

Developments in inflationary cosmology

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Abstract. This talk presents some recent work that has been done in inflationary cosmology. First a brief review is given of the inflation scenario and its basic models. After that, one of the main problems in developing inflationary models has been the requirement of a very flat inflation potential. In solving this problem, supersymmetry has played a major role, and the reasons will be discussed and a specific example of the SUSY hybrid model will be examined. Some problems introduced by SUSY such as the η and gravitino problems will then be discussed. Then in a different direction, the quintessential inflation model will be examined as a proposal where a single scalar field plays the role of both the inflaton at early time and the dark energy field later. The final topic covered is developments in understanding dissipation and particle production processes during the inflationary phase.

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1. Introduction

Inflationary cosmology continues to develop in many interesting directions. Inflation remains the most compelling explanation to the cosmological puzzles, in other words the horizon and flatness problems [1]. Its appeal is both the simplicity of the solution, essentially kinematic, and its realizability from quantum field theory particle physics models. Here a few recent developments in the subject will be discussed, in particular, problems in SUSY models of inflation, the ‘ η ’ and gravitino problems, the quintessential inflation model and dissipation processes during inflation.

2. Basics of inflation

Inflation is a regime in which the cosmic scale factor has an accelerated growth $\ddot{a} > 0$. From the general relativity scale factor equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \quad (1)$$

where G is the Newton constant, inflation is realized when $p < -\rho/3$, thus for a fluid with negative pressure. The most common example of such a fluid is vacuum energy $p = -\rho$. Such an equation of state can be realized from scalar fields. The energy and pressure density of a scalar field are

$$\begin{aligned}\rho &= \frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2a^2}, \\ p &= \frac{\dot{\phi}^2}{2} - V(\phi) - \frac{(\nabla\phi)^2}{6a^2},\end{aligned}\tag{2}$$

so that a state which is dominated by the potential energy of a scalar field has a negative pressure. Thus the idea that has generally been adopted in realizing inflation from particle physics is to get the potential energy of some scalar field to dominate the energy density of the universe for some short time period in the early phases of the universe, thereby generating the requisite amount of inflation necessary to solve the cosmological puzzles. Then once enough inflation has occurred, somehow put the universe back into a radiation-dominated hot Big Bang regime. The scalar field that performs the task of driving inflation is called the inflaton.

There are two dynamical realizations of inflation, cold and warm, which differ in how particle production occurs. The cold inflation picture was the initial picture and the standard inflation paradigm. This will be reviewed here and then warm inflation will be discussed in §5.

In the cold inflation picture, the scalar inflaton field is assumed to be essentially in isolation, thus interacting with nothing else besides gravity, during the inflation phase. Also, the scalar field must carry a large potential energy. The evolution of the scalar field in the FRW universe is described by the general relativistic version of the Klein–Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2(t)}\nabla^2\phi - \frac{\partial V}{\partial\phi} = 0.\tag{3}$$

In this equation the Hubble damping term, $3H\dot{\phi}$, formally acts like a friction term that damps the inflaton evolution. However, this $3H\dot{\phi}$ term does not lead to dissipative energy production, since its origin is from the coupling of the scalar field with the background FRW metric. Once the system starts within an inflationary regime, the field rapidly smoothens out and then the $\nabla^2\phi$ term quickly becomes negligible. During inflation the slow-roll conditions must be satisfied ($V'' < H^2$ and $V > \dot{\phi}^2/2$). These conditions are usually summarized in terms of two parameters

$$\begin{aligned}\epsilon &\equiv \frac{m_{\text{P}}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1, \\ \eta &\equiv m_{\text{P}}^2 \left(\frac{V''}{V} \right) \ll 1,\end{aligned}\tag{4}$$

where the primes denote the derivative with respect to ϕ . These slow-roll conditions ensure that the inflaton is moving slowly and is potential energy dominated. The scalar field not only solves the cosmological puzzles through its background

component driving inflation, but its quantum fluctuations will generate the primordial seeds of density perturbations. The fluctuations of the inflation field in cold inflation go as [2],

$$\delta\phi^2 \sim H^2, \quad \text{cold inflation} \quad T < m_\phi. \quad (5)$$

Here we have specifically specified that in cold inflation in particular the temperature of the universe during inflation is below the inflaton mass. The density perturbations resulting from these fluctuations go as

$$\frac{\delta\rho}{\rho} \sim \frac{H\delta\phi}{\dot{\phi}}. \quad (6)$$

There are three basic cold inflation models. The first and original cold inflation model is new inflation [3], which is based on the picture of inflation occurring during a phase transition. The typical example is the Coleman–Weinberg potential

$$V(\phi) = B\sigma^4 + B\sigma^4 \left[\ln(\phi^2/\sigma^2) - \frac{1}{2} \right] + g^2 T^2 \phi^2. \quad (7)$$

In such models there is a critical temperature T_c and for $T > T_c$ the inflaton potential has its global minima at $\phi = 0$, and for $T \leq T_c$ a new minimum appears and during the initial transition period, as the inflaton moves to the new minimum, inflation is expected to occur.

The next class of inflation models are those with monomial potentials, typically called chaotic inflation [4],

$$V = c\phi^n. \quad (8)$$

In such models, inflation starts when the inflaton field is displaced far from the origin, for example for $n = 2, 4$, $\phi \gtrsim m_P$. The potential is relatively flat and initially $V'' < H^2$, so that in eq. (3) slow-roll motion occurs. Eventually, the inflaton reaches a point where $V'' \gtrsim H^2$ and the inflaton field becomes kinetic energy-dominated, now oscillating about the origin. These oscillations lead to particle creation that reheat the universe [6].

The final class of cold inflation models is hybrid inflation, in which there is a second bosonic field required [5],

$$V = (M^2 - \chi^2)^2 + g^2 \phi^2 \chi^2 + m^2 \phi^2. \quad (9)$$

The idea in such models is that one field ϕ is responsible for inflation and a separate field χ is responsible for reheating. Inflation occurs when the inflaton $\phi^2 > \phi_c^2 \equiv 4M^2/g^2$ and in this regime $\chi = 0$. For $\phi^2 = \phi_c^2$, the χ field becomes unstable and a new minimum develops at $\chi^2 = M^2$. In reaching this new minimum, the χ field oscillates, thereby reheating the universe and ending the inflation phase.

3. Solutions and problems of supersymmetry

In order for slow-roll inflation to occur and in order to obtain observationally consistent amplitude of density perturbations, it requires a very flat inflaton potential.

In cold inflation the potential must have a curvature much smaller than the Hubble scale $V'' \ll 9H^2$. This is a problem, since in any quantum field theory model, quantum fluctuations will induce corrections to the inflaton effective potential. Thus either the couplings of the inflaton model have to be very tiny or supersymmetry (SUSY) must be invoked to cancel the radiative corrections. Having all tiny couplings is unnatural, thus SUSY offers the best prospects for building realistic inflation models. A typical inflaton model will consist of an inflaton potential and interaction of the inflaton field with some set of other fields. These interactions are essential, since they permit the inflaton to convert its vacuum energy into radiation and thus reheat the universe [6]. Thus a very standard interaction structure for an inflaton model is

$$W = \frac{1}{3}\sqrt{\lambda}\Phi^3 + g\Phi X^2 + 4mX^2, \quad (10)$$

where $\Phi = \phi + \psi\theta + \theta^2 F$ and $X = \chi + \theta\psi_\chi + \theta^2 F_\chi$ are chiral superfields. The field ϕ represents the inflaton in this model with $\phi = \varphi + \sigma$ and $\langle\phi\rangle = \varphi$. This is the simplest SUSY model for monomial inflaton potentials, in this case

$$V_0(\varphi) = \frac{\lambda}{4}\varphi^4, \quad (11)$$

and which includes the standard reheating interaction term to an additional boson $g^2\phi^2\chi^2$ and fermion $g\phi\psi_\chi\psi_\chi$. To obtain inflation and observationally consistent density perturbations, one finds it requires the coupling [4] to be very tiny

$$\lambda \approx 10^{-14}. \quad (12)$$

In order to preserve this tiny coupling, SUSY is needed. When $\varphi \neq 0$, there is a nonzero vacuum energy and so SUSY is broken. This manifests in the splitting of masses between the χ and ψ_χ SUSY partners with in particular

$$\begin{aligned} m_{\psi_\chi}^2 &= [2g^2\varphi^2 + 16\sqrt{2}mg\varphi + 64m^2] \\ m_{\chi_1}^2 &= \left[\frac{1}{8} \left(g^2 + \frac{1}{2}\sqrt{\lambda}g \right) \varphi^2 + \sqrt{2}mg\varphi + 4m^2 \right] = m_{\psi_\chi}^2 + \sqrt{\lambda}g\varphi^2 \\ m_{\chi_2}^2 &= \left[\frac{1}{8} \left(g^2 - \frac{1}{2}\sqrt{\lambda}g \right) \varphi^2 + \sqrt{2}mg\varphi + 4m^2 \right] = m_{\psi_\chi}^2 - \sqrt{\lambda}g\varphi^2. \end{aligned} \quad (13)$$

One can check that the one-loop zero temperature effective potential correction in this case is not significant to alter the flatness of the tree-level inflation potential,

$$V_1(\varphi) \approx \frac{9}{128\pi^2}\lambda g^2\varphi^4 \left[\ln \frac{m_{\psi_\chi}^2}{m^2} - 2 \right] \ll V_0(\varphi) = \frac{\lambda}{4}\varphi^4. \quad (14)$$

This shows how SUSY helps to control radiative corrections to the inflaton effective potential.

Alongside the advantage of SUSY in developing inflationary models comes two difficulties called the η [7] and gravitino [8] problems. Addressing the former first,

only global SUSY is needed to define an inflaton potential. However, a realistic global SUSY potential would be embedded within a local SUSY theory, supergravity (SUGRA). SUGRA contributes to the inflaton model some additional terms which in general can generate masses of order and bigger than the Hubble scale, and this is what is called the ‘ η ’-problem. A generic SUGRA potential has two parts, the F -term part which leads to the global SUSY potential and a Kähler potential

$$V_F = e^{\mathcal{K}/m_P^2} [K_{ij}^{-1} D_{z_i} W D_{z_j}^* W^* - 3m_P^{-2} |W|^2],$$

where \mathcal{K} is the Kähler potential, z_i are the bosonic components of the superfields and $D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_P^{-2} \frac{\partial \mathcal{K}}{\partial z_i} W$, $K_{ij} \equiv \frac{\partial^2 \mathcal{K}}{\partial z_i \partial z_j^*}$.

In general, the Kähler potential, \mathcal{K} , cannot be arbitrary in an inflation model, but rather must be very special in order to preserve the flatness of the F -term potential. For example, consider the hybrid SUSY model superpotential [9]

$$W = \kappa \Phi (X \bar{X} - M^2), \quad (15)$$

where Φ is a gauge singlet and X and \bar{X} are the conjugate pair of superfields transforming as a non-trivial representation of some gauge group G . For this model the simplest Kähler potential, often called the minimal Kähler potential, is

$$\mathcal{K}_{\min} = |\phi|^2 + |\chi|^2 + |\bar{\chi}|^2, \quad (16)$$

with ϕ , χ and $\bar{\chi}$ being bosonic components of the superfields. This model leads to the inflaton potential

$$V \approx 2\kappa^2 |\phi|^2 |\chi|^2 + \kappa^2 (|\chi|^2 - M^2)^2 \left(1 + 2 \frac{|\chi|^2}{m_P^2} + \frac{|\phi|^4}{2m_P^4} + \frac{|\chi|^4}{m_P^4} \right) + \dots, \quad (17)$$

in which, as can be seen, the leading order term in the SUGRA expansion, the mass term of the inflaton field ϕ has been cancelled. There are extensions to the minimal Kähler potential, and this can be exploited to build inflation models with different predictions. Recently, in [10] a non-minimal Kähler potential was suggested

$$\mathcal{K} = \mathcal{K}_{\min} + \kappa_\phi \frac{|\phi|^4}{4m_P^2} + \kappa_{\phi\chi} \frac{|\phi|^2 |\chi|^2}{m_P^2} + \kappa_{\phi\bar{\chi}} \frac{|\phi|^2 |\bar{\chi}|^2}{m_P^2} + \kappa_{\phi\phi} \frac{|\phi|^6}{6m_P^4} + \dots, \quad (18)$$

which modified the prediction of the spectral index in a range consistent with recent WMAP measurements. It would also be interesting to examine logarithmic terms added to the Kähler potential such as

$$\mathcal{K} = \mathcal{K}_{\min} + a_S m_P^2 \ln \left(\frac{|\phi|^2}{m_P^2} \right) + a_\chi m_P^2 \ln \left(\frac{|\chi|^2}{m_P^2} \right) + a_{\bar{\chi}} m_P^2 \ln \left(\frac{|\bar{\chi}|^2}{m_P^2} \right). \quad (19)$$

The other problem mentioned above that is caused by SUSY in inflation models is the gravitino problem [8]. The gravitino is the superpartner to the graviton. The

gravitino can have a lifetime longer than ~ 1 s if its mass is lighter than $\sim O(10 \text{ TeV})$. Thus thermally produced gravitinos in the early universe may decay after Big-Bang nucleosynthesis. Such a situation would alter the abundances of light elements produced from BBN, since the decay products of the gravitino induce electromagnetic and hadronic showers which will cause dissociation of the light elements. Since BBN is quantitatively highly successful in predicting the light element abundances, the presence of a large abundance of gravitinos at the time of BBN is not acceptable. The problem arises because first if SUSY breaks at around the TeV scale, then in most models it would produce gravitinos with masses around this scale and thus the decay rate would be long enough to affect nucleosynthesis. Detailed calculations show that in order to control gravitino abundances enough to not affect nucleosynthesis, inflation should end at a low enough reheat temperature. This scale varies depending on gravitino mass, but it ranges from around 10^5 to 10^9 GeV [11].

4. Quintessential inflation

An interesting idea for associating both primordial inflation and the present day accelerated expansion has been for the same field to be driving both phases. This idea, termed quintessential inflation, was first suggested by Peebles and Vilenkin in [12]. In order for this to work, it would require a scalar field potential $V(\phi)$ which in one region is large and flat, where the primordial inflation could occur and another region where the potential becomes very flat and tending towards zero, with a tiny potential energy tuned to the present-day observed value, $\sim 10^{-10} \text{ eV}^4$. Therefore, such a potential would have to be a monotonic function of ϕ . An immediate problem one might anticipate in such a case is how to implement reheating after inflation, since generally this requires oscillations of the inflaton. The suggestion made in [12] is that entropy production after primordial inflation occurs through gravitational particle production, based on the mechanism of Ford [13].

The potential suggested in [12] is

$$V = \begin{cases} \lambda(\phi^4 + M^4) & \text{for } \phi < 0 \\ \frac{\lambda M^8}{\phi^4 + M^4} & \text{for } \phi \geq 0. \end{cases} \quad (20)$$

Their scenario has four distinct regimes. First is the inflation period. This period ends into a kinetic energy-dominated era, in which the inflaton is moving fast. During this period particle creation by gravitational particle production occurs. This produced radiation eventually overtakes the kinetic energy, leading to the radiation-dominated era, thus getting into the standard hot Big Bang evolution. The radiation-dominated era then goes over into a matter-dominated era. Finally at later times, corresponding to the present, the matter-dominated era ends into a cosmological constant-dominated era. Below, these different expansion periods are explained in detail.

Initially the inflaton field is pictured to be very large $-\phi \gg M$. In this regime the potential eq. (20) is a chaotic inflation form [4], $V(\phi) \sim \lambda\phi^4$. As one seeks an inflationary solution, one knows from the chaotic inflation model that the coupling is tiny (eq. (12)) and that inflation will occur in the regime $-\phi \gtrsim m_{\text{P}}$. The mass

parameter M will be fixed later to fit observational constraints on the present-day cosmological constant. The above solution requires $M \ll m_P$ and this will be confirmed later in the calculation. In this inflation regime, the evolution equation of the homogeneous part of the inflaton field is of the slow-roll form

$$3H\dot{\phi} = -\frac{dV}{d\phi}, \quad (21)$$

where the Hubble parameter $H = \sqrt{V/(3m_P^2)}$. Inflation will end when the slow-roll motion ceases, which occurs when $9H^2 \sim m_\phi^2 = 12\lambda\phi^2$, which implies $-\phi \sim m_P$.

Once $-\phi \lesssim m_P$, slow-roll motion ends and the energy density of the universe ceases to be dominated by the ϕ potential energy and starts to be dominated by rather its kinetic energy $\dot{\phi}^2/2$. In this regime, the inflaton evolution equation is to a good approximation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = 0, \quad (22)$$

which has the solution $\dot{\phi} \sim a^{-3}$ and so the energy density of the universe goes as

$$\rho_\phi \approx \frac{\dot{\phi}^2}{2} \sim \lambda m_P \left(\frac{a_{\text{EI}}}{a}\right)^6, \quad (23)$$

where a_{EI} is the scale factor at the end of inflation. Using eq. (23) in the Friedmann equation

$$H^2 = (\dot{a}/a)^2 = \rho_\phi/(3m_P^2) \quad (24)$$

gives the scale factor behaviour $a \sim t^{1/3}$, which in turn leads to the solution

$$\phi = \sqrt{2}m_P \ln(a/a_{\text{EI}}). \quad (25)$$

The ϕ field now evolves like this in the kinetic energy-dominated regime until large ϕ , where the potential starts flattening out with $V(\phi) \sim M^8/\phi^4$ and at that point, ϕ re-enters at $a = a_{\text{DE}}$ a potential (or dark) energy-dominated regime. Ignoring for the moment the presence of radiation, one finds

$$\frac{a_{\text{DE}}}{a_{\text{EI}}} \sim \left(\frac{m_P}{M}\right)^{4/3} \ln^{2/3}\left(\frac{m_P}{M}\right). \quad (26)$$

Since the potential eq. (20) does not have any minimum about which the inflaton field can oscillate, reheating of the universe cannot occur via the standard reheating mechanism [6] of an oscillating scalar field which is radiatively producing particles. Rather in this scenario gravitational production of particles [13] produces the subsequent radiation. This is a very weak process which as will be discussed, produces radiation at a very low scale. However, the key observation is that the inflaton kinetic energy density is falling as a^{-6} and so eventually the radiation energy density produced by this mechanism, which will fall as a^{-4} , will take over. Consider the mechanism of gravitational production of particles for a massless scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}\xi \mathcal{R}\chi^2. \quad (27)$$

Assuming that space-time expansion is close to de Sitter and inflation is immediately followed by radiation-dominated expansion, the energy density of the created χ particles is found to be

$$\rho_r = RH_{\text{EI}}^4 (a_{\text{EI}}/a)^4, \quad (28)$$

where $R \sim 10^{-2}(1 - 6\xi)^2$. The contribution to the energy density ρ_r from massless fermions and gauge fields is suppressed, since they are conformally invariant fields, and so the main contribution to ρ_r comes from the total number N_χ of massless scalar fields with $R \sim N_\chi$. Thus eq. (28) is the radiation energy density after inflation alongside the inflation kinetic energy with ratio

$$\frac{\rho_r}{\rho_\phi} = \lambda R \left(\frac{a}{a_{\text{EI}}} \right)^2, \quad (29)$$

meaning the radiation-dominated era at $a = a_r$ begins when

$$\frac{a_r}{a_{\text{EI}}} = (\lambda R)^{-1/2}. \quad (30)$$

The radiation-dominated phase must of course occur before the late time potential energy-dominated phase $a_r < a_{\text{DE}}$, which for $R \sim 0.01$ – 1 implies $M < 10^{13}$ GeV, so the upper bound is well above that needed for the present-day dark energy, and so this solution so far is consistent.

At the end of the kinetic energy-dominated phase, ϕ is moving very slowly. At this point $a = a_r$, eqs (25) and (30) imply

$$\phi_r \equiv \phi(t = t_r) = \sqrt{2}m_{\text{P}} \ln(\lambda R)^{-1/2} \quad (31)$$

and

$$\dot{\phi}_r = R^{3/2} \lambda^2 m_{\text{P}}^2. \quad (32)$$

In this regime the potential energy term in the inflaton evolution equation is negligible and ϕ evolves to a good approximation by eq. (22), which leads to the solution

$$\phi(t) = \phi_r - m_{\text{P}} \left(\frac{t_r}{t} \right)^{1/2}. \quad (33)$$

Thus for $t > t_r$, ϕ remains almost constant at $\phi \approx \phi_r$. In [12] they argue that this evolution behaviour of ϕ remains until the present era. Thus the potential energy is

$$V(\phi) = \frac{\lambda M^8}{\phi_r^4} = \frac{\lambda M^8}{m_{\text{P}}^4 [\ln(\lambda R)^{-1/2}]^4}. \quad (34)$$

Equating the above to the present-day observed dark energy density leads to the constraint

$$M \sim \lambda^{-1/8} m_{\text{P}}^{3/4} H_0^{1/4} [\ln(\lambda R)^{-1/2}]^{1/2} \sim 10^6 \text{ GeV}. \quad (35)$$

With this the model parameters λ and M are completely constrained and the model is shown to be able to solve both the primordial inflation and late time dark energy problems. In [12] they go further to examine gravitational wave production in the model. One of the features of this model is that it gives a very low temperature of the universe after inflation. This can be attractive for other model building purposes such as constraining the over-production of gravitinos [8,11]. However, having a high temperature after inflation also has advantages such as for leptogenesis. For achieving this, one modification to the model that is suggested here is to couple the inflaton field in this model to another scalar field which goes unstable once ϕ is below some mass scale. At this point, the other field could undergo oscillations which thereby reheat the universe.

5. Dissipative inflaton dynamics

As mentioned in §2, there are two dynamical realizations of inflation. One is the original or standard picture, also referred to as isentropic or cold inflation [1,3,4], which has been discussed in the earlier sections. In this picture inflationary expansion occurs with the universe in a supercooled phase, which subsequently ends with a reheating period that introduces radiation into the universe. The fluctuations created during inflation are effectively zero-point ground state fluctuations and the evolution of the inflaton field is governed by a ground state evolution eq. (3). The other picture of inflation dynamics is non-isentropic or warm inflation [14] (for a review, see [15]). In this picture, inflationary expansion and radiation production occur concurrently. Moreover, the fluctuations created during inflation emerge from some excited statistical state and the evolution of the inflaton has dissipative terms arising from the interaction of the inflaton with other fields.

The dividing point between warm and cold inflation is roughly at $\rho_r^{1/4} \approx H$, where ρ_r is the radiation energy density during inflation and H is the Hubble parameter. Thus $\rho_r^{1/4} > H$ is the warm inflation regime and $\rho_r^{1/4} \lesssim H$ is the cold inflation regime. This criterion is independent of thermalization, but if such were to occur, one sees this criterion basically amounts to the warm inflation regime corresponding to when $T > H$. This is easy to understand since the typical inflaton mass during inflation is $m_\phi \approx H$ and so when $T > H$, thermal fluctuations of the inflaton field will become important. This criterion for entering the warm inflation regime required the dissipation of a very tiny fraction of the inflaton vacuum energy during inflation. For example, for inflation with vacuum (i.e. potential) energy at the GUT scale ($\sim 10^{15-16}$ GeV), in order to produce radiation at the scale of the Hubble parameter, which is $\approx 10^{10-11}$ GeV, it just requires dissipating one part in 10^{20} of this vacuum energy density into radiation. Thus, energetically not a very significant amount of radiation production is required to move into the warm inflation regime. In fact the levels are so small, and their eventual effects on density perturbations and inflaton evolution are so significant, that care must be taken to account for these effects in the analysis of any inflation models.

In warm or non-isentropic inflation [14], dissipative effects are important during the inflation period, so that radiation production occurs concurrently with inflationary expansion. The basic equation for describing the evolution of an inflaton field that dissipates energy was proposed in [16] to be of a Langevin form, in which there is a fluctuation–dissipation relation which uniquely relates the field fluctuations and energy dissipation. The simplest such equation would be one in which the dissipation is temporally local,

$$\ddot{\phi} + [3H + \Upsilon]\dot{\phi} - \frac{1}{a^2(t)}\nabla^2\phi - \frac{\partial V}{\partial\phi} = \zeta. \quad (36)$$

In this equation, $\Upsilon\dot{\phi}$ is a dissipative term and ζ is a fluctuating force. Both are effective terms, arising due to the interaction of the inflaton with other fields. In general, these two terms will be related through a fluctuation–dissipation theorem, which would depend on the statistical state of the system and the microscopic dynamics.

Focusing on the background inflaton field, from above its evolution equation is then,

$$\ddot{\phi} + [3H + \Upsilon(\phi)]\dot{\phi} + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0. \quad (37)$$

For $\Upsilon = 0$, this equation reduces the familiar background inflaton evolution equation for cold inflation eq. (3), but for a non-zero Υ , it corresponds to the case where the inflaton field is dissipating energy into the universe, thus creating a radiation component. The conditions for slow-roll inflation are modified in the presence of the extra friction term Υ , and we have now

$$\epsilon_{\Upsilon} = \frac{\epsilon}{(1 + Q)} < 1, \quad (38)$$

$$\eta_{\Upsilon} = \frac{\eta}{(1 + Q)} < 1, \quad (39)$$

where

$$Q \equiv \frac{\Upsilon}{3H}, \quad (40)$$

and ϵ, η are the slow-roll parameters without dissipation given in eq. (4). In addition, when the friction term Υ depends on the value of the inflaton field, we can define a third slow-roll parameter

$$\epsilon_{H\Upsilon} = \frac{\beta_{\Upsilon}}{(1 + Q)} < 1, \quad (41)$$

with

$$\beta_{\Upsilon} = \frac{V'}{3H^2} \frac{\Upsilon'}{\Upsilon}. \quad (42)$$

In this slow-roll regime of warm inflation the inflaton evolution equation is well-approximated by

$$[3H + \Upsilon(\varphi)]\dot{\varphi} + \xi R\varphi + \frac{dV_{\text{eff}}(\varphi)}{d\varphi} = 0. \quad (43)$$

The dissipation of the inflaton's motion is associated with the production of entropy. The entropy density of the radiation $s(\phi, T)$ is defined by a thermodynamic relation in terms of the thermodynamic potential,

$$s = -V_{,T}. \quad (44)$$

The rate of entropy production can be deduced from the conservation of energy-momentum. The total density ρ and pressure p are given by

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + Ts \quad (45)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V. \quad (46)$$

Energy-momentum conservation,

$$\dot{\rho} + 3H(p + \rho) = 0, \quad (47)$$

now implies entropy production. Making use of eq. (37) (with $\xi = 0$) we get

$$T(\dot{s} + 3Hs) = \Upsilon\dot{\phi}^2. \quad (48)$$

The zero curvature Friedman equation completes the set of differential equations for ϕ , T and the scale factor a ,

$$3H^2 = 8\pi G(\frac{1}{2}\dot{\phi}^2 + V + Ts). \quad (49)$$

The entropy production has been described in a slightly different way in the initial warm inflation papers [14,17]. We can recover an alternative equation in the case when the temperature corrections to the potential are negligible. If we set $\delta m_T = 0$ in the finite temperature effective potential $V_{\text{eff}}(\phi, T)$, then the radiation density $\rho_r = 4sT/3$, and eq. (48) becomes

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2. \quad (50)$$

This equation is only valid when $\delta m_T = 0$.

The evolution of the inflaton fluctuations is determined by eq. (36) most conveniently studied by transforming into momentum space,

$$\ddot{\delta\phi}(\mathbf{k}, t) + (3H + \Upsilon)\dot{\delta\phi}(\mathbf{k}, t) + (\mathbf{k}^2 a^{-2} + m^2)\delta\phi(\mathbf{k}, t) = \xi(\mathbf{k}, t). \quad (51)$$

The quantity of main interest for inflation calculations is the inflaton fluctuation as a given mode freezes-out during inflation. In cold inflation the freeze-out scale is $\sim H$, as governed by the damping term in the inflaton evolution equation. Similar

effect occurs in warm inflation, except now the damping term $\Upsilon + 3H$ controls this scale. As shown in [17], the inflaton fluctuation at freeze-out has the form

$$\delta\phi^2(k_F) \approx \frac{k_F T}{2\pi^2}. \quad (52)$$

To determine k_F , one must determine when the homogeneous solution to eq. (51) for mode $\delta\phi(\mathbf{k}, t)$ effectively stops decaying within a Hubble time. This homogeneous solution contains the past memory of the mode and once it stops decaying the mode essentially does not change. It is clear that as the physical wavenumber in eq. (51) decreases, the respective mode decays more slowly, with characteristic decay time $\sim 3H(1+Q)/k_p^2$. This decay time becomes less than $1/H$ at the freeze-out scale $k_F^2 \sim 3H^2(1+Q)$. Thus in the strong dissipative regime $Q \gg 1$, this implies $k_F \sim \sqrt{H\Upsilon}$ and in the weak dissipative regime $Q \ll 1$, $k_F \sim H$, the latter being consistent with what occurs in cold inflation. In particular for the strong dissipative regime [17],

$$\delta\phi^2 \sim \sqrt{H\Upsilon} T \text{ warm inflation } (\Upsilon > 3H), \quad T > m_\phi. \quad (53)$$

For comparison, recall for cold inflation, where inflaton fluctuations are exclusively quantum, its fluctuations go as eq. (5). In warm inflation just as in cold inflation, the relation of density perturbations to the inflaton fluctuations are given by eq. (6).

Non-Gaussian effects from warm inflation have also been computed [18–20]. In the strong dissipative regime it was shown in [19] that entropy fluctuations during warm inflation play an important role in generating non-Gaussianity, with the prediction

$$-15 \ln\left(1 + \frac{Q}{14}\right) - \frac{5}{2} \lesssim f_{\text{NL}} \lesssim \frac{33}{2} \ln\left(1 + \frac{Q}{14}\right) - \frac{5}{2}, \quad (54)$$

where f_{NL} is the non-linearity parameter and Q is defined in eq. (40).

There are several phenomenological warm inflation models [21]. More interestingly, there have been several calculations of warm inflation dynamics from first principles, which in particular means determining the dissipative coefficient Υ from a first principles quantum field theory model. The assumption in such calculations is that the underlying microscopic dynamics determining Υ operates at time scales much faster than the macroscopic motion of the inflaton and universe expansion, $\Gamma_i > \dot{\phi}/\phi, H$, where Γ_i represents all relevant decay widths of the fields responsible for dissipation [22]. The earliest first principles warm inflation model was the distributed mass model in which the inflaton was coupled to a set of bosonic fields χ_i through shifted couplings as $g^2(\phi - M_i)^2 \chi_i^2$ [23]. In this model when $\phi \sim M_i$ the χ_i field would become light and so thermally excited and this would lead to a dissipative effect from this field [17, 22, 23]. This model has an interesting string interpretation, in which the mass distribution M_i arises from a fine structure splitting of a single highly degenerate string mass level [24]. This was the only model constructed where the fields that are directly coupled to the inflaton become thermally excited. Such a situation proved too difficult in maintaining a flat inflaton potential, due to the loop corrections from the thermally excited fields, which cannot be cancelled by SUSY.

This has led to the development of the two-stage dissipative mechanism of warm inflation [25] in which the inflaton ϕ is coupled to a set of heavy fields χ and ψ_χ which in turn are coupled to light fields y and ψ_y . The main point is that the heavy fields are not thermally excited and so the loop corrections to the inflaton potential are only from vacuum fluctuations, which can be controlled by SUSY. A generic superpotential that realizes the two-stage mechanism is

$$W_I = \sum_{i=1}^{N_\chi} \sum_{j=1}^{N_{\text{decay}}} [g\Phi X_i^2 + 4mX_i^2 + hX_i Y_j^2], \quad (55)$$

where $\Phi = \phi + \psi\theta + \theta^2 F$, $X = \chi + \theta\psi_\chi + \theta^2 F_\chi$ and $Y = y + \theta\psi_y + \theta^2 F_y$ are chiral superfields. The field ϕ will be identified as the inflaton in this model. In the context of the two-stage mechanism X is the heavy fields to which the inflaton is directly coupled and these fields in turn are coupled to light Y fields. The dissipative coefficients for the two-stage mechanism were calculated in [26]. Several first principles warm inflation models have been developed for the two-stage mechanism, which include monomial and hybrid warm inflation models [27], and warm hilltop models [28]. These models have shown a range of predictions within the observationally consistent region for density perturbations. In addition, in the strong dissipative regime of these models, they predict moderate to large non-Gaussian effects based on eq. (54).

Warm inflation models such as these have two unique and attractive model building features: (a) in the strong dissipative regime since $m_\phi \gg H$, they do not have the ‘ η ’-problem as discussed for cold inflation models in §3, (b) in warm inflation models for monomial potentials, the amplitude of the inflaton field, $\langle\phi\rangle$, is always below the Planck scale. In cold inflation, for monomial potentials, usually called chaotic inflation scenarios [4], the inflaton amplitude during inflation is larger than the Planck scale. This is a problem for model building. Quantum field theory models are generally regarded as low energy effective theories of some higher more fundamental theory, such as possibly strings, and there is some upper energy scale to which this low energy approximation is valid. Above this scale, the theory would be modified by additional, usually an infinite number of, operator corrections. The highest scale can be the Planck scale, and so for any quantum theory above this scale one expects an infinite number of non-renormalizable operator corrections, $\sim \sum_{n=1}^{\infty} g_n \phi^4 (\phi/m_P)^n$, which have to be retained [29–32]. In such a regime the low energy approximation to the theory is essentially not useful. Thus from a model building perspective cold inflation chaotic inflation models are difficult to implement, whereas warm inflation models do not suffer this complication [17,27,28].

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