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The corrections to scaling within Mazenko's theory in the limit of low and high dimensions

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Abstract. We consider corrections to scaling within an approximate theory developed by Mazenko for nonconserved order parameter in the limit of low ($d \to 1$) and high ($d \to \infty$) dimensions. The corrections to scaling considered here follows from the departures of the initial condition from the scaling morphology. Including corrections to scaling, the equal time correlation function has the form: $C(r,t) = f_0(r/L) + L^{-\omega} f_1(r/L) + \cdots$, where L is a characteristic length scale (i.e. domain size). The correction-to-scaling exponent ω and the correction-to-scaling functions $f_1(x)$ are calculated for both low and high dimensions. In both dimensions the value of ω is found to be $\omega = 4$ similar to 1D Glauber model and OJK theory (the theory developed by Ohta, Jasnow and Kawasaki).

Keywords. Morphological instability; phase changes; nonequilibrium and irreversible thermodynamics.

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1. Introduction

Phase-ordering kinetics or 'domain coarsening' is the subject concerned with the growth of the order parameter when the system is rapidly quenched from the high temperature phase (disordered phase) into the region of two- or more-ordered phases [1]. The scaling theory in phase-ordering kinetics asserts that when all length scales are scaled by the characteristic length scale $L(t)$ (e.g. domain size), quantities of interest such as the one-time correlation function $C(\mathbf{r},t)$ become time-independent in the scaling limit. The characteristic length scale $L(t)$ usually increases according to a power law, $L(t) \sim t^b$, where b is the growth exponent or scaling exponent. This simply means that quantities such as the pair correlation function, $C(\mathbf{r},t)$, are given by scaling forms [1], e.g.

$$C(\mathbf{r},t) = f(r/L), \tag{1}$$

and the quantity which unites theory, simulations and experiments, the structure factor $S(\mathbf{k},t)$, which is the Fourier transform of $C(\mathbf{r},t)$, becomes

$$S(\mathbf{k},t) = L^d g(kL), \tag{2}$$

where d is the dimensionality of the system, $f(r/L)$ and $g(kL)$ are 'scaling functions'.

In fact both the scaling functions and scaling exponents describe only the leading behaviour in the theory of scaling phenomena. There may be, and usually are, subdominant corrections, known as corrections to scaling. These corrections cannot be neglected in practice if more accurate values for exponents and scaling functions are required.

Here we consider corrections to scaling associated with the departures of the initial state from the scaling morphology [2,3]. However, there are other sources of corrections to scaling such as corrections due to the finite size of the 'defect core' ξ (ξ = domain wall thickness when the order parameter is a scalar [1]) and thermal fluctuations [1,4]. The result for the one-time pair correlation function has the form [3]

$$C(r,t) = f_0(r/L) + L^{-\omega} f_1(r/L), \tag{3}$$

where L is a characteristic length scale ('domain size') extracted from the energy, $f_0(r/L)$ is the scaling function, ω is the correction-to-scaling exponent which is in general non-trivial and $f_1(r/L)$ is the correction-to-scaling function.

The paper is organized as follows: The next section introduces the Mazenko theory with corrections to scaling due to noninitial condition. Section 3 deals with the calculations for corrections to scaling function and exponent for $d \to 1$. The corrections to scaling for $d \to \infty$ are considered in §4. Concluding remarks are given in §5.

2. Mazenko theory with corrections to scaling

A 'Gaussian closure' theory (Mazenko theory) proposed by Mazenko [5] following earlier work by Ohta, Jasnow and Kawasaki (OJK theory) [6] has proved to be quite useful in the study of coarsening dynamics. For nonconserved scalar fields, the pair correlation function $C(\mathbf{r},t)$ satisfies the following closed equation [1,5] within Mazenko Theory:

$$\frac{1}{2}\frac{\partial C}{\partial t} = \nabla^2 C + \frac{1}{\pi S_0(t)} \tan\left(\frac{\pi}{2} C\right). \tag{4}$$

The function $S_0(t)$ is defined as $\langle m(\mathbf{r},t)^2\rangle$, where m is an auxiliary Gaussian field [5]. For the present purposes, however, it is sufficient to note that S_0 has dimensions $(\text{length})^2$. It is convenient to define the coarsening length scale $L(t)$ by $S_0 = L^2/\lambda$, where λ is a constant whose value is fixed by physical requirements [1,5]. This definition of L is in accord with previous definitions [1,3,7]. Since initial conditions contain only short-range spatial correlations, the parameter λ is fixed by the requirement that only exponential decay for large-x is present in $C(\mathbf{r},t)$ [5].

Writing $S_0 = L^2/\lambda$ in (4), setting $C(\mathbf{r}, t) = f_0(r/L) + L^{-\omega} f_1(r/L) + \cdots$, $\mathrm{d}L/\mathrm{d}t = 1/2L + b/L^{1+\omega} + \cdots$, and equating leading and next-to-leading powers of L in the usual way gives the following equations for the functions $f_0(x)$ and $f_1(x)$ [3]:

$$f_0'' + \left(\frac{d-1}{x} + \frac{x}{4}\right) f_0' + \frac{\lambda}{\pi} \tan\left(\frac{\pi}{2} f_0\right) = 0 \tag{5}$$

$$f_1'' + \left(\frac{d-1}{x} + \frac{x}{4}\right) f_1' + \frac{\lambda}{2} \sec^2\left(\frac{\pi}{2} f_0\right) f_1 + \frac{\omega}{4} f_1 + \frac{b}{2} x f_0' = 0, \tag{6}$$

where $x = r/L$ is the scaling variable, $C(\mathbf{r}, t) \to f_0(x)$ (scaling function) in the limit $t \to \infty$ while $f_1(x)$ is the correction to scaling function, b is the constant which fixes amplitude of $f_1(x)$. The correction-to-scaling exponent ω is determined in the same way as the parameter λ [3].

Equation (5) provides both the scaling function $f_0(x)$ and the parameter λ while solution to (6) gives both the correction-to-scaling function $f_1(x)$ and the correction to scaling exponent ω.

3. Results for low dimensionality

When $d \to 1$, the scaling function $f_0(x)$ is not regular (as highlighted in §3.1) at small-x unless $\lambda \to 0$ faster than the rate at which $d \to 1$. In this limit we consider solving eqs (5) and (6) perturbatively in $\epsilon = d - 1$. That is, we are looking for the solutions of the form:

$$f_0(x) = u_0(x) + \epsilon u_1(x) + \epsilon^2 u_2(x) + \cdots \tag{7}$$

$$\lambda = \lambda_0 + \epsilon\lambda_1 + \epsilon^2\lambda_2 + \cdots \tag{8}$$

$$f_1(x) = v_0(x) + \epsilon v_1(x) + \epsilon^2 v_2(x) + \cdots \tag{9}$$

$$\omega = \omega_0 + \epsilon\omega_1 + \epsilon^2\omega_2 + \cdots. \tag{10}$$

3.1 *Scaling results in the limit $d \to 1$*

In order to solve eq. (5), the small-x analysis of $f_0(x)$ from (5) is important and is shown as follows:

$$f_0(x) = 1 - \frac{x}{\pi}\sqrt{\frac{2\lambda}{d-1}} + O(x^3) \tag{11}$$

$$= 1 - \frac{x}{\pi}\sqrt{\left[\frac{2(\lambda_0 + \epsilon\lambda_1 + \epsilon^2\lambda_2(x) + \cdots)}{\epsilon}\right]} + O(x^3). \tag{12}$$

The last equation is obtained by substituting (8) in (11). For $f_0(x)$ to be regular at small-x for $d \to 1$, $\lambda_0 = 0$ and eq. (11) reduces to

$$f_0(x) = 1 - \frac{x}{\pi}\sqrt{2\lambda_1} - \frac{x}{\pi}\frac{\lambda_2}{\sqrt{2\lambda_1}}\epsilon + O(\epsilon^2). \tag{13}$$

Comparing (7) and (13) the small-x analysis leads to: $u_0(x) = 1 - \frac{x}{\pi}\sqrt{2\lambda_1}$ and $u_1(x) = -\frac{x}{\pi}\frac{\lambda_2}{\sqrt{2\lambda_1}}$. Substituting eqs (7) and (8) in (5), and considering the terms of order $O(1)$ we get

$$u_0''(x) + \frac{x}{4}u_0'(x) = 0 \tag{14}$$

with solution

$$u_0(x) = 1 - \mathrm{erf}\left[\frac{x}{2\sqrt{2}}\right] = \mathrm{erfc}\left[\frac{x}{2\sqrt{2}}\right]. \tag{15}$$

The conditions $u_0(0) = 1$ and $u_0'(0) = -\frac{1}{\pi}\sqrt{2\lambda_1}$ have been employed in (15) above which also leads to $\lambda_1 = \pi/4$. The result for $u_0(x)$ is similar to 1D Glauber model [3,8,9]. We now consider terms of order $O(\epsilon)$ following substitution of (7) and (8) in (5) which gives

$$u_1''(x) + \frac{x}{4}u_1'(x) = R(x)\,, \tag{16}$$

with $R(x) = -\frac{u_0'}{x} - \frac{\lambda_1}{\pi}\tan\left(\frac{\pi}{2}u_0\right)$.

The above differential equation must be solved with initial conditions $u_1(0) = 0$ and $u_1'(0) = -\sqrt{\frac{2}{\pi^3}}\lambda_2$. The solution for (16) follows:

$$\begin{aligned} u_1(x) = &-\sqrt{\frac{2}{\pi^3}}\,\lambda_2 \times \int_0^x \exp\left(-\frac{y^2}{8}\right)\mathrm{d}y \\ &+ \int_0^x \left[\exp\left(-\frac{y^2}{8}\right) \times \int_0^y \exp\left(\frac{z^2}{8}\right) R(z)\mathrm{d}z\right]\mathrm{d}y. \end{aligned} \tag{17}$$

The parameter λ_2 is fixed by the condition that as $x \to \infty$, $u_1(x) \to 0$. The value for the parameter λ_2 is found to be $\lambda_2 = -0.0934$. Therefore, we have

$$\lambda = \frac{\pi}{4}\epsilon - 0.0934\epsilon^2 + \cdots\;. \tag{18}$$

For $d = 2$, the estimate $\lambda = 0.692$ is very close to the value 0.711 obtained by direct numerical solution [1,3] of eq. (5). The scaling function is given by

$$f_0(x) = u_0(x) + \epsilon u_1(x), \tag{19}$$

where $u_0(x)$ and $u_1(x)$ are given by eqs (15) and (17) respectively. The scaling functions for $d = 1$ (exact result) and $d = 2$ (from eq. (19)) are shown in figure 1.

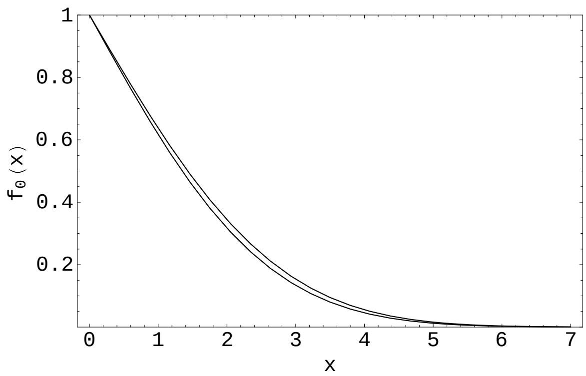

Figure 1. Scaling function $f_0(x)$: curves from left to right are the exact results for $d = 1$ and $d = 2$.

3.2 *Corrections to scaling results in the limit* $d \to 1$

The small-x analysis of eq. (6) gives

$$f_1(x) = \frac{1}{\pi}\sqrt{\frac{2\lambda}{d-1}}\,\frac{b}{(8d+4)}\,x^3 + O(x^5) \tag{20}$$

$$= \frac{b}{12\pi}\sqrt{2\lambda_1}\,x^3 + \epsilon\,\frac{b}{12\pi}\left(\frac{\lambda_2}{\sqrt{2\lambda_1}} - \frac{2}{3}\sqrt{2\lambda_1}\right)x^3 + O(\epsilon^2), \tag{21}$$

from which it follows that for small-x,

$$v_0(x) = \frac{b}{12\sqrt{2\pi}}\,x^3 + O(x^5) \tag{22}$$

$$v_1(x) = \frac{b}{12\sqrt{2\pi}}\left(\frac{2\lambda_2}{\pi} - \frac{2}{3}\right)x^3 + O(x^5)\,. \tag{23}$$

Substituting eqs (7)–(10) in (6), terms of order $O(1)$ leads to

$$v_0'' + \frac{x}{4}v_0' + \frac{\omega_0}{4}v_0 + \frac{b}{2}xu_0' = 0, \tag{24}$$

where $u_0'(x) = -\frac{1}{\sqrt{2\pi}}\exp(-x^2/8)$. Writing $v_0(x) = \exp(-x^2/8)g(x)$ and substituting this in eq. (24) gives

$$g'' - \frac{x}{4}g' + \frac{(\omega_0 - 1)}{4}g = Bx\,, \tag{25}$$

where $B = b/\sqrt{8\pi}$. What are the boundary conditions on $g(x)$? Clearly $g(0) = 0$, because $C(0,t) = 1$ is already implemented by $u_0(0) = 1$. Solution to (25) can be expressed in a series form: $g(x) = \sum_{n=0}^{\infty} g_n x^n$. However, the conditions $u_0(0) = 1$ and $v_0'(0) = 0$ lead to $g_0 = g_1 = g_2 = 0$ and as a result the series expansion for $g(x)$ starts at $O(x^3)$. Inserting the series solution $g(x) = \sum_{n=3}^{\infty} g_n x^n$ gives $g_3 = B/6$, and the recurrence relation $g_{n+2} = [a(n+1-\omega_0)/(n+1)(n+2)]g_n$ for the higher-order odd coefficients, all even coefficients vanishing. In order that $v_0(x)$ decreases faster than a power-law for large-x, as required on physical grounds for initial conditions with only short-range correlations, the series expansion for $g(x)$ must terminate. This gives the condition $\omega_0 = n+1 = 4, 6, 8, \ldots$. We conclude that the leading correction-to-scaling exponent for $d \to 1$ within Mazenko theory is $\omega_0 = 4$ with corresponding correction-to-scaling function

$$v_0(x) = \frac{B}{6}\, x^3 \exp\left(-\frac{x^2}{8}\right) = \frac{b}{12\sqrt{2\pi}}\, x^3 \exp\left(-\frac{x^2}{8}\right). \tag{26}$$

The results obtained here for corrections to scaling are similar to the ones we obtained in 1D Glauber model [3]. This shows that Mazenko theory reduces to 1D Glauber model in the limit $d \to 1$.

The $O(\epsilon)$ terms from eq. (6) on substituting (7)–(10) leads to

$$v_1'' + \frac{x}{4} v_1' + \frac{\omega_0}{4} v_1 = -\frac{v_0'}{x} - \frac{\lambda_1}{2} \sec^2\left(\frac{\pi}{2} u_0\right) v_0 - \frac{\omega_1}{4} v_0 - \frac{b}{2} x u_1'. \tag{27}$$

Since $v_0 \propto x^3 \exp(-x^2/8)$, it is clear that $v_1 = A_1 x^\alpha \exp(-x^2/8)$ and one has to determine α while A_1 follows from (23). Substituting $v_1 = A_1 x^\alpha \exp(-x^2/8)$ leads to $\alpha = 3$, while consideration of the dominant terms for $x \to \infty$ leads to $\omega_1 = 1 - 2\lambda_1 = 1 - \pi/2$. Note here that the observation $\sec^2\left(\frac{\pi}{2} u_0\right) \to 1$ as $x \to \infty$ has been used. The correction to scaling exponent then follows

$$\omega = 4 + \epsilon(1 - \pi/2) + \cdots . \tag{28}$$

For $d = 2$, the above gives $w = 3.429$, which is very close to the value 3.884 obtained through direct numerical solution of Mazenko theory in $d = 2$ [3]. The correction to scaling function $f_1(x) = v_0(x) + \epsilon v_1(x)$ is given by

$$f_1(x) = \frac{b}{12\sqrt{2\pi}} \left[1 - 0.726\epsilon\right] x^3 \exp\left(-\frac{x^2}{8}\right). \tag{29}$$

4. Results for high dimension

In order to make analysis of eq. (4) in the limit $d \to \infty$, we make the following change of variables: $\gamma(r,t) = \sin(\frac{\pi}{2} C(r,t))$ and apply $S_0 = L^2/\lambda$ as before. Then eq. (4) becomes

$$\frac{1}{2}\frac{\partial \gamma}{\partial t} = \frac{\mathrm{d}^2\gamma}{\mathrm{d}r^2} + \gamma \frac{(\mathrm{d}\gamma/\mathrm{d}r)^2}{(1-\gamma^2)} + \frac{d-1}{r}\frac{\mathrm{d}\gamma}{\mathrm{d}r} + \frac{\lambda}{2L^2}\gamma. \tag{30}$$

Setting $\gamma(r,t) = \gamma_0(r/L) + L^{-\omega}\gamma_1(r/L) + \cdots$, $\mathrm{d}L/\mathrm{d}t = 1/2L + b/L^{1+\omega} + \cdots$, and equating leading and next-to-leading powers of L in the usual way gives the following equations for the scaling function $\gamma_0(x)$ and correction-to-scaling function $\gamma_1(x)$:

$$\gamma_0'' + \frac{\gamma_0\gamma_0'}{1-\gamma_0^2} + \left(\frac{d-1}{x} + \frac{x}{4}\right)\gamma_0' + \frac{\lambda}{2}\gamma_0 = 0 \tag{31}$$

$$\gamma_1'' + \frac{2\gamma_0\gamma_1'\gamma_0'^2 + \gamma_0'^2\gamma_1}{1-\gamma_0^2} + \left(\frac{d-1}{x} + \frac{x}{4}\right)\gamma_1' + \left(\frac{\lambda}{2} + \frac{\omega}{4}\right)\gamma_1 + \frac{bx}{2}\gamma_0' = 0. \tag{32}$$

Since we are interested in the limit $d \to \infty$, we shall consider both eqs (31) and (32) perturbatively in $\tilde{\epsilon} = 1/d$. We now let

$$\lambda = \frac{1}{\tilde{\epsilon}\lambda_1 + \tilde{\epsilon}^2\lambda_2 + \cdots} = \frac{1}{\tilde{\epsilon}\lambda_1} - \frac{\lambda_2}{\lambda_1^2}\tilde{\epsilon} + \cdots \tag{33}$$

$$\gamma_0(x) = h_0(x) + \tilde{\epsilon}h_1(x)\cdots \tag{34}$$

$$\gamma_1(x) = p_0(x) + \tilde{\epsilon}p_1(x)\cdots \tag{35}$$

$$\omega = \omega_0 + \tilde{\epsilon}\omega_1\cdots. \tag{36}$$

4.1 *Scaling results in the limit $d \to \infty$*

We first consider the scaling equation (31) with substitution of eqs (33) and (34) by considering terms $O(\tilde{\epsilon}^{-1})$ and later terms of order $O(1)$. Terms of order $O(\tilde{\epsilon}^{-1})$ leads to $\frac{h_0'}{x} + \frac{h_0}{2\lambda_1} = 0$ with solution

$$h_0(x) = \exp\left[-\frac{x^2}{4\lambda_1}\right]. \tag{37}$$

The condition $h_0(0) = 1$ has been used. Since for large-x, $\gamma_0 \approx \exp[-x^2/8]$ then $\lambda_1 = 2$. For the terms of order $O(1)$ we have

$$\frac{h_1'}{x} + \frac{h_1}{2\lambda_1} + \left[\frac{x^2}{4\lambda_1^2}\left(1 - \frac{\lambda_1}{2}\right) - \frac{\lambda_2}{2\lambda_1} + \frac{x^2h_0^2}{4\lambda_1^2(1-h_0^2)}\right]h_0 = 0\,. \tag{38}$$

Since h_1 decays to zero for large-x, then $\lambda_1 = 2$ and $\lambda_2 = 0$. The solution to eq. (38) is then given by

$$h_1(x) = -\frac{h_0}{16} \times \int_0^x \frac{y^3}{[\exp\left(\frac{y^2}{4}\right) - 1]}\mathrm{d}y, \tag{39}$$

with condition $h_1(0) = 0$. The solution to eq. (31) follows

$$\gamma_0 = \exp\left(-\frac{x^2}{8}\right) \times \left[1 - \frac{1}{16d} \times \int_0^x \frac{y^3}{[\exp\left(\frac{y^2}{4}\right) - 1]}\mathrm{d}y\right] \tag{40}$$

with

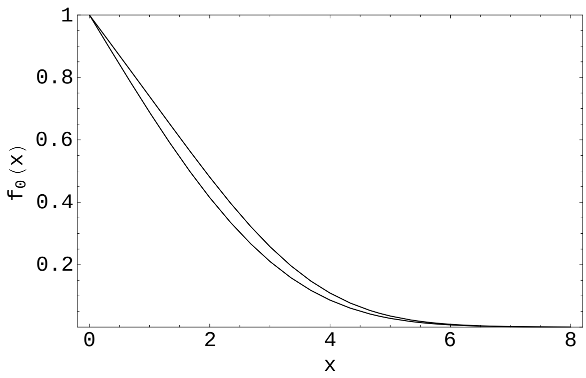

Figure 2. Scaling function $f_0(x)$: curves from left to right are the results for $d = 3$ and the exact results for $d = \infty$

$$\lambda = \frac{d}{2}\left[1 + 0\left(\frac{1}{d^2}\right)\right]. \tag{41}$$

The scaling function $f_0(x)$ is then given by

$$f_0(x) = \frac{2}{\pi}\sin^{-1}\left(\gamma_0(x)\right). \tag{42}$$

For $d = \infty$ we recover the OJK result [6]. This is in agreement with earlier conclusions that coarsening dynamics reduces to OJK result in the limit of $d \to \infty$ [5,10,11]. The scaling functions for $d = \infty$ (exact result) and $d = 3$ (from (42)) are shown in figure 2.

4.2 *Correction to scaling results in the limit $d \to \infty$*

The next step is to consider corrections to scaling with the help of eq. (32) with substitution of eqs (33)–(36). Terms of order $O(\tilde{\epsilon}^{-1})$ leads to $\frac{p_0'}{x} + \frac{p_0}{2\lambda_1} = 0$ with a solution $p_0(x) = 0$ (the condition $p_0(0) = 0$) has been used. We now consider the next terms of order $O(1)$ which leads to

$$\frac{p_1'}{x} + \frac{p_1}{2\lambda_1} + \frac{b}{2}xh_0' = 0. \tag{43}$$

Setting $p_1 = qh_0$ and substituting it in (43) gives $q = bx^4/16\lambda_1$. Hence the solution to (43) is given by

$$p_1(x) = \frac{bx^4}{16\lambda_1}\exp\left[-\frac{x^2}{4\lambda_1}\right] = \frac{bx^4}{32}\exp\left[-\frac{x^2}{8}\right]. \tag{44}$$

The next step is to consider terms of order $O(\tilde{\epsilon})$ which gives the following equation:

$$p_2' + \frac{xp_2}{4} = M(x), \tag{45}$$

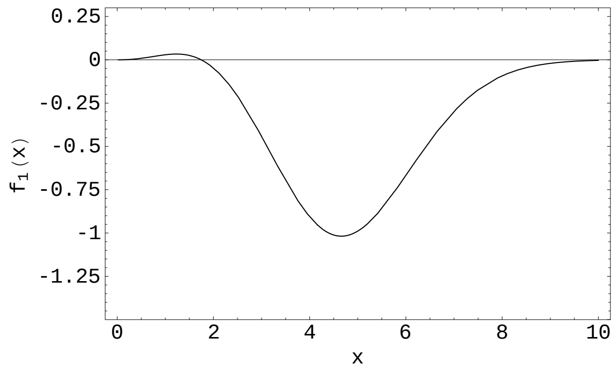

Figure 3. The correction to scaling function $f_1(x)$ for $d = 3$.

where

$$M(x) = -xp_1'' - \left(\frac{x^2}{4} - 1\right)p_1' - \frac{\omega_0}{4}xp_1 - \frac{bx}{2}h_1' - \frac{2xh_0'^2 h_0 p_1}{1 - h_0^2}. \tag{46}$$

In order to extract the value of ω_0 we consider the large-x analysis of (45). In this limit the above equation reduces to (as the last two terms in (45) are subdominant)

$$p_2' + \frac{xp_2}{4} = -xp_1'' - \left(\frac{x^2}{4} - 1\right)p_1' - \frac{\omega_0}{4}xp_1, \tag{47}$$

whose solution is (with condition $p_2(0) = 0$)

$$p_2(x) = \exp\left(-\frac{x^2}{8}\right)\int_0^x \exp(y^2/8) \times \left[-yp_1''(y) - \left(\frac{y^2}{4} - 1\right)p_1'(y) - \frac{\omega_0}{4}yp_1(y)\right]\mathrm{d}y. \tag{48}$$

The value of ω_0 is found from the above equation with a physical condition that $p_2(x)$ decays to 0 faster than any other possible $p_2(x)$ with another value of ω_0. The value $\omega_0 = 4$ satisfies this requirement. The solution to (45) follows:

$$p_2(x) = \exp\left(-\frac{x^2}{8}\right)\int_0^x \exp\left(y^2/8\right)M(y)\mathrm{d}y, \tag{49}$$

with $\omega_0 = 4$. The correction to scaling function $f_1(x)$ in the limit $d \to \infty$ is

$$f_1(x) = \frac{2}{\pi\sqrt{1 - \gamma_0^2}} \times \left[\tilde{\epsilon}p_1(x) + \tilde{\epsilon}^2 p_2(x)\right]. \tag{50}$$

The correction to scaling function $f_1(x)$ is shown in figure 3 for $d = 3$ using eq. (50). The results for $f_1(x)$ and ω_0 reproduce the corrections to scaling results for OJK [3]. This reinstate the conclusions drawn earlier that OJK is recovered from coarsening dynamical models in the limit of large-d [5,10,11]. In order to find ω_1 one has to consider terms beyond $O(\tilde{\epsilon}^2)$ and the equations become intractable as the number of parameters to be fixed increases and as a result it is not possible to find ω_1 in the large-d limit within the current study.

5. Concluding remarks

The understanding of corrections to scaling is critical in the analysis of data from experiments and simulations. We have shown that correction-to-scaling function $f_1(x)$ and correction-to-scaling exponent ω interpolate well between $d = 1$ and $d = \infty$ as known results are recovered: 1D Glauber model as $d \to 1$ and OJK result as $d \to \infty$. Our results further reinstate the conclusions drawn earlier that OJK is exact in the limit $d \to \infty$ [10–12].

The parameter λ is related to autocorrelation exponent $\bar{\lambda}$ introduced by Fisher and Huse [13] and later shown to be nontrivial by Newman and Bray [14]. The relation is as follows [15]: $\lambda = d - \bar{\lambda}$. We also note the fact that similar to OJK theory in coarsening dynamics, Mazenko theory utilizes the Gaussian field. In future, we hope to carry out the current study beyond the Gaussian field approximation.

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