxwellian plasma with drifts, collisions and in the presence of beams.

Transverse instability is a rather general name which is used for some particular electromagnetic modes such as e.g., the WI. This instability is due to the anisotropy of the electron velocity distribution in an unmagnetized (or magnetized) plasma [11]. Davidson and Hammer [12] studied the wave instabilities which included transverse electromagnetic WI driven by kinetic energy anisotropy in an unmagnetized plasma (e.g., electromagnetic instabilities driven by thermal anisotropy or directed counter-streaming motion).

Zaki [13] studied the excitation of electromagnetic instability for REB penetrating an infinite collisionless plasma with drifts and in the presence of the beam. We consider the same problem but with an infinite collisional plasma.

The purpose of this paper is to investigate the effect of electron–ion collisions on the relativistic WI in the limit of high plasma temperature for two different models of the equilibrium distribution functions.

2. Mathematical model

2.1 *Excitation of the WI in REB model*

Let us consider a homogeneous, spatially infinite collisional plasma. Ion motions are neglected throughout this paper. The dynamics of the system under consideration is described by the relativistic Vlasov equation with a Krook collision term combined with Maxwell's equations [14]:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} + q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)\cdot\frac{\partial f}{\partial \vec{p}} = -\nu(f - f_0), \tag{1}$$

$$\mathrm{rot}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\operatorname{rot}\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi \vec{J}}{c}, \tag{3}$$

where CGS Gaussian units are used. In these equations, f is the electron distribution function at position $\vec{r}$ and momentum $\vec{p}$ at time t, ν is the effective collision frequency, q denotes the charge (including sign), c is the velocity of light, $\vec{J}$ is the current, $\vec{v}$ and $\vec{p}$ are related by $\vec{v} = \vec{p}/m\gamma$, $\gamma = (1 + p^2/(mc)^2)^{1/2}$, m is the electron rest mass, and $\vec{E}$ and $\vec{B}$ represent the electromagnetic field. Every quantity is expressed in terms of its complex Fourier amplitude in a field-free plasma.

Let us now consider a current neutral beam-plasma system. The REB propagates with the velocity $\vec{v}_{\mathrm{d}}^{\,\mathrm{b}}$ and the plasma return current flows with velocity $\vec{v}_{\mathrm{d}}^{\,\mathrm{p}}$. Here we assume the case in which an electromagnetic mode has $\vec{K}$ normal to $\vec{v}_{\mathrm{d}}^{\,\mathrm{b}}$, perturbed electric field $\vec{E}_{\vec{K}}$ parallel to $\vec{v}_{\mathrm{d}}^{\,\mathrm{b}}$, and perturbed magnetic field $\vec{B}_{\vec{K}}$ normal to both $\vec{v}_{\mathrm{d}}^{\,\mathrm{b}}$ and $\vec{E}_{\vec{K}}$.

Consider the model where the total equilibrium distribution function $f_0(\vec{p})$ is described by nonrelativistic background electron and relativistic electron beam (REB). One finds [15]

$$\begin{aligned} f_0(\vec{p}) = {}& \frac{n_{\mathrm{p}}}{2\pi m(\theta_x^{\mathrm{p}}\theta_y^{\mathrm{p}})^{1/2}} \exp\left[-\frac{(p_x + p_{\mathrm{d}}^{\mathrm{p}})^2}{2m\theta_x^{\mathrm{p}}} - \frac{p_y^2}{2m\theta_y^{\mathrm{p}}}\right] \\ & + \frac{n_{\mathrm{b}}}{2\pi m\gamma(\theta_x^{\mathrm{b}}\theta_y^{\mathrm{b}})^{1/2}} \exp\left[-\frac{(p_x - p_{\mathrm{d}}^{\mathrm{b}})^2}{2m\gamma\theta_x^{\mathrm{b}}} - \frac{p_y^2}{2m\gamma\theta_y^{\mathrm{b}}}\right]. \end{aligned} \tag{4}$$

Here θ_x and θ_y are the temperature components parallel to x and y directions, p_{d} is the drift momentum, superscripts p and b represent the plasma electron and the beam electron, respectively.

From the linearized Vlasov equation with the collision term (1) and using eqs (2)–(4) , we obtain the linear dispersion equation for a purely transverse mode as follows [10,16,17]:

$$1 + \chi_{\mathrm{p}}^{\mathrm{L}}(k,\omega) + \chi_{\mathrm{b}}^{\mathrm{L}}(k,\omega) = \frac{k^2c^2}{\omega^2}, \tag{5}$$

where

$$\chi_{\mathrm{p}}^{\mathrm{L}}(k,\omega) = -\frac{\omega_{\mathrm{p}}^2}{\omega(\omega + i\nu)}\left[1 - AW(\xi) - \frac{i\nu}{\omega}(A-1)W(\xi)\right], \tag{6}$$

$$\chi_{\mathrm{b}}^{\mathrm{L}}(k,\omega) = -\frac{\omega_{\mathrm{b}}^2}{\omega^2}[1 - BW(\eta)], \tag{7}$$

$$\omega_{\mathrm{p}}^2 = \frac{4\pi npq^2}{m}, \quad \omega_{\mathrm{b}}^2 = \frac{4\pi n_{\mathrm{b}}q^2}{m\gamma}, \quad A = \frac{((\theta_x^{\mathrm{p}}) + p_{\mathrm{d}}^{\mathrm{p}^2}/m)}{\theta_y^{\mathrm{p}}},$$

$$B = \frac{((\theta_x^{\mathrm{b}}) + p_{\mathrm{d}}^{\mathrm{b}^2}/m\gamma)}{\theta_y^{\mathrm{b}}}, \quad \xi = \frac{(\omega + i\nu)}{k\sqrt{\theta_y^{\mathrm{p}}/m}} \quad \text{and} \quad \eta = \frac{\omega}{k\sqrt{\theta_y^{\mathrm{b}}/m\gamma}}.$$

$\chi_{\mathrm{p}}^{\mathrm{L}}$ and $\chi_{\mathrm{b}}^{\mathrm{L}}$ are the plasma and beam linear susceptibilities, and for convenience $(p_{\mathrm{d}}^{\mathrm{p}})^2$ is written as $p_{\mathrm{d}}^{\mathrm{p}^2}$ throughout. In this investigation we have taken into consideration the collisional effect only in the background plasma. The function $W(z)$ is defined as

$$W(z) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \frac{y}{y - z} \exp(-y^2/2)\mathrm{d}y. \tag{8}$$

It is clear that, when the collision frequency $\nu = 0$, eq. (5) is in agreement with that obtained in ref. [13] for the collisionless plasma in the REB model.

The nature of the analytic solution depends, to a large extent, on the beam and the background plasma temperatures. We obtain the following analytical solution in the limit of high plasma temperature and high beam temperature limits, defined as $|\xi| < 1$ and $|\eta| < 1$ in eq. (5). In this situation the following asymptotic expansion for the function $W(x)$ can be used:

$$W(x) = i\left(\frac{\pi}{2}\right)^{1/2} \times \exp(-x^2/2) + 1 - x^2 + \cdots. \tag{9}$$

By using the expansion (9), eq. (5) can be approximately rewritten as

$$\begin{aligned} 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega + i\nu)} & \left[1 - A\left(1 + i\sqrt{\frac{\pi}{2}}\frac{(\omega + i\nu)}{kV_y^{\mathrm{p}}}\right)\right. \\ & \left. + \frac{i\nu}{\omega}(1 - A) \cdot \left(1 + i\sqrt{\frac{\pi}{2}}\frac{(\omega + i\nu)}{kV_y^{\mathrm{b}}}\right)\right] \\ & - \frac{\omega_{\mathrm{b}}^2}{\omega^2}\left[1 - B\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{kV_y^{\mathrm{b}}}\right)\right] \\ & = \frac{k^2c^2}{\omega^2}, \end{aligned} \tag{10}$$

where $V_y^{\mathrm{p}} = (\theta_y^{\mathrm{p}}/m)^{1/2}$ and $V_y^{\mathrm{b}} = (\theta_y^{\mathrm{b}}/m\gamma)^{1/2}$. In this paper, we are concerned about the Weibel-type electromagnetic instability, so that we put

$$\omega = i\delta, \tag{11}$$

where δ is the growth rate of the instability. We assume that δ and ν are all below the plasma frequency ω_{p}, $A \geq 1$ and $B \geq 1$. Inserting (11) into (10), we obtain the equation for $\delta_{(\mathrm{REB})}$ approximately as follows:

$$\delta_{(\mathrm{REB})} \cong \delta_0 - \left[\frac{(A-1)\cdot(\nu/\varsigma)}{(A/\varsigma) + (B/\varsigma)(\omega_{\mathrm{b}}/\omega_{\mathrm{p}})^2}\right], \tag{12}$$

where

$$\delta_0 = [(A-1)\omega_{\rm p}^2 + (B-1)\omega_{\rm b}^2 - k^2c^2] \cdot \left[\sqrt{\frac{\pi}{2}}\omega_{\rm p}^2 \cdot \frac{A}{\varsigma} + \sqrt{\frac{\pi}{2}}\omega_{\rm b}^2 \cdot \frac{B}{\varsigma}\right]^{-1} \tag{13}$$

and $\varsigma = kV_y^{\rm p}$, where δ_0 is the growth rate in the case of collisionless case [13] in the REB model. From eq. (12), it appeared that the effect of the plasma collision frequency is to reduce the growth rate of WI in the limit of high plasma temperature, i.e.,

$$\frac{\delta_{\rm (REB)}}{\delta_0} < 1. \tag{14}$$

WI is appeared when wave numbers satisfy

$$k\left[A - 1 - \frac{c^2k^2}{\omega_{\rm p}^2}\right] > \sqrt{\frac{\pi}{2}}B\left[\frac{(A-1)\cdot(\nu/\varsigma)}{(A/\varsigma) + (B/\varsigma)\cdot(\omega_{\rm b}/\omega_{\rm p})^2}\right]. \tag{15}$$

The maximum growth rate is obtained as

$$\begin{aligned}\delta_{{\rm (REB)}_{\rm max}} = {} & (8/27\pi)^{1/2}\left(\frac{\omega_{\rm p}}{c}\right) \\ & \times \left[\frac{(\theta_x^{\rm p} + \frac{p_{\rm d}^{{\rm p}^2}}{m})}{\theta_y^{\rm p}} + \left(\frac{\omega_{\rm b}}{\omega_{\rm p}}\right)^2 \cdot \frac{(\theta_x^{\rm b} + \frac{p_{\rm d}^{{\rm b}^2}}{m\gamma})}{\theta_y^{\rm b}} - \left(\frac{\omega_{\rm b}}{\omega_{\rm p}}\right)^2 - 1\right]^{3/2} \\ & \times \left[\frac{(\theta_x^{\rm p} + \frac{p_{\rm d}^{{\rm p}^2}}{m})}{\theta_y^{\rm p}\sqrt{\theta_y^{\rm p}/m}} + \left(\frac{\omega_{\rm b}}{\omega_{\rm p}}\right)^2 \frac{(\frac{p_{\rm d}^{{\rm b}^2}}{m\gamma})}{\theta_y^{\rm b}\sqrt{\theta_y^{\rm b}/m\gamma}}\right]^{-1} \\ & - \left[\frac{(A-1)\cdot(\nu/\varsigma)}{(A/\varsigma) + (B/\varsigma)\cdot(\omega_{\rm b}/\omega_{\rm p})^2}\right]\end{aligned} \tag{16}$$

at

$$\begin{aligned} & k^2_{{\rm (REB)}_{\rm max}} \\ & = \frac{1}{3}\cdot\left(\frac{\omega_{\rm p}}{c}\right)^2 \cdot \left[\frac{(\theta_x^{\rm p} + \frac{p_{\rm d}^{{\rm p}^2}}{m})}{\theta_y^{\rm p}} + \left(\frac{\omega_{\rm b}}{\omega_{\rm p}}\right)^2 \cdot \frac{(p_{\rm d}^{{\rm b}^2}/m\gamma)}{\theta_y^{\rm b}} - \left(\frac{\omega_{\rm b}}{\omega_{\rm p}}\right)^2 - 1\right]. \end{aligned} \tag{17}$$

In eq. (12), it is clear that when $\nu = 0$, we get $\delta_{\rm (REB)} \cong \delta_0$, i.e., eq. (12) is in agreement with that obtained in ref. [13] for the collisionless case in the REB model. It is also clear that when $\nu = 0$, eq. (16) is in agreement with that obtained in ref. [13] for the collisionless case in REB model.

2.2 *Excitation of the WI in RMB model*

When a parallel velocity spread is not included, e.g., by replacing the bi-Maxwellian $\alpha\exp[-(p_x - p_{\rm d}^{\rm b})^2/2m\gamma\theta_x^{\rm b}]$ in (4), by $\delta(p_x - p_{\rm d}^{\rm b})$, one finds the model where the total equilibrium distribution function $f_0(\vec{p})$ is approximated by nonrelativistic background electron and relativistic monoenergetic beam (RMB) [13] distribution functions. The equilibrium distribution function in the RMB model can be written as follows:

$$f_0(\vec{p}) = \frac{n_{\mathrm{p}}}{2\pi m(\theta_x^{\mathrm{p}}\theta_y^{\mathrm{p}})^{1/2}} \exp\left[-\frac{(p_x + p_{\mathrm{d}}^{\mathrm{p}})^2}{2m\theta_x^{\mathrm{p}}} - \frac{p_y^2}{2m\theta_y^{\mathrm{p}}}\right] + \frac{n_{\mathrm{b}}}{(2\pi m\gamma\theta_y^{\mathrm{b}})^{1/2}}\delta(p_x - p_{\mathrm{d}}^{\mathrm{b}}) \exp\left[-\frac{p_y^2}{2m\gamma\theta_y^{\mathrm{b}}}\right]. \tag{18}$$

From the linearized Vlasov equation with the collision term (1) and using eqs (2), (3) and (18), we obtain the linear dispersion equation for a purely transverse mode as follows:

$$1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega + i\nu)}\left[1 - AW(\xi) - \frac{i\nu}{\omega}(A-1)W(\xi)\right] - \frac{\omega_{\mathrm{b}}^2}{\omega^2}[1 - CW(\eta)] = \frac{k^2c^2}{\omega^2}, \tag{19}$$

where

$$\omega_{\mathrm{p}}^2 = \frac{4\pi n_{\mathrm{p}} q^2}{m}, \quad \omega_{\mathrm{b}}^2 = \frac{4\pi n_{\mathrm{b}} q^2}{m\gamma},$$

$$A = \frac{((\theta_x^{\mathrm{p}}) + p_{\mathrm{d}}^{\mathrm{p}^2}/m)}{\theta_y^{\mathrm{p}}}, \quad C = \frac{(p_{\mathrm{d}}^{\mathrm{b}^2}/m\gamma)}{\theta_y^{\mathrm{b}}},$$

$$\xi = \frac{(\omega + i\nu)}{k\sqrt{\theta_y^{\mathrm{p}}/m}} \quad \text{and} \quad \eta = \frac{\omega}{k\sqrt{\theta_y^{\mathrm{b}}/m\gamma}}.$$

In this investigation we have taken into consideration the collisional effect only in the background plasma.

Equation (19) is similar to that in ref. [10] for the collisionless WI of a bi-Maxwellian plasma without drifts and beams, in the isotropic plasma (i.e., $\theta_x^{\mathrm{p}} = \theta_y^{\mathrm{p}}$) and for low plasma temperature (i.e., at $|\xi| \gg 1$) except that in ref. [10], we have: $\omega^2 \approx \omega_{\mathrm{p}}^2 + k^2c^2$, the familiar light waves in plasma.

Following the same procedure applied in the REB model, we can get the maximum growth rate at collisional WI of a bi-Maxwellian plasma in RMB model in the form

$$\delta_{(\mathrm{RMB})_{\max}} = (8/27\pi)^{1/2}\left(\frac{\omega_{\mathrm{p}}}{c}\right) \times \left[\frac{(\theta_x^{\mathrm{p}} + \frac{p_{\mathrm{d}}^{\mathrm{p}^2}}{m})}{\theta_y^{\mathrm{p}}} + \left(\frac{\omega_{\mathrm{b}}}{\omega_{\mathrm{p}}}\right)^2 \cdot \frac{(p_{\mathrm{d}}^{\mathrm{b}^2}/m\gamma)}{\theta_y^{\mathrm{b}}} - \left(\frac{\omega_{\mathrm{b}}}{\omega_{\mathrm{p}}}\right)^2 - 1\right]^{3/2} \times \left[\frac{(\theta_x^{\mathrm{p}} + \frac{p_{\mathrm{d}}^{\mathrm{p}^2}}{m})}{\theta_y^{\mathrm{p}}\sqrt{\frac{\theta_y^{\mathrm{p}}}{m}}} + \left(\frac{\omega_{\mathrm{b}}}{\omega_{\mathrm{p}}}\right)^2 \frac{(p_{\mathrm{d}}^{\mathrm{b}^2}/m\gamma)}{\theta_y^{\mathrm{b}}\sqrt{\theta_y^{\mathrm{b}}/m\gamma}}\right]^{-1}$$

$$-\left[\frac{(A-1)\cdot(\nu/\varsigma)}{(A/\varsigma)+(C/\varsigma)\cdot(\omega_{\mathrm{b}}/\omega_{\mathrm{p}})^2}\right]. \tag{20}$$

Also, eq. (20) is similar to that in the collisionless WI of bi-Maxwellian plasma without drifts and beams [10] except that in ref. [10], we have

$$\delta_{\max} = (8/27\pi)^{1/2}\cdot\left(\frac{\omega_{\mathrm{p}}}{c}\right)\cdot\left(\frac{\theta_x^{\mathrm{p}}}{\theta_y^{\mathrm{p}}}-1\right)^{3/2}\cdot\sqrt{\theta_y^{\mathrm{p}}/m}\cdot\left[\frac{\theta_y^{\mathrm{p}}}{\theta_x^{\mathrm{p}}}\right]. \tag{21}$$

It is clear that at the beam temperature, when $\nu = 0$ and the component parallel to x-direction is low compared to the component parallel to y-direction, i.e., $\theta_x^{\mathrm{b}} \to 0$ (e.g., Zayed and Kitsenko [18]) we find that eq. (16) is in agreement with that obtained in the collisionless case in RMB model [13]. Equation (16) in the REB model is similar to eq. (20) obtained for a maximum growth rate in the RMB model, except that in eq. (20) the value of $\theta_x^{\mathrm{b}} \to 0$.

From eqs (16) and (20), we find that

$$\frac{\delta_{(\mathrm{REB})_{\max}}}{\delta_{(\mathrm{RMB})_{\max}}} > 1 \tag{22}$$

i.e., comparing the growth rate of WI in RMB and REB models, we come to the conclusion that growth rate of the collisional plasma is more in the second model.

3. Conclusion

The Weibel (or filamentation [5]) instability for REB penetrating an infinite collisional plasma was studied. From the results of the analytic solution, it appeared that the effect of the plasma collision frequency was to reduce the growth rate of WI at high plasma temperature (as shown from inequality (14)). That is to say, collisions are inversely proportional to the WI growth rate. This leads to the important result that WI can be stabilized by increasing the plasma temperature.

On comparing the growth rate of WI in RMB and REB models, we came to the conclusion that growth rate of the collisional plasma is more in the second case (as shown in inequality (22)).

Results obtained in REB model (e.g. fast ignition [15]) agree with those in RMB model (e.g., focussed beam [17]) provided that $\theta_x^{\mathrm{b}} \to 0$ (e.g., eq. (16) in the REB model is similar to eq. (20) obtained for a maximum growth rate in the RMB model, except that in eq. (20) the value of $\theta_x^{\mathrm{b}} \to 0$). Finally, to investigate the electromagnetic instabilities in a dense plasma, one has to take into account the collisional effect as well as the ion motion.

References

[1] M Brambilla, *Kinetic theory of plasma waves: Homogeneous plasma* (Oxford Science Publications, New York, 1998)

[2] J Vranjas, S Poedts and B P Pandey, *Proc. of the 2007 ICTP Summer College on Plasma Phys.* (Trieste, Italy, 30 July–24 August 2007)

[3] V P Krainov, *Annual Moscow Workshop, Physics of Nonideal Plasmas* (Moscow, 3–4 Dec. 2002)
[4] T B Yang, J Arons and A B Langdon, *Phys. Plasmas* **1**, 3059 (1994)
[5] K Nishikawa, *International Workshop on Magnetohydrodynamic Accretion Flow and Jets* (YIP, Kyote, 25–27 Jan. 2005)
[6] A Sid, A Ghzal, A Soudani and M Bekhouche, *Plasma and Fusion Research* **5**, 007 (2010)
[7] R Lee and M Lampe, *Phys. Rev. Lett.* **31**, 1390 (1973)
[8] B Shokri and M Ghorbanalilu, *Phys. Plasmas* **11**, 5398 (2004)
[9] R L Morse and C W Nielson, *Phys. Fluids* **14**, 830 (1971)
[10] N A Krall, and A W Trivelpiece, *Principles of plasma physics* (McGraw-Hill Inc, New York, 1973)
[11] F F Cap, *Handbook on plasma instabilities* (Academic Press, New York, 1978) Vol. 2
[12] R C Davidson and D A Hammer, *Phys. Fluids* **15**, 317 (1972)
[13] N G Zaki, *Egypt. J. Phys.* **32(1)**, 59 (2001)
[14] A Bret and C Deutsch, *Phys. Plasmas* **12**, 082109 (2005)
[15] C Deutsh, A Bret and P Fromy, *Laser and particle beams* **23**, 5 (2005)
[16] R C Davidson, *Basic plasma physics: Selected chapters* edited by A A Galeev and R N Sudan, *Handbook of plasma physics* edited by M N Rosenbluth and R Z Sagdeev (Elsevier Science Publishers B.V., 1989)
[17] T Okada, T Yabe and K Niu, *J. Phys. Soc. Jpn* **43**, 1042 (1977)
[18] K E Zayed and A B Kitsenko, *Plasma Phys.* **10**, 673 (1968)