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# Stimulated Brillouin scattering of an electromagnetic wave in weakly magnetized plasma with variably charged dust particles

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**Abstract.** We have investigated analytically the stimulated Brillouin scattering (SBS) of an electromagnetic wave in non-dissipative weakly magnetized plasma in the presence of dust particles with variable charge.

**Keywords.** Stimulated Brillouin scattering; dusty plasmas; ion acoustic wave; dust accoustic wave.

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## 1. Introduction

The importance of the laser–plasma interaction becomes an active field of research in the laser-fusion experiments, charged particle acceleration experiments, X-ray generation, propagation of EM waves in ionosphere etc. and the stimulated Brillouin scattering (SBS) plays an important role in laser–plasma interaction as it produces a backscattered light, and therefore this process is one of the real threat to the inertial confinement fusion research. After the invention of the ruby laser in 1960, Chiao *et al* [1] was the first to observe SBS. SBS is also a very important and interesting phenomenon in non-linear optics where the field of SBS has led to the birth of the fascinating subject like optical phase conjugation. Dusty plasmas, in contrast to ordinary plasmas, contain additionally large dust grains of radii in the range $10^{-2}$–$10^{-6}$ m. These particles become negatively charged and the magnitude of this charge is of the order of $10^3e$–$10^5e$ for micrometer-sized dust particles [2]. The presence of these massive and highly charged particles can significantly change the collective behaviour of the plasma in which they are suspended.

Shukla and Stenflo [3] investigated the non-linear coupling between large amplitude electromagnetic wave and the slow background motion in dusty plasma. Shukla and Stenflo [4] studied the non-linear coupling between large amplitude electromagnetic waves and dust acoustic waves in dust electron plasma, taking into account the combined effect of the radiation pressure and thermal force. Using kinetic theory, Salimullah *et al* [5] examined the currents of electrons and ions to a spherical dust grain in a uniform strongly magnetized dusty plasma. They found that the external magnetic field reduces the charging current thereby decreasing the dust charge fluctuation damping of a low frequency electrostatic wave in dusty plasma. Recently, Mahmoud [6] studied stimulated Brillouin backscattering at relativistic laser power.

In our theoretical estimation of SBS of an EM wave in weakly magnetized plasma, we have considered basic equations along with variably charged dust particles to obtain the dispersion relation in §2. Next, we evaluated the growth rate of the excited dust acoustic wave in magnetized plasma, in the presence of dust particles of variable charge so as to find out the dependency of different parameters on growth rate of the excited low frequency wave in §3. Finally, in §4 we have given the conclusion.

## 2. Dispersion relation

We consider weakly magnetized dusty plasma consisting of electrons, ions and dust particles. The following considerations have been made to describe the dynamics of dust in the plasma system: (i) The charging of dust particles is due only to electron attachment and ion collection currents. (ii) The dust particles are spherical in shape and of uniform radii. (iii) The number density of dust grains is not very high, i.e. $n_{\mathrm{e}0}$ and $n_{\mathrm{i}0} > n_{\mathrm{d}0}$ where $n_{\mathrm{i}0}$ is the equilibrium number density of ions, $n_{\mathrm{e}0}$, $n_{\mathrm{d}0}$ are the equilibrium number density for electrons and dust particles respectively. $(-Z_{\mathrm{d}}e)$ is the charge on the dust grain. The equations of continuity are

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha V_\alpha) = 0, \tag{1}$$

where $V$ and $n$ stand for the velocity and density of different particles and subscripts $\alpha = \mathrm{i}$, e and d where i, e and d represent ion, electron and dust species respectively.

The equations of momentum conservation can be written as

$$\frac{\partial V_\alpha}{\partial t} = \frac{1}{m_\alpha}(eE + F_{p\alpha}) + \frac{e}{m_\alpha c}(V_\alpha \times B_0) - \frac{V_{\theta\alpha}^2 \nabla n_\alpha}{n_{\alpha 0}}, \tag{2}$$

where $B_0$ is the externally applied magnetic field along the $z$ direction. $V_{\theta\alpha}$, $Z_d$ and $m_\alpha$ represent thermal velocity, charge number density and mass respectively, $c$ is the velocity of light and $E$ is the electric field.

$$F_{p\alpha} = -e\nabla\phi_{p\alpha} = \frac{e^2\nabla(E_0 \cdot E_1)}{4m_\alpha\omega_0\omega_1}, \tag{3}$$

$F_{p\alpha}$ and $\phi_{p\alpha}$ are the ponderomotive force and ponderomotive potential respectively.

The Poisson's equation, determining the potential, can be written as

$$\nabla^2\phi = 4\pi e(n_{\mathrm{e}} + Z_{\mathrm{d}} n_{\mathrm{d}} - n_{\mathrm{i}}). \tag{4}$$

Here, $E$ can be expressed as $-\nabla\phi$ and all quantities can be taken to vary as $\mathrm{e}^{i(kx-\omega t)}$. We obtain relation for $n_\alpha$ using eq. (2) as

$$n_\alpha = \frac{k^2 \chi_\alpha}{4\pi e}(\phi + \phi_{p\alpha}), \tag{5}$$

$$\chi_\alpha = \frac{\omega_{p\alpha}^2}{\omega^2 + \omega_{c\alpha}^2 - 2k^2 V_{\theta\alpha}^2}, \tag{6}$$

where $\chi_\alpha$ is the susceptibility. Susceptibility of a material or a substance describes its response to an applied field.

We then consider the dust charge dynamics as given by Jana *et al* [7] as follows:

$$\frac{\mathrm{d}q_\mathrm{d}}{\mathrm{d}t} + \eta q_\mathrm{d} = |I_{\mathrm{e}0}| \left( \frac{n_\mathrm{i}}{n_{\mathrm{i}0}} - \frac{n_\mathrm{e}}{n_{\mathrm{e}0}} \right), \tag{7}$$

$$q_\mathrm{d} = \frac{|I_{\mathrm{e}0}|}{(-i\omega + \eta)} \left( \frac{n_\mathrm{i}}{n_{\mathrm{i}0}} - \frac{n_\mathrm{e}}{n_{\mathrm{e}0}} \right), \tag{8}$$

$$\eta = \frac{e|I_{\mathrm{e}0}|}{C} \left( \frac{1}{T_\mathrm{e}} + \frac{1}{\varpi_0} \right), \tag{9}$$

where $\eta^{-1}$ is the typical time-scale of decay and $C$ is the grain capacitance. $\varpi_0 = T_\mathrm{i} - e\phi_0$, equilibrium current is denoted by $|I_{\mathrm{e}0}|$, $Z_\mathrm{d} = q_\mathrm{d}/e$ and $\phi_0$ is the floating grain potential of the dust grains.

Next, we have considered the number density of dust particles from [8] as

$$n_\mathrm{d} = -\frac{n_{\mathrm{d}0}\phi_0 e Z_\mathrm{d}}{m_\mathrm{d} V_{\theta\mathrm{d}}^2}. \tag{10}$$

Using eqs (10) and (7) we get the following dust number density:

$$n_\mathrm{d} = -\frac{n_{\mathrm{d}0}\phi_0}{m_\mathrm{d} V_{\theta\mathrm{d}}^2} \left[ \frac{|I_{\mathrm{e}0}|}{(-i\omega + \eta)} \left( \frac{n_\mathrm{i}}{n_{\mathrm{i}0}} - \frac{n_\mathrm{e}}{n_{\mathrm{e}0}} \right) \right] \tag{11}$$

and

$$\chi_\mathrm{d} = -\frac{Z_\mathrm{d} n_{\mathrm{d}0}\phi_0 |I_{\mathrm{e}0}|}{m_\mathrm{d} V_{\theta\mathrm{d}}^2 (-i\omega + \eta)} \left( \frac{\chi_\mathrm{i}}{n_{\mathrm{i}0}} + \frac{\chi_\mathrm{e}}{n_{\mathrm{e}0}} \right), \tag{12}$$

where $\chi, n$ are susceptibility and number density.

Using eqs (4), (5) and (11) we find $n_\mathrm{e}$ as

$$n_\mathrm{e} = -\frac{k^2 \chi_\mathrm{e}}{4\pi e \varepsilon} \left\{ \phi_{pe} + \chi_\mathrm{i}[\phi_{pi} - \phi_{pe}] \left[ 1 - \frac{Z_\mathrm{d} n_{\mathrm{d}0}\phi_0 |I_{\mathrm{e}0}|}{m_\mathrm{d} V_{\theta\mathrm{d}}^2 (-i\omega + \eta) n_{\mathrm{i}0}} \right] \right\}. \tag{13}$$

The non-linear current density can be expressed as $J_1 = -en_{\mathrm{e}0}V_{\mathrm{e}1} - en_\mathrm{e}V_{\mathrm{e}0}$. Considering the $x$-component, we get [9]

$$J_{1x} = \frac{-ie^2 n_{\mathrm{e}0} E_{1x} \omega_1}{m_\mathrm{e}(\omega_{ce}^2 - \omega_1^2)} - \frac{ie^2 n_\mathrm{e} E_{0x} \omega_0}{m_\mathrm{e}(\omega_{ce}^2 - \omega_0^2)}. \tag{14}$$

Therefore, the equation for a scattered electromagnetic wave along $x$-direction can be written as

$$(\omega_1^2 - k_1^2c^2)E_{1x} = -4\pi i\omega_1 J_{1x}. \tag{15}$$

Substituting the value of $J_{1x}$ in eq. (15) we get

$$(\omega_1^2 - k_1^2c^2)E_{1x} = -\frac{4\pi e^2 n_{\mathrm{e}0}E_{1x}\omega_1^2}{m_{\mathrm{e}}(\omega_{ce}^2-\omega_1^2)} + \frac{4\pi e^2\omega_1\omega_0 E_{0x}}{m_{\mathrm{e}}(\omega_{ce}^2-\omega_0^2)}\left\{\frac{k^2\chi_{\mathrm{e}}}{4\pi e\varepsilon}\left[\phi_{pe}+\chi_{\mathrm{i}}(\phi_{pi}-\phi_{pe})\right.\right.$$
$$\left.\left.\times\left(1-\frac{Z_{\mathrm{d}}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|}{m_{\mathrm{d}}V_{\theta\mathrm{d}}^2(-i\omega+\eta)n_{\mathrm{i}0}}\right)\right]\right\}. \tag{16}$$

Substituting the value of $\Phi$ and $\chi$ for ion, electron and dust we get the dispersion relation as $\varepsilon D_1 = \mu$.

$$\varepsilon = 1-\chi_{\mathrm{i}}\left[1-\frac{Z_{\mathrm{d}}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|}{m_{\mathrm{d}}V_{\theta\mathrm{d}}^2(-i\omega+\eta)n_{\mathrm{i}0}}\right]-\chi_{\mathrm{e}}\left[1-\frac{Z_{\mathrm{d}}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|}{m_{\mathrm{d}}V_{\theta\mathrm{d}}^2(-i\omega+\eta)n_{\mathrm{e}0}}\right], \tag{17}$$

$$D_1 = \omega_1^2 - k_1^2c^2 + \frac{\omega_{pe}^2\omega_1^2}{\omega_{ce}^2-\omega_1^2}, \tag{18}$$

$$\mu = \frac{e^2k^2E_{0x}^2\omega_{pe}^2}{4m_e(\omega_{ce}^2-\omega_0^2)(\omega^2+\omega_{ce}^2-2k^2V_{\theta e}^2)}$$
$$\times\left\{\chi_{\mathrm{i}}\left[\frac{1}{m_{\mathrm{i}}}-\frac{1}{m_{\mathrm{e}}}\right]\left[1-\frac{Z_{\mathrm{d}}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|}{m_{\mathrm{d}}V_{\theta\mathrm{d}}^2(-i\omega+\eta)n_{\mathrm{i}0}}\right]-\frac{1}{m_{\mathrm{e}}}\right\}. \tag{19}$$

## 3. Growth rate

Next, we analyse equation considering the following conditions: $\omega = \omega_r + i\gamma_{\mathrm{L}}$; $\omega_{cd} < \omega_{ci} < \omega_{ce}$; $kV_{\theta\mathrm{d}} < kV_{\theta i} < \omega < kV_{\theta\mathrm{e}} < \omega_{ce}$; $\omega_0 \sim \omega_{1r} > \omega_{ce}$; $\gamma_{\mathrm{L}} < \omega$ where $\gamma_{\mathrm{L}}$ is the linear damping rate of the excited low frequency wave.

To find out the linear damping rate, $\gamma_{\mathrm{L}}$, we apply the above approximations along with $\varepsilon(\omega,k)=0$ to get

$$\gamma_{\mathrm{L}} = \frac{-\left(\frac{\omega_{pi}^2}{\omega_r^2 n_{\mathrm{i}0}}+\frac{\omega_{pe}^2}{\omega_{ce}^2 n_{\mathrm{e}0}}+\frac{2\omega_{pi}^2\eta}{n_{\mathrm{i}0}}\right)}{2\omega_{pi}^2\left(\frac{m_{\mathrm{d}}V_{\theta\mathrm{d}}^2}{\omega_r^2 Z_{\mathrm{d}}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|}+\frac{1}{n_{\mathrm{i}0}}\right)}. \tag{20}$$

In the absence of linear damping, the growth rate can be determined by

$$\gamma_0^2 = \frac{\mu}{\left(\frac{\partial\varepsilon_r}{\partial\omega_r}\right)\left(\frac{\partial D_{1r}}{\partial\omega_{1r}}\right)}. \tag{21}$$

Hence the growth rate of the excited dust acoustic wave in the presence as well as in absence of dust particles is given by $\gamma_1$ and $\gamma_2$, respectively and can be expressed, using $\gamma_{\mathrm{L}} \approx \eta$, as

$$\gamma_1^2 = \frac{e^2k^2E_{0x}^2\omega_{pe}^2\left\{\frac{\omega_{pi}^2}{\omega_r^2}\left[\frac{1}{m_\mathrm{e}}-\frac{1}{m_\mathrm{i}}\right]\left[1-\frac{Z_\mathrm{d}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|(2\eta)}{m_\mathrm{d}V_{\theta\mathrm{d}}^2\omega_r^2 n_{\mathrm{i}0}}\right]-\frac{1}{m_\mathrm{e}}\right\}}{4m_\mathrm{e}(\omega_{ce}^2-\omega_{1r}^2)(2\omega_{1r})\omega_{ce}^2\left[\frac{2\omega_{pi}^2}{\omega_r^3}-\frac{4\omega_{pi}^2Z_\mathrm{d}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|(2\eta)}{m_\mathrm{d}V_{\theta\mathrm{d}}^2n_{\mathrm{i}0}\omega_r^5}-\frac{2\omega_{pe}^2Z_\mathrm{d}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|(2\eta)}{\omega_{ce}^2m_\mathrm{d}V_{\theta\mathrm{d}}^2n_{\mathrm{e}0}\omega_r^3}\right]}. \tag{22}$$

The growth rate is directly proportional to both pump electric field and number density of plasma particles. This is similar to the work described by Chen [10].

$$\gamma_2^2 = \frac{e^2k^2E_{0x}^2\omega_{pe}^2\left[\frac{\omega_{pi}^2}{\omega_r^2}\left(\frac{1}{m_\mathrm{e}}-\frac{1}{m_\mathrm{i}}\right)-\frac{1}{m_\mathrm{e}}\right]}{4m_\mathrm{e}\omega_{ce}^2(\omega_{ce}^2-\omega_{1r}^2)(2\omega_{1r})\left(\frac{2\omega_{pi}^2}{\omega_r^3}\right)}. \tag{23}$$

From eq. (23), one can infer that the growth rate of the excited dust acoustic wave depends on the threshold electric field, the mass and number density of the variably charged dust particles inside magnetized plasma. This completely matches with the conclusions of Jain *et al* [9]. Comparison of eqs (22) and (23) yields

$$\left(\frac{\gamma_1}{\gamma_2}\right)^2 = \frac{1-\frac{Z_\mathrm{d}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|(2\eta)}{m_\mathrm{d}V_{\theta\mathrm{d}}^2n_{\mathrm{i}0}\omega_r^2}}{1-\frac{Z_\mathrm{d}n_{\mathrm{d}0}\phi_0|I_{\mathrm{e}0}|(2\eta)}{m_\mathrm{d}V_{\theta\mathrm{d}}^2n_{\mathrm{i}0}}\left[\frac{2}{\omega_r^2}+\frac{1}{\omega_{ci}^2}\right]}. \tag{24}$$

Next, we calculate the ratio of the growth rates, eq. (24), with the help of a typical dusty plasma experiment with the following parameters given by Barkan *et al* [11] or Singh and Bora [12]: $m_\mathrm{i}/m_\mathrm{p} \sim 40$, where $m_\mathrm{i}$ and $m_\mathrm{p}$ are the ion and proton mass respectively. Dust mass, $m_\mathrm{d} \sim 10^{-12}$ g; $Z_\mathrm{d} = 10^2$. Electron and ion temperatures denoted by $T_\mathrm{e}$ and $T_\mathrm{i}$ take the same value 0.4 eV and dust grain temperature, $T_\mathrm{d} \sim 0.03$ eV. The characteristic frequency of the waves is given by $\omega = 2\pi f \sim 1\times 10^6$ rad/s. Magnetic field $B_0 \sim 0.4$ Tesla. Dust radius, $r_a = 10^{-3}$–$10^{-5}$ cm. $n_{\mathrm{e}0} = 10^9$ cm$^{-3}$, $n_{\mathrm{d}0} = 10^4 - 9\times 10^9$ cm$^{-3}$, $n_{\mathrm{i}0} = 3\times 10^9$ cm$^{-3}$. Here, we have considered that the charge in the dust particles is due to electron attachment only.

## 4. Conclusion

Here, we have considered that the charge on the dust particles is solely due to electron attachment. Equations (22) and (23) also reveal that the growth rate is directly proportional to the pump electric field as well as the number density of plasma particles. This is similar to the work described by Chen [10] while discussing the phenomena of stimulated Raman scattering and SBS where he has shown that for SBS the growth rate is proportional to the pump electric field amplitude and depends very weakly on the carrier density. Also, from the above expressions (eqs (22)–(24)) we can arrive at a conclusion that as the concentration of dust particles inside plasma is increased, the ratio of growth rate of the excited dust acoustic wave to the IAW (i.e. without dust particles) during the SBS process of an EM wave is decreased. This may be due to the fact that as the concentration of dust particles increases, the inertia of the system will become higher and higher. Therefore, this

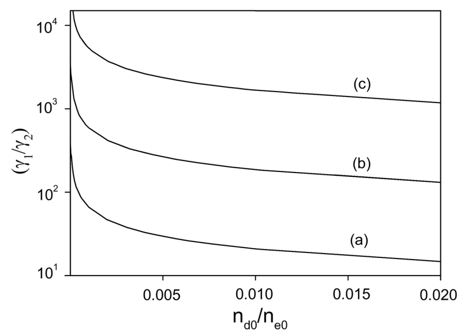


**Figure 1.** The plot is done for the ratio of growth rates ($\gamma_1/\gamma_2$) vs. ratio of equilibrium number density of dust and electron ($n_{\rm d0}/n_{\rm e0}$) for different values of dust radius ($r_a$). The curves labelled as (**a**), (**b**) and (**c**) are for $r_a = 10^{-3}$, $10^{-4}$, $10^{-5}$ cm respectively. The other parameters are $Z_{\rm d} = 10^2$, $n_{\rm e0} = 10^9$ cm$^{-3}$, $n_{\rm d0} = 10^6$–$2 \times 10^7$ cm$^{-3}$, $n_{\rm i0} = 3 \times 10^9$ cm$^{-3}$.

resistive force drives the growth rate of the low frequency wave to be lesser. From figure 1, we can arrive at a conclusion that as the concentration of dust particles inside the plasma increases the ratio of the growth rate of excited dust acoustic wave to the IAW (i.e. without dust particles) during the SBS process of an EM wave is decreased. Results show similar pattern of figure 1, when $Z_{\rm d}$ is varied from 10 to $10^5$ (not shown). This result is similar to the one obtained by Jain *et al* [9] for the case of a negative ion plasma.

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