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Switching behaviour of a nonlinear Mach–Zehnder interferometer

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Abstract. In the present paper, a detailed investigation on the switching behaviour of a nonlinear Mach–Zehnder interferometer (NMZI) has been carried out using beam propagation method (BPM). A thorough investigation on input vs. output characteristic has been carried out by varying different parameters like length of the arms, refractive index of the linear/nonlinear arm, wavelength of the input beams and nonlinear coefficient of the material of the nonlinear arm. The input vs. output characteristic has also been investigated by shifting the balance point of the NMZI. The present paper provides a physically intuitive understanding of the effect of change in different parameters of the NMZI on its switching behaviour.

Keywords. Mach–Zehnder interferometer; Kerr effect; self-phase modulation; all-optical switching.

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0. Introduction

Mach–Zehnder Interferometer is one of the most important components in integrated optical circuits and has been extensively utilized for optical devices such as switches, modulators, and so on. In order to control the optical output, extra phase shifts are added to one of its arms, for example, by changing the refractive index [1], by changing the optical-path length [2,3], by thermo-optic effect [4], and by using liquid crystals [5]. The NMZI using the optical Kerr effect seems to be more suitable for faster transmission systems because of its ultrafast response. Proposals for a variety of applications using MZI-based devices are existing in the literature, for example, multiplexing and modulation [6], low-loss combiner [7], WDM applications [8], optical power limiter [9] etc.

In addition to the above-mentioned, NMZI structure has also been investigated for all-optical logic [10–18] due to its application in telecommunication systems. All-optical AND gate can perform the bit-level functions such as address recognition, packet-header modification, and data-integrity verification. The all-optical

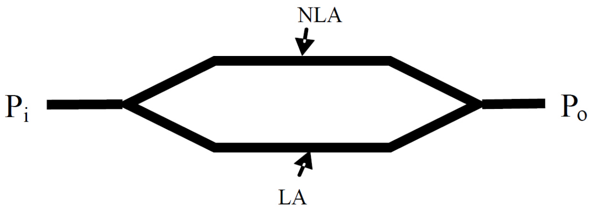

Figure 1. Schematics of a nonlinear MZI switch.

XOR gate is a key technology to implement primary systems for binary address and header recognition, binary addition and counting, decision and comparison, encoding and encryption, and pattern matching.

Although numerous papers on a variety of applications are existing on NMZI-based devices, a detailed analysis on the effect of changing geometrical (length) and physical (refractive index, nonlinear coefficient etc.) parameters of an NMZI on its output characteristic is missing in the literature. In view of it, a detailed investigation on the switching behaviour of a NMZI (with its one arm of a Kerr nonlinear material) has been presented in this paper. Using our own written code of beam propagation method (BPM), a thorough investigation on input vs. output has been carried out by varying different parameters of the NMZI such as length of the arms, refractive index of the linear/nonlinear arm, wavelength of the input beams and nonlinear coefficient of the material of the nonlinear arm.

The present paper provides a physically intuitive understanding of the effect of change in different parameters of the NMZI on its switching behaviour.

1. Theory and numerical investigation

The NMZI considered here is with one arm (NLA) made up of a Kerr nonlinear material and the other arm (LA) of a linear material as shown in figure 1. P_{i} is the input port of the NMZI and P_{o} is the output port.

When an input beam is launched at P_{i}, it splits into two equal parts. One part propagates through the NLA and its counterpart through the LA. The part propagating through the NLA experiences self-phase modulation (SPM). If the split parts of the beam recombine with 'in-phase' condition, whole of the input beam reaches at P_{o} and if those recombine with 'out-of-phase' condition, the input beam does not reach at P_{o} giving zero output.

We consider a laser (coherent) beam injected into the MZI, and the beam, in the paraxial approximation, could be described by the equation [19]

$$\frac{\partial E}{\partial z} = -i\frac{1}{2kn_0}\frac{\partial^2 E}{\partial x^2} - ik\left[n\left(x,z\right) - n_0\right]E. \tag{1}$$

Here, E is the transverse field envelop $[=\sqrt{I}\exp(-x^2/2x_0^2)]$ of the beams, I is the axial intensity, x is the transverse coordinate, x_0 is the width of the input beam, $k = 2\pi/\lambda$ is the free space propagation constant and n_0 is the refractive index of the substrate on which NMZI of refractive index $n(x,z)$ is fabricated. $n(x,z)$ is expressed as

$$n(x,z) = \begin{cases} n_{\mathrm{L}}(x,z), & \text{for LA,} \\ n_{\mathrm{NL}}(x,z) + \Delta n(x,z), & \text{for NLA,} \end{cases} \tag{2}$$

where $n_{\mathrm{L}}(x,z)$ is the refractive index of the linear arm, $n_{\mathrm{NL}}(x,z)$ is the constant part of the refractive index of the nonlinear arm and $\Delta n(x,z)$ is its intensity-dependent change in refractive index which is expressed as [19]

$$\Delta n(x,z) \approx n_2(|Et|^2). \tag{3}$$

Here n_2 is the nonlinear coefficient of the material of NLA. In the present paper, input vs. output characteristic of the NMZI has been obtained by solving eq. (1) using split step Fourier method or beam propagation method (BPM) [20].

1.1 *The NMZI*

The chosen parameters of the NMZI to examine the NMZI are core width $2a = 8\,\mu$m, separation of two waveguides is 25 μm, branching angle at the combining/splitting region $\theta_{\mathrm{B}} = 1.2^{\circ}$ and the chosen length (Z) of LA and NLA is 1200 μm. Nonlinear coefficient n_2 of NLA material is considered to be equal to 2×10^{-14} m^2/W.

Before numerical investigations, it is worth to explain the meaning of balanced and imbalanced NMZI

1.1.1 *Initially balanced NMZI*

By initially balanced NMZI, we mean that the optical path lengths of the linear and the nonlinear arms are equal for vanishingly small optical powers. In other words, the optical path lengths of the two arms are equal when nonlinearity of the nonlinear arm is not effective. Obviously, an initially balanced NMZI is transparent for a vanishingly small input power.

By considering $\Delta_{\mathrm{L}} = [n_{\mathrm{L}}(x,z) - n_0] = 0.3\%$ (of the LA), an initially balanced NMZI at $\lambda = 1.55$ μm may be obtained by using the mentioned parameters as shown in figure 2.

1.1.2 *Initially imbalanced NMZI*

Optical path lengths of the linear and the nonlinear arms of an initially imbalanced NMZI differ by half of the wavelength at vanishingly small optical powers, and hence, NMZI becomes opaque for vanishingly small optical powers. Initially imbalanced NMZI (at $\lambda = 1.55$ μm) may be obtained using the mentioned parameters and by considering $\Delta_{\mathrm{L}} = [n_{\mathrm{L}}(x,z) - n_0] = 0.38\%$ (as shown in figure 3).

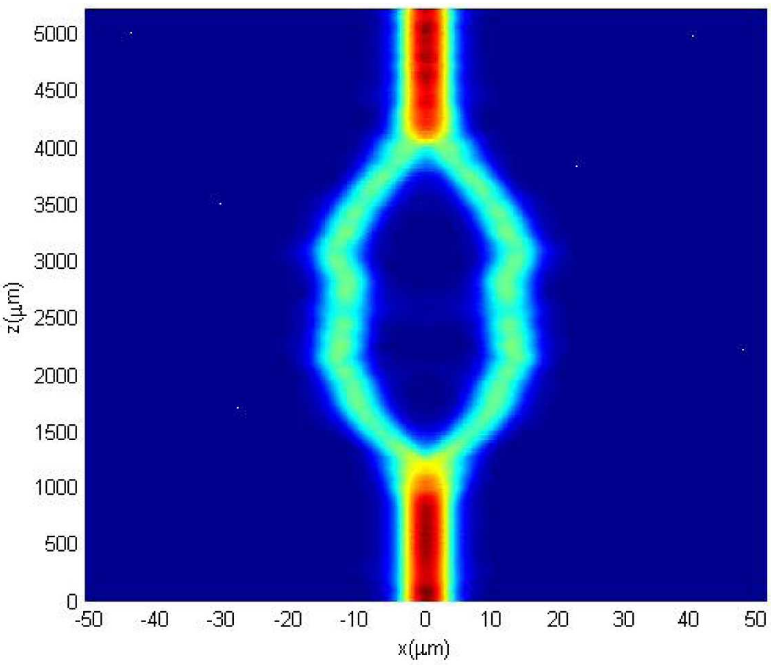

Figure 2. An initially balanced NMZI is transparent for a vanishingly small input power.

1.2 *Effect of input power*

To examine the effect of input power (I) on the output power (I_o) of the NMZI, Gaussian beam has been considered at the input and transmission coefficient T ($= I_o/I$) of the NMZI is determined by varying I in the range (0–35) $\times$ 10^{10} W/m^2.

It is worth mentioning here that I_o could be estimated by knowing the beam's axial electric field at the output of the NMZI (by solving eq. (1)) and by assuming that a Gaussian mode with the same axial electric field would couple if any single-moded waveguide is connected at the output of the NMZI.

Solid line of figure 4 shows variation of the transmission coefficient T of the mentioned balanced NMZI with respect to the input power I and figure 5 shows the variation of the output power I_o of the NMZI with the input power I. It may be mentioned here that for a given NMZI and wavelength, axial intensity is proportional to the power of the beam and therefore we have crudely used the term power in place of the axial intensity in this paper.

As the NMZI is initially balanced, for vanishingly small input power I, the split parts of the input beam recombine with 'in-phase' condition (or with zero phase difference) at the NMZI output and give maximum T. When the input power is gradually increased, the magnitude of self-phased modulation (SPM) of the part propagating through NLA increases and the balanced NMZI gradually turns into

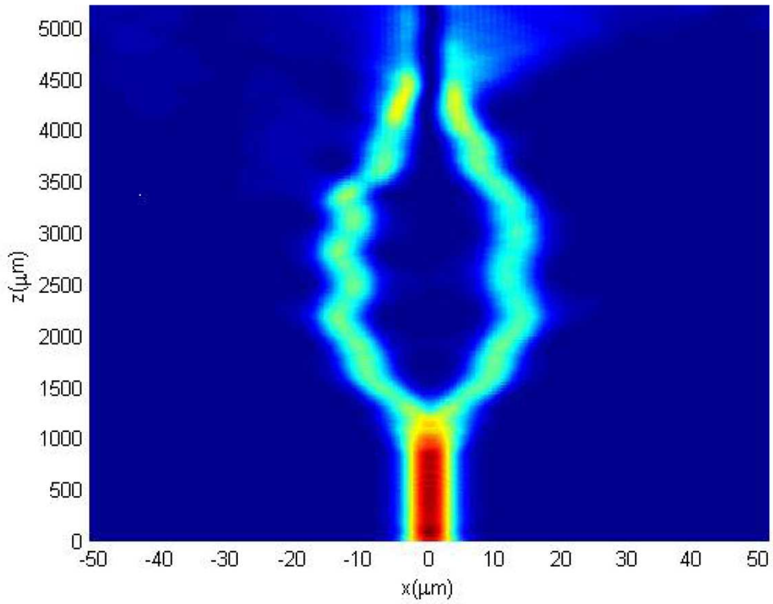

Figure 3. An initially imbalanced NMZI is opaque for a vanishingly small input power.

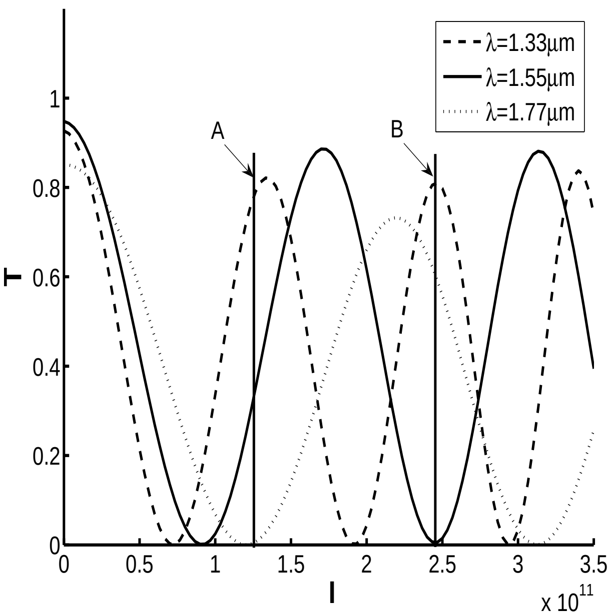

Figure 4. Variation of transmission coefficient of the balanced NMZI with the input power for wavelengths 1.33, 1.55 and 1.77 μm.

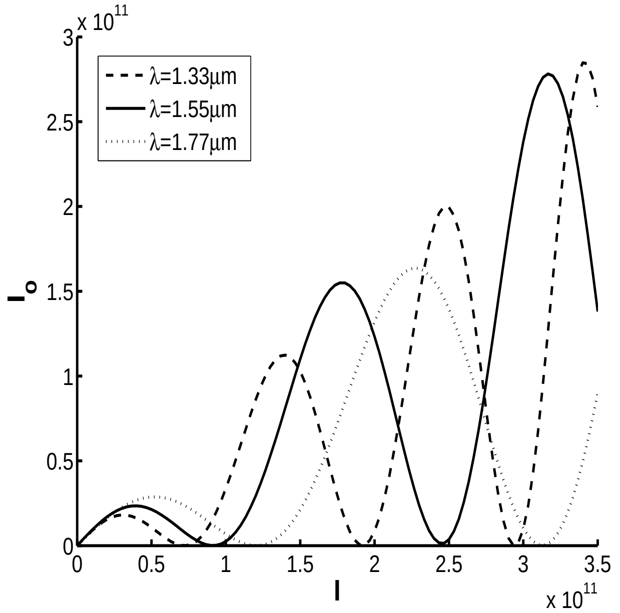

Figure 5. Variation of output power I_o of the balanced NMZI with the input power I for wavelengths 1.33, 1.55 and 1.77 μm.

the imbalanced state (split parts of the input beam recombine with 'out-of-phase' condition or with a phase difference of π) resulting in minimum T. If the optical power is further increased, T again increases and becomes unity at the optical power that generates a phase difference of 2π between the split parts of the recombining beam. In other words, the transmission coefficient oscillates between maxima and minima with the increase in I.

It may be pointed out here that ideally T should oscillate between 0 and 1. However, it is not so in the figures of this paper due to numerical errors.

1.3 *Effect of wavelength*

To understand the effect of changing wavelength, variation of T with I has been plotted in figure 4 for three different wavelengths, i.e., $\lambda = 1.55$, 1.33 and 1.77 μm. It is obvious from the figure that for lower wavelengths, transmission coefficient changes more rapidly when the input power is gradually increased.

Variation of the transmission coefficient of the imbalanced NMZI with respect to the input power has been shown in figure 6. Many features of the curves of this figure remain the same as in figure 4, i.e., T oscillates between a maximum and a minimum; moreover, T varies rapidly for lower wavelength. However, the 'initially imbalanced state' is wavelength-dependent, i.e., a NMZI which is imbalanced at 1.55 μm will not be perfectly imbalanced at 1.33 or 1.77 μm. The variation of I_o with I of the initially balanced and imbalanced NMZI are shown in figures 5 and 7 respectively.

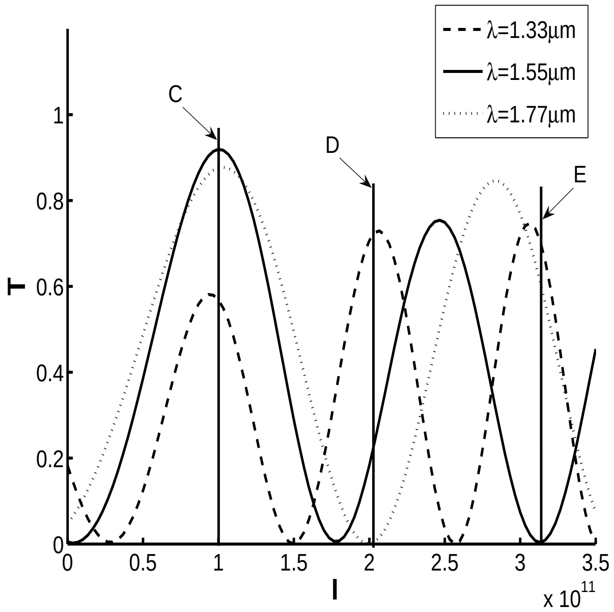

Figure 6. Variation of transmission coefficient of the imbalanced NMZI with the input power I for wavelengths 1.33, 1.55 and 1.77 μm.

1.4 *Wavelength- and power-dependent transparency*

It can be clearly seen in figures 4 and 6 that transparency of a NMZI is wavelength- and power-dependent. For example, at I corresponding to line A, the NMZI of figure 4 is transparent to 1.33 μm while it is opaque to 1.77 μm and for I corresponding to line B, it is transparent to 1.33 μm and opaque to 1.55 μm. Similarly, the NMZI of figure 6 is transparent to all three wavelengths, i.e., 1.33, 1.55 and 1.77 μm for I corresponding to line C. The same NMZI is transparent to 1.33 μm and opaque to 1.77 μm for I corresponding to line D. Moreover, it is opaque to 1.55 μm and transparent to 1.33 μm for I corresponding to line E.

1.5 *Effect of NMZI length*

To examine the effect of changing length of a NMZI on it switching behaviour, initially balanced NMZI has been considered and T with varying I have been obtained for different arm lengths of NMZI ($L = 1100$, 1200 and 1300 μm) as shown in figure 8. The $I_{\rm o}$ vs. I curves are shown in figure 9.

The same for initially imbalanced NMZI are shown in figures 10 and 11.

It is clearly seen in the figures that T changes more rapidly with I for longer arms of NMZI. In other words, the NMZI becomes more sensitive to the optical input with increase in its length. It is due to the increased light–matter interaction in lengthier nonlinear arm resulting in increased SPM.

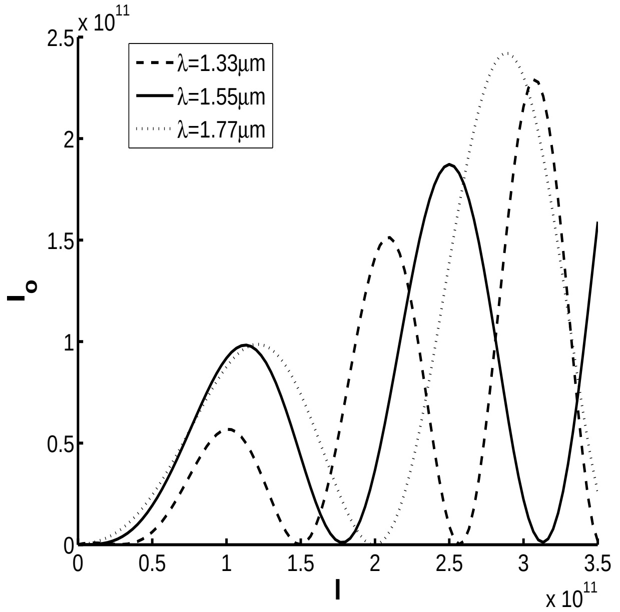

Figure 7. Variation of output power I_o of the imbalanced NMZI with the input power I for wavelengths 1.33, 1.55 and 1.77 μm.

1.6 *Effect of nonlinearity coefficient*

To examine the effect of changing the nonlinearity coefficient of nonlinear arm of a NMZI on its switching behaviour, different nonlinearity coefficients have been considered for the balanced and imbalanced NMZI. Figure 12 shows the variation of T with I of the initially balanced NMZI. I has been varied in the range (0–35) $\times$ 10^{10} W/m^2. Dashed, solid and dotted curves show T vs. I for nonlinearity coefficients equal to 1.5×10^{-14}, 2.0×10^{-14} and 2.5×10^{-14} m^2/W respectively. It is obvious from the figure that for higher nonlinearity coefficient, transmission coefficient changes more rapidly with the input power. The corresponding variation of I_o with I is shown in figure 13.

The same for initially imbalanced NMZI are shown in figures 14 and 15. In this case also for higher nonlinearity coefficient, transmission coefficient changes more rapidly with the input power.

1.7 *Effect of shifting the balance point*

Lastly, we investigate the effect of shifting the balance point of a NMZI on its switching behaviour.

For this purpose, initially balanced NMZI (at $\lambda = 1.55$ μm) of figure 4 is chosen with $\Delta_L = 0.30\%$, the NMZI is initially balanced as shown by the solid line plot of T with I in figure 16. The balance point of the mentioned NMZI could be slightly shifted on either side by changing the refractive index of the linear arm, i.e. by

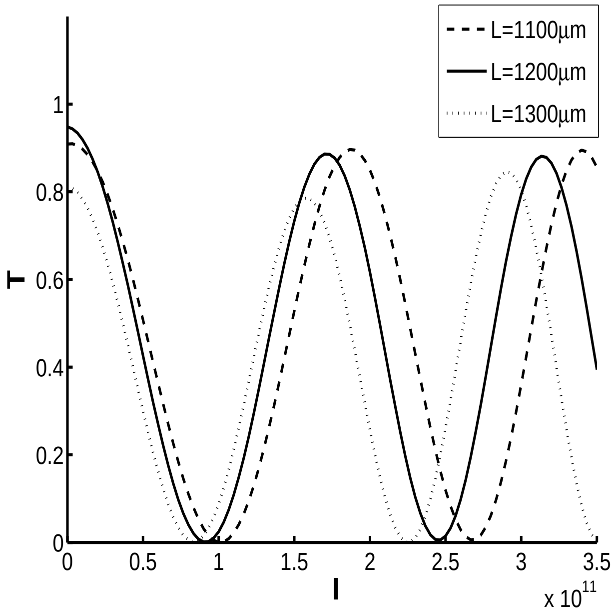

Figure 8. T vs. I curves of the initially balanced NMZI for NMZI's arm lengths $L = 1100$, 1200 and 1300 μm.

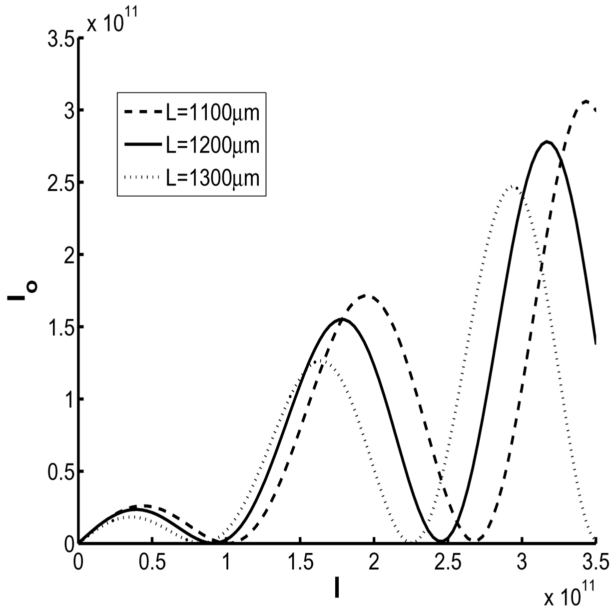

Figure 9. I_o vs. I curves of the initially balanced NMZI for NMZI's arm lengths $L = 1100$, 1200 and 1300 μm.

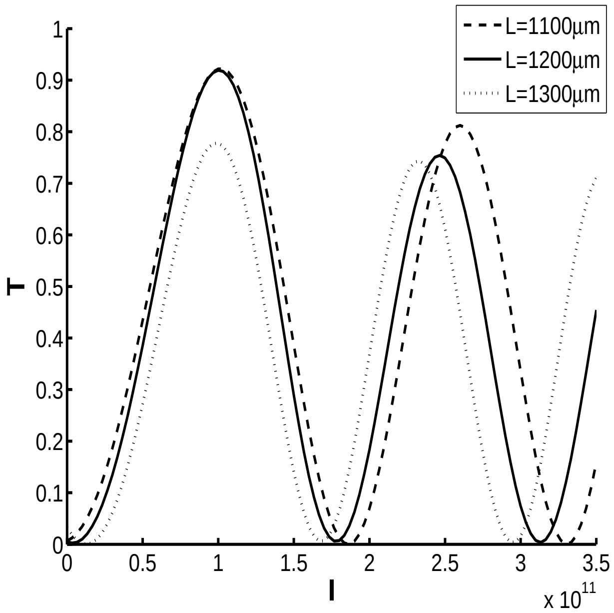

Figure 10. T vs. I curves of the initially imbalanced NMZI for NMZI's arm lengths $L = 1100$, 1200 and 1300 μm.

changing Δ_L. By increasing Δ_L, the balance point could be shifted towards right, while it shifts towards left by decreasing the Δ_L as shown in figure by dotted and dashed curves respectively. The corresponding variation of I_o with I of the NMZI is shown in figure 17.

The same exercise has been carried out for the initially imbalanced NMZI as shown in figures 18 and 19.

On comparing figures 16 with 18 and 17 with 19, one can notice some interesting points such as T with I plot of the initially balanced NMZI is just 180° out-of-phase to T with I plot of the initially imbalanced NMZI (see figures 16 and 18). However, the nature of I_o with I plot is quite different (see figures 17 and 19). It is of extreme importance to note that the switching-on or switching-off power of the NMZI could be manipulated by shifting the balance point. For example, the switching-on power of the NMZI corresponding to dotted curve of figure 19 is more than that of NMZI corresponding to the solid and dashed curves. This fact could give tremendous liberty to a fabricator to adjust the operating power levels to some extent when geometry and the nonlinear material of the NMZI are fixed.

Lastly, we must add that the present work is applicable where the response time of the nonlinear medium is much smaller than the pulse duration. For short duration pulses, where response time of the nonlinear medium is comparable to the duration of the pulses, the same NMZI gives different switching behaviour. Moreover, to analyse the reliable performance and the tolerance of a device, investigation of the effect of intensity fluctuations when the parameters are very close to the threshold of nonlinearity are important. These studies require extensive work and would be taken up in future.

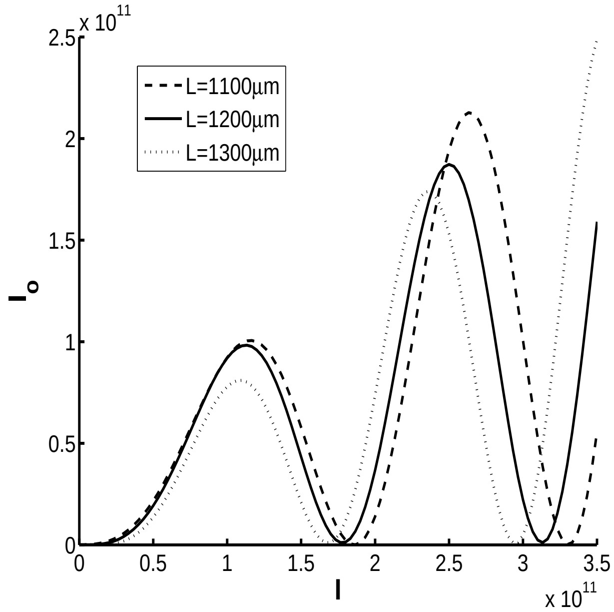

Figure 11. I_o vs. I curves of the initially imbalanced NMZI for NMZI's arm lengths $L = 1100$, 1200 and 1300 μm.

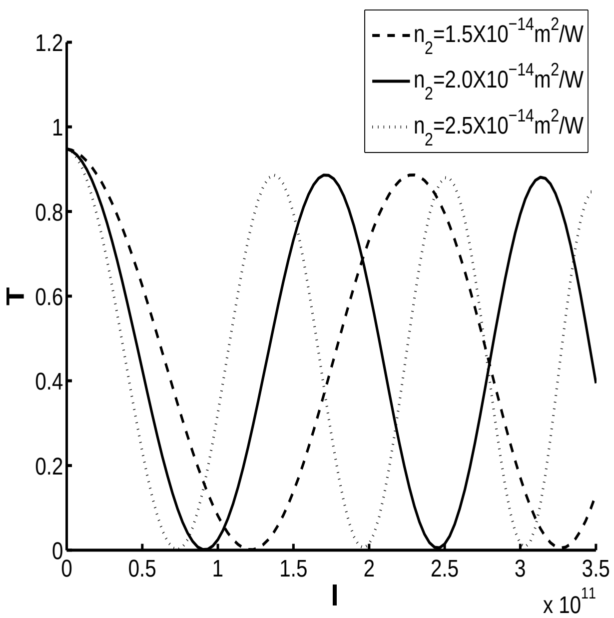

Figure 12. Variation of T with I of the initially balanced NMZI for different nonlinearity coefficients of the nonlinear arm.

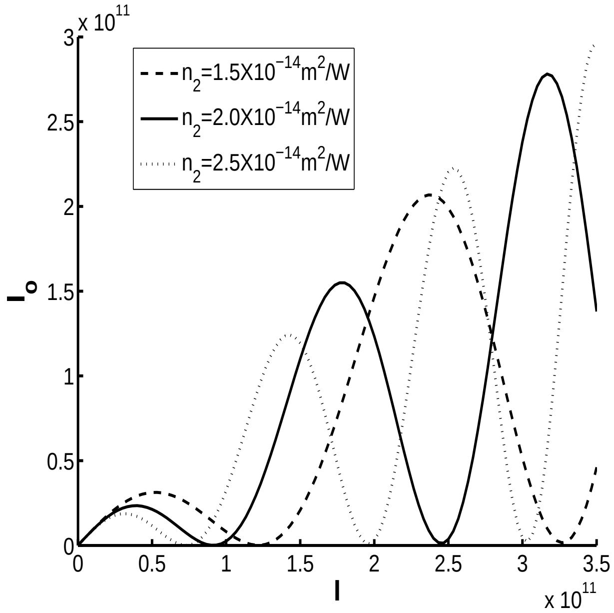

Figure 13. Variation of I_o with I of the initially balanced NMZI for different nonlinearity coefficients of the nonlinear arm.

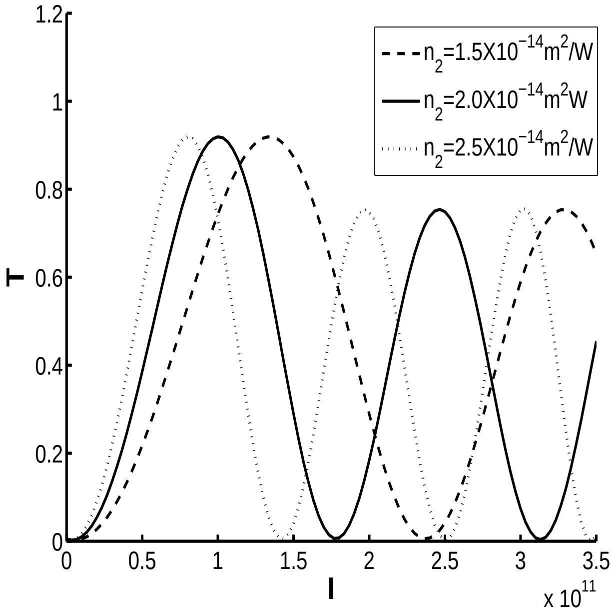

Figure 14. Variation of T with I of the initially imbalanced NMZI for different nonlinearity coefficients of the nonlinear arm.

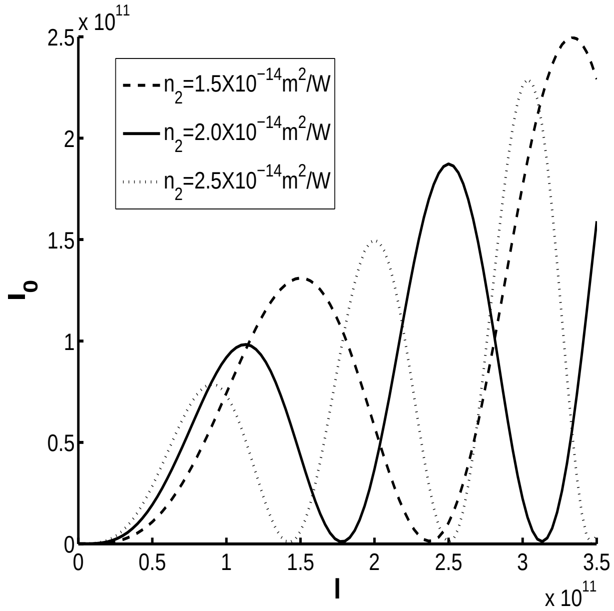

Figure 15. Variation of I_o with I of the initially imbalanced NMZI for different nonlinearity coefficients of the nonlinear arm.

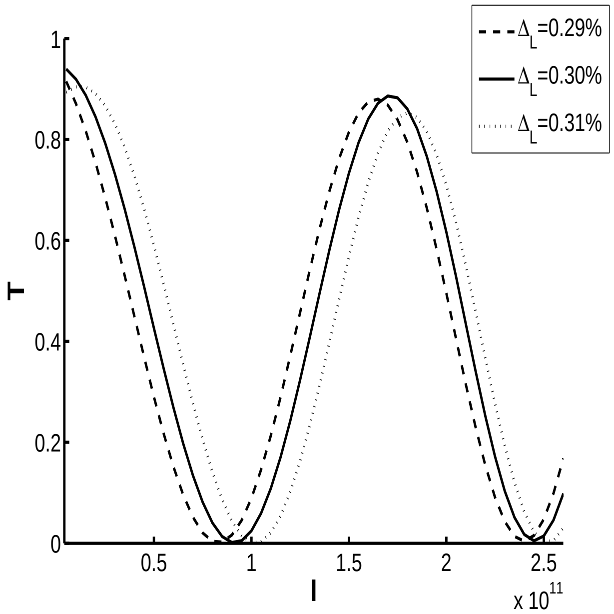

Figure 16. Variation of T with I of the initially balanced NMZI for shifted balance points.

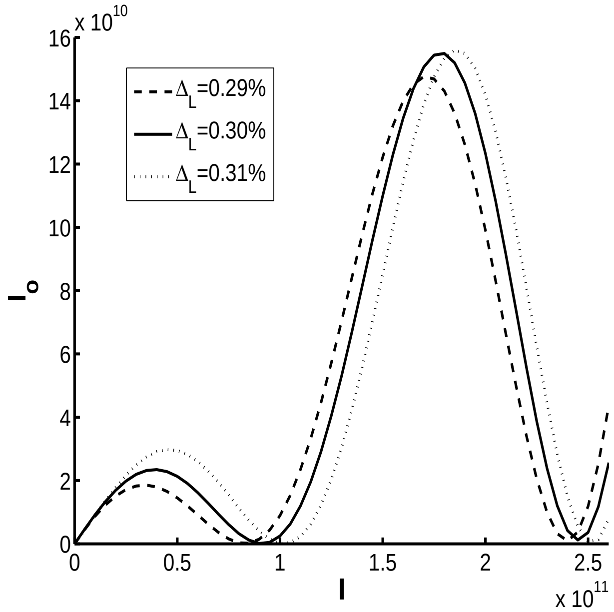

Figure 17. Variation of I_o with I of the initially balanced NMZI for shifted balance points.

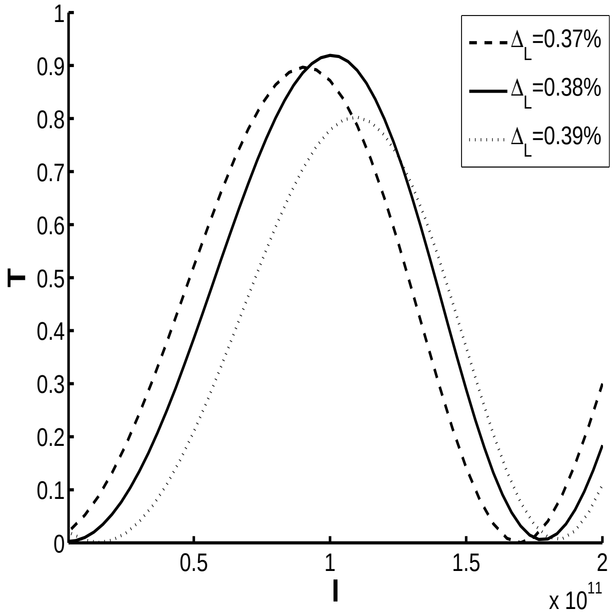

Figure 18. Variation of T with I of the initially imbalanced NMZI for shifted balance points.

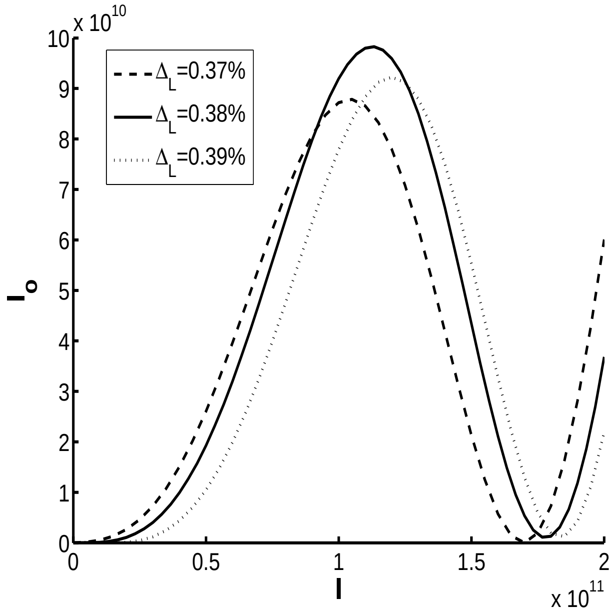

Figure 19. Variation of I_o with I of the initially imbalanced NMZI for shifted balance points.

2. Conclusion

In the present paper, a detailed investigation on the switching behaviour of a nonlinear Mach–Zehnder interferometer (NMZI) has been carried out using beam propagation method (BPM). A thorough investigation on input vs. output characteristics has been carried out by varying different parameters like length of the arms, refractive index of the linear/nonlinear arm, wavelength of the input beams and nonlinear coefficient of the material of the nonlinear arm. It is shown that the transmission coefficient of a NMZI oscillates between maxima and minima with the increase in the input power. Moreover, for lower wavelengths, transmission coefficient changes more rapidly when the input power is gradually increased. Also, a NMZI becomes more sensitive to the optical input with increase in its length and/or nonlinearity coefficient.

The most important finding of the present work is that the switching on/off power of the NMZI could be manipulated by shifting the balance point. This fact could give tremendous liberty to a fabricator to adjust the operating power levels when geometry and the nonlinear material of the NMZI are fixed.

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