PRAMANA — journal of physics

Vol. 70, No. 5
May 2008
pp. 911–933

Analytical solutions for some defect problems in 1D hexagonal and 2D octagonal quasicrystals

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MS received 7 November 2007; revised 4 December 2007; accepted 10 December 2007

Abstract. We study some typical defect problems in one-dimensional (1D) hexagonal and two-dimensional (2D) octagonal quasicrystals. The first part of this investigation addresses in detail a uniformly moving screw dislocation in a 1D hexagonal piezoelectric quasicrystal with point group $6mm$. A general solution is derived in terms of two functions φ_1, φ_2, which satisfy wave equations, and another harmonic function φ_3. Elementary expressions for the phonon and phason displacements, strains, stresses, electric potential, electric fields and electric displacements induced by the moving screw dislocation are then arrived at by employing the obtained general solution. The derived solution is verified by comparison with existing solutions. Also obtained in this part of the investigation is the total energy of the moving screw dislocation. The second part of this investigation is devoted to the study of the interaction of a straight dislocation with a semi-infinite crack in an octagonal quasicrystal. Here the crack penetrates through the solid along the period direction and the dislocation line is parallel to the period direction. We first derive a general solution in terms of four analytic functions for plane strain problem in octagonal quasicrystals by means of differential operator theory and the complex variable method. All the phonon and phason displacements and stresses can be expressed in terms of the four analytic functions. Then we derive the exact solution for a straight dislocation near a semi-infinite crack in an octagonal quasicrystal, and also present the phonon and phason stress intensity factors induced by the straight dislocation and remote loads.

Keywords. Quasicrystal; dislocation; crack; stress intensity factors; piezoelectricity; general solution.

PACS Nos 72.10.Fk; 61.72.Bb; 61.72.Nn; 62.40+i; 77.65.-j; 71.23.Ft

1. Introduction

Quasicrystals, which were first discovered in 1984 by Shechtman *et al* [1], possess a type of ordered structure characterized by crystallographically disallowed long-range orientational symmetry and by long-range quasiperiodic translational order. Since the discovery of quasicrystals, the elastic theory of quasicrystals continues to attract investigators' attention [2–7]. It has been experimentally verified that quasicrystals can really exist as stable phases, then it is necessary to take quasicrystals as a thermodynamic system and to establish the corresponding thermodynamics of equilibrium properties. Yang *et al* [8] first generalized the thermodynamics of equilibrium thermal, electrical, magnetic and elastic properties to the case of quasicrystals. By group theory, they derived physical property tensors for two-dimensional pentagonal, octagonal, decagonal and dodecagonal and three-dimensional icosahedral and cubic quasicrystals. Recently Li and Liu [9] derived physical property tensors for one-dimensional quasicrystals according to group representation theory. They presented particular matrix forms of the thermal expansion coefficient tensors and piezoelectric coefficient tensors under 31 point groups for the 1D quasicrystals. Most recently Rao *et al* [10] determined the maximum number of non-vanishing and independent second-order piezoelectric coefficients in pentagonal and icosahedral quasicrystals also by using group representation theory. With the development of the elasticity theory of quasicrystals, theoretical investigations of dislocation and crack problems in quasicrystals also receive focused attention. Ding *et al* [11] obtained the displacement fields induced by a straight dislocation line along the period direction of decagonal quasicrystals. Yang *et al* [12] derived an analytic expression for the elastic displacement fields induced by a dislocation in an icosahedral quasicrystal. Li *et al* [13] considered an infinite decagonal quasicrystal containing a Griffith crack which penetrates through the solid along the period direction. Zhou and Fan [14] studied an octagonal quasicrystal weakened by a Griffith crack. Wang and Zhong [6] studied the interaction between a semi-infinite crack and a line dislocation in a decagonal quasicrystalline solid. Liu *et al* [15] addressed the interaction between a screw dislocation and a semi-infinite crack in one-dimensional quasicrystals.

In the first part of this paper, we investigate the problems of dislocation dynamics in a one-dimensional hexagonal piezoelectric quasicrystal with point group $6mm$ by employing the results of Li and Liu [9] as our starting step. A general solution is derived in terms of two functions φ_1, φ_2, which satisfy wave equations, and one harmonic function φ_3. Elementary expressions for the phonon and phason displacements, strains, stresses, electric potential, electric fields and electric displacements induced by a straight screw dislocation line parallel to the quasiperiodic axis moving along a period direction in this piezoelectric quasicrystal are then obtained by employing the obtained general solution. Also derived is the total energy of the moving dislocation.

In the second part of this paper, we address in detail the interaction problem between a straight dislocation and a semi-infinite crack in an octagonal quasicrystalline solid. We first present the general solution for plane strain problems in octagonal quasicrystals. All of the phonon and phason fields can be expressed in terms of four analytic functions. We then derive the field potentials for (i) a straight dislocation in an infinite octagonal quasicrystal; (ii) asymptotic fields around a semi-infinite crack in an octagonal quasicrystal; and (iii) a straight dislocation near a semi-infinite crack in an octagonal quasicrystal. We also derive analytic expressions of the phonon and phason stress intensity factors induced by the straight dislocation.

2. A uniformly moving screw dislocation in a 1D hexagonal piezoelectric quasicrystal with point group 6 *mm*

2.1 *Basic formulations*

The generalized Hooke's law for 1D hexagonal piezoelectric quasicrystal with point group 6 mm, whose period plane is the (x_1, x_2)-plane and whose quasiperiodic direction is the x_3-axis, is given by [5,9]

$$
\begin{aligned}
&\sigma_{11} = c_{11}\varepsilon_{11} + c_{12}\varepsilon_{22} + c_{13}\varepsilon_{33} + R_1 w_{33} - e_{31}^{(1)} E_3,\\
&\sigma_{22} = c_{12}\varepsilon_{11} + c_{22}\varepsilon_{22} + c_{13}\varepsilon_{33} + R_1 w_{33} - e_{31}^{(1)} E_3,\\
&\sigma_{33} = c_{13}\varepsilon_{11} + c_{13}\varepsilon_{22} + c_{33}\varepsilon_{33} + R_2 w_{33} - e_{33}^{(1)} E_3,\\
&\sigma_{23} = \sigma_{32} = 2c_{44}\varepsilon_{32} + R_3 w_{32} - e_{15}^{(1)} E_2,\\
&\sigma_{13} = \sigma_{31} = 2c_{44}\varepsilon_{31} + R_3 w_{31} - e_{15}^{(1)} E_1,\\
&\sigma_{12} = \sigma_{21} = 2c_{66}\varepsilon_{12},\\
&H_{33} = R_1(\varepsilon_{11} + \varepsilon_{22}) + R_2\varepsilon_{33} + K_1 w_{33} - e_{33}^{(2)} E_3,\\
&H_{31} = 2R_3\varepsilon_{31} + K_2 w_{31} - e_{15}^{(2)} E_1,\\
&H_{32} = 2R_3\varepsilon_{32} + K_2 w_{32} - e_{15}^{(2)} E_2,\\
&D_3 = e_{31}^{(1)}(\varepsilon_{11} + \varepsilon_{22}) + e_{33}^{(1)}\varepsilon_{33} + e_{33}^{(2)} w_{33} + \in_{33} E_3,\\
&D_1 = 2e_{15}^{(1)}\varepsilon_{31} + e_{15}^{(2)} w_{31} + \in_{11} E_1,\\
&D_2 = 2e_{15}^{(1)}\varepsilon_{32} + e_{15}^{(2)} w_{32} + \in_{11} E_2,
\end{aligned}
\tag{1}
$$

where σ_{ij} and H_{ij} are the phonon and phason stress components, D_i are the electric displacements; ε_{ij} and w_{3j} are the phonon and phason strains, E_i are the electric fields; $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}, c_{66}$ are six elastic constants in the phonon field and $c_{66} = (c_{11} - c_{12})/2$; K_1 and K_2 are two elastic constants in the phason field; R_1, R_2, R_3 are three phonon–phason coupling elastic constants; $e_{ij}^{(1)}$ and $e_{ij}^{(2)}$ are piezoelectric coefficients and $\in_{11}$ and $\in_{33}$ are two dielectric coefficients.

The strain–displacement and electric field–electric potential relations are given by

$$
\begin{aligned}
&\varepsilon_{11} = u_{1,1}, \quad \varepsilon_{22} = u_{2,2}, \quad \varepsilon_{33} = u_{3,3},\\
&\varepsilon_{12} = \frac{1}{2}(u_{1,2} + u_{2,1}), \quad \varepsilon_{31} = \frac{1}{2}(u_{1,3} + u_{3,1}), \quad \varepsilon_{32} = \frac{1}{2}(u_{2,3} + u_{3,2}),\\
&w_{31} = w_{3,1}, \quad w_{32} = w_{3,2}, \quad w_{33} = w_{3,3},\\
&E_1 = -\phi_{,1}, \quad E_2 = -\phi_{,2}, \quad E_3 = -\phi_{,3},
\end{aligned}
\tag{2}
$$

where $u_i (i = 1–3)$ are three phonon displacement components, w_3 is the phason displacement, ϕ is the electric potential; a comma followed by $i (i = 1, 2, 3)$ denotes partial derivative with respect to the ith spatial coordinate.

In the absence of body force and electric charge density, the equations of motion and the charge equilibrium equation are [4,16]

$$\begin{aligned}
&\sigma_{11,1}+\sigma_{12,2}+\sigma_{13,3}=\rho\ddot{u}_1,\\
&\sigma_{21,1}+\sigma_{22,2}+\sigma_{23,3}=\rho\ddot{u}_2,\\
&\sigma_{31,1}+\sigma_{32,2}+\sigma_{33,3}=\rho\ddot{u}_3,\\
&H_{31,1}+H_{32,2}+H_{33,3}=\rho\ddot{w}_3,\\
&D_{1,1}+D_{2,2}+D_{3,3}=0,
\end{aligned}\tag{3}$$

where the superdot means the differentiation with respect to time, and ρ is the mass density of the piezoelectric quasicrystalline solid.

For the anti-plane shear problem in which the non-trivial displacements u_3, w_3 and the electric potential ϕ are independent of x_3, the equations of motion and the charge equilibrium equation can be expressed in terms of u_3, w_3 and ϕ as

$$\begin{aligned}
&c_{44}\nabla^2 u_3+R_3\nabla^2 w_3+e_{15}^{(1)}\nabla^2\phi=\rho\ddot{u}_3,\\
&R_3\nabla^2 u_3+K_2\nabla^2 w_3+e_{15}^{(2)}\nabla^2\phi=\rho\ddot{w}_3,\\
&e_{15}^{(1)}\nabla^2 u_3+e_{15}^{(2)}\nabla^2 w_3-\in_{11}\nabla^2\phi=0,
\end{aligned}\tag{4}$$

where $\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}$ is the two-dimensional Laplace operator.

It follows from $(4)_3$ that $\nabla^2\phi$ can be expressed in terms of $\nabla^2 u_3$ and $\nabla^2 w_3$ as

$$\nabla^2\phi=\frac{e_{15}^{(1)}}{\in_{11}}\nabla^2 u_3+\frac{e_{15}^{(2)}}{\in_{11}}\nabla^2 w_3.\tag{5}$$

Inserting the above into $(4)_{1,2}$ and eliminating $\nabla^2\phi$, we arrive at

$$\begin{aligned}
&\tilde{c}_{44}\nabla^2 u_3+\tilde{R}_3\nabla^2 w_3=\rho\ddot{u}_3,\\
&\tilde{R}_3\nabla^2 u_3+\tilde{K}_2\nabla^2 w_3=\rho\ddot{w}_3,
\end{aligned}\tag{6}$$

where $\tilde{c}_{44}=c_{44}+e_{15}^{(1)2}/\in_{11}$ is the piezoelectrically stiffened elastic constant in the phonon field, $\tilde{K}_2=K_2+e_{15}^{(2)2}/\in_{11}$ is the piezoelectrically stiffened elastic constant in the phason field, and $\tilde{R}_3=R_3+e_{15}^{(1)}e_{15}^{(2)}/\in_{11}$ is the piezoelectrically stiffened phonon–phason coupling elastic constant.

Next we introduce two new functions φ_1 and φ_2 given by

$$u_3=\alpha\varphi_1-\tilde{R}_3\varphi_2,\quad w_3=\tilde{R}_3\varphi_1+\alpha\varphi_2,\tag{7}$$

where

$$\alpha=\frac{1}{2}\left[\tilde{c}_{44}-\tilde{K}_2+\sqrt{(\tilde{c}_{44}-\tilde{K}_2)^2+4\tilde{R}_3^2}\right].\tag{8}$$

Consequently eq. (6) can be rewritten in the following canonical form:

$$\nabla^2\varphi_1=\frac{1}{s_1^2}\frac{\partial^2\varphi_1}{\partial t^2},\quad \nabla^2\varphi_2=\frac{1}{s_2^2}\frac{\partial^2\varphi_2}{\partial t^2},\tag{9}$$

where

$$s_1 = \sqrt{\frac{(\tilde{c}_{44} + \tilde{K}_2) + \sqrt{(\tilde{c}_{44} - \tilde{K}_2)^2 + 4\tilde{R}_3^2}}{2\rho}},$$
$$s_2 = \sqrt{\frac{(\tilde{c}_{44} + \tilde{K}_2) - \sqrt{(\tilde{c}_{44} - \tilde{K}_2)^2 + 4\tilde{R}_3^2}}{2\rho}} \tag{10}$$

are two wave speeds under anti-plane shear conditions. Meanwhile if we introduce a third function φ_3 given by

$$\phi = \varphi_3 + \frac{e_{15}^{(1)}}{\in_{11}} u_3 + \frac{e_{15}^{(2)}}{\in_{11}} w_3 = \varphi_3 + \frac{e_{15}^{(1)}\alpha + e_{15}^{(2)}\tilde{R}_3}{\in_{11}}\varphi_1 + \frac{e_{15}^{(2)}\alpha - e_{15}^{(1)}\tilde{R}_3}{\in_{11}}\varphi_2, \tag{11}$$

then eq. (5) can be written as

$$\nabla^2\varphi_3 = 0. \tag{12}$$

The nontrivial stresses and electric displacements can be expressed in terms of three new functions φ_1, φ_2 and φ_3 as

$$\begin{aligned}
&\sigma_{13} = \sigma_{31} = (\tilde{c}_{44}\alpha + \tilde{R}_3^2)\varphi_{1,1} + \tilde{R}_3(\alpha - \tilde{c}_{44})\varphi_{2,1} + e_{15}^{(1)}\varphi_{3,1},\\
&\sigma_{23} = \sigma_{32} = (\tilde{c}_{44}\alpha + \tilde{R}_3^2)\varphi_{1,2} + \tilde{R}_3(\alpha - \tilde{c}_{44})\varphi_{2,2} + e_{15}^{(1)}\varphi_{3,2},\\
&H_{31} = \tilde{R}_3(\alpha + \tilde{K}_2)\varphi_{1,1} + (\tilde{K}_2\alpha - \tilde{R}_3^2)\varphi_{2,1} + e_{15}^{(2)}\varphi_{3,1},\\
&H_{32} = \tilde{R}_3(\alpha + \tilde{K}_2)\varphi_{1,2} + (\tilde{K}_2\alpha - \tilde{R}_3^2)\varphi_{2,2} + e_{15}^{(2)}\varphi_{3,2},\\
&D_1 = - \in_{11}\varphi_{3,1},\ D_2 = - \in_{11}\varphi_{3,2},
\end{aligned} \tag{13}$$

Consequently eqs (7) and (11) for the displacements and electric potential and eq. (13) for stresses and electric displacements give a general solution for a kind of elasticity dynamic problems in a 1D hexagonal piezoelectric quasicrystal with point group $6mm$, where the dislocation line is parallel to the quasiperiodic axis. The three unknown functions φ_1, φ_2 and φ_3 can be determined from the appropriate boundary or initial-boundary conditions.

2.2 *Electroelastic fields induced by a moving piezoelectric screw dislocation*

Now consider a straight screw dislocation line parallel to the quasiperiodic x_3 axis. The screw dislocation suffers a finite discontinuity in the displacements and in the electric potential across the slip plane. We further assume that the screw dislocation moves at a constant velocity V along the x_1-axis. We choose a new coordinate system (x, y) which moves together at the same velocity V as the dislocation, and the moving and the fixed coordinate systems coincide at $t = 0$.

Make the following transformation to transform the fixed coordinate system (x_1, x_2) to the moving coordinate system (x, y):

$$x = x_1 - Vt, \quad y = x_2 \tag{14}$$

then eqs (9) and (12) can be transformed to the following equations in the moving coordinates

$$\beta_1^2 \frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2} = 0, \quad \beta_2^2 \frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2} = 0, \quad \frac{\partial^2 \varphi_3}{\partial x^2} + \frac{\partial^2 \varphi_3}{\partial y^2} = 0, \tag{15}$$

where

$$\beta_1 = \sqrt{1 - V^2/s_1^2}, \quad \beta_2 = \sqrt{1 - V^2/s_2^2}. \tag{16}$$

The general solution of eq. (15) can be immediately arrived at

$$\varphi_1 = \mathrm{Im}\,\{f_1(z_1)\}, \quad \varphi_2 = \mathrm{Im}\,\{f_2(z_2)\}, \quad \varphi_3 = \mathrm{Im}\,\{f_3(z)\}, \tag{17}$$

where $z_1 = x + \mathrm{i}\beta_1 y$, $z_2 = x + \mathrm{i}\beta_2 y$, $z = x + \mathrm{i}y$.

The screw dislocation investigated here is defined as

$$\oint \mathrm{d}u_3 = b, \quad \oint \mathrm{d}w_3 = d, \quad \oint \mathrm{d}\phi = \Delta\phi, \tag{18}$$

or equivalently

$$\oint \mathrm{d}\varphi_1 = \frac{\alpha b + \tilde{R}_3 d}{\alpha^2 + \tilde{R}_3^2}, \quad \oint \mathrm{d}\varphi_2 = \frac{\alpha d - \tilde{R}_3 b}{\alpha^2 + \tilde{R}_3^2},$$
$$\oint \mathrm{d}\varphi_3 = \Delta\phi - \frac{e_{15}^{(1)} b + e_{15}^{(2)} d}{\in_{11}}, \tag{19}$$

where b is the phonon displacement jump across the slip plane, d is the phason displacement jump across the slip plane, $\Delta\phi$ is the electric potential jump across the slip plane.

Consequently the three analytic functions $f_1(z_1), f_2(z_2)$ and $f_3(z)$ take the forms

$$f_1(z_1) = \frac{\alpha b + \tilde{R}_3 d}{2\pi(\alpha^2 + \tilde{R}_3^2)} \ln z_1,$$
$$f_2(z_2) = \frac{\alpha d - \tilde{R}_3 b}{2\pi(\alpha^2 + \tilde{R}_3^2)} \ln z_2,$$
$$f_3(z) = \frac{\in_{11}\Delta\phi - e_{15}^{(1)} b - e_{15}^{(2)} d}{2\pi\in_{11}} \ln z. \tag{20}$$

In view of eqs (7), (11), (17) and (20), the phonon and phason displacements and electric potential can be expressed in terms of the mechanical and electric dislocations as follows:

$$
\begin{aligned}
u_3 &= \frac{\alpha(\alpha b+\tilde{R}_3 d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\tan^{-1}\frac{\beta_1 x_2}{x_1-Vt}+\frac{\tilde{R}_3(\tilde{R}_3 b-\alpha d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\tan^{-1}\frac{\beta_2 x_2}{x_1-Vt},\\
w_3 &= \frac{\tilde{R}_3(\alpha b+\tilde{R}_3 d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\tan^{-1}\frac{\beta_1 x_2}{x_1-Vt}+\frac{\alpha(\alpha d-\tilde{R}_3 b)}{2\pi(\alpha^2+\tilde{R}_3^2)}\tan^{-1}\frac{\beta_2 x_2}{x_1-Vt},\\
\phi &= \frac{\in_{11}\Delta\phi-e_{15}^{(1)}b-e_{15}^{(2)}d}{2\pi\in_{11}}\tan^{-1}\frac{x_2}{x_1-Vt}\\
&\quad+\frac{(e_{15}^{(1)}\alpha+e_{15}^{(2)}\tilde{R}_3)(\alpha b+\tilde{R}_3 d)}{2\pi\in_{11}(\alpha^2+\tilde{R}_3^2)}\tan^{-1}\frac{\beta_1 x_2}{x_1-Vt}\\
&\quad+\frac{(e_{15}^{(2)}\alpha-e_{15}^{(1)}\tilde{R}_3)(\alpha d-\tilde{R}_3 b)}{2\pi\in_{11}(\alpha^2+\tilde{R}_3^2)}\tan^{-1}\frac{\beta_2 x_2}{x_1-Vt},
\end{aligned}
\tag{21}
$$

Similarly, the phonon and phason strains are given by

$$
\begin{aligned}
\gamma_{31} = 2\varepsilon_{31} &= -\frac{\alpha(\alpha b+\tilde{R}_3 d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_1 x_2}{(x_1-Vt)^2+\beta_1^2x_2^2}\\
&\quad-\frac{\tilde{R}_3(\tilde{R}_3 b-\alpha d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_2 x_2}{(x_1-Vt)^2+\beta_2^2x_2^2},\\
\gamma_{32} = 2\varepsilon_{32} &= \frac{\alpha(\alpha b+\tilde{R}_3 d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_1 (x_1-Vt)}{(x_1-Vt)^2+\beta_1^2x_2^2}\\
&\quad+\frac{\tilde{R}_3(\tilde{R}_3 b-\alpha d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_2 (x_1-Vt)}{(x_1-Vt)^2+\beta_2^2x_2^2},
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
w_{31} &= -\frac{\tilde{R}_3(\alpha b+\tilde{R}_3 d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_1 x_2}{(x_1-Vt)^2+\beta_1^2x_2^2}\\
&\quad-\frac{\alpha(\alpha d-\tilde{R}_3 b)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_2 x_2}{(x_1-Vt)^2+\beta_2^2x_2^2},\\
w_{32} &= \frac{\tilde{R}_3(\alpha b+\tilde{R}_3 d)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_1 (x_1-Vt)}{(x_1-Vt)^2+\beta_1^2x_2^2}\\
&\quad+\frac{\alpha(\alpha d-\tilde{R}_3 b)}{2\pi(\alpha^2+\tilde{R}_3^2)}\frac{\beta_2 (x_1-Vt)}{(x_1-Vt)^2+\beta_2^2x_2^2},
\end{aligned}
\tag{23}
$$

the electric fields by

$$
\begin{aligned}
E_1 &= \frac{\in_{11}\Delta\phi-e_{15}^{(1)}b-e_{15}^{(2)}d}{2\pi\in_{11}}\frac{x_2}{(x_1-Vt)^2+x_2^2}\\
&\quad+\frac{(e_{15}^{(1)}\alpha+e_{15}^{(2)}\tilde{R}_3)(\alpha b+\tilde{R}_3 d)}{2\pi\in_{11}(\alpha^2+\tilde{R}_3^2)}\frac{\beta_1 x_2}{(x_1-Vt)^2+\beta_1^2x_2^2}\\
&\quad+\frac{(e_{15}^{(2)}\alpha-e_{15}^{(1)}\tilde{R}_3)(\alpha d-\tilde{R}_3 b)}{2\pi\in_{11}(\alpha^2+\tilde{R}_3^2)}\frac{\beta_2 x_2}{(x_1-Vt)^2+\beta_2^2x_2^2},
\end{aligned}
$$

$$E_2 = -\frac{\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d}{2\pi\in_{11}}\frac{(x_1 - Vt)}{(x_1 - Vt)^2 + x_2^2} - \frac{(e_{15}^{(1)}\alpha + e_{15}^{(2)}\tilde{R}_3)(\alpha b + \tilde{R}_3 d)}{2\pi\in_{11}(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_1(x_1 - Vt)}{(x_1 - Vt)^2 + \beta_1^2 x_2^2} - \frac{(e_{15}^{(2)}\alpha - e_{15}^{(1)}\tilde{R}_3)(\alpha d - \tilde{R}_3 b)}{2\pi\in_{11}(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_2(x_1 - Vt)}{(x_1 - Vt)^2 + \beta_2^2 x_2^2}, \tag{24}$$

the phonon and phason stresses by

$$\sigma_{13} = \sigma_{31} = -\frac{(\tilde{c}_{44}\alpha + \tilde{R}_3^2)(\alpha b + \tilde{R}_3 d)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_1 x_2}{(x_1 - Vt)^2 + \beta_1^2 x_2^2} - \frac{\tilde{R}_3(\alpha - \tilde{c}_{44})(\alpha d - \tilde{R}_3 b)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_2 x_2}{(x_1 - Vt)^2 + \beta_2^2 x_2^2} - \frac{e_{15}^{(1)}(\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d)}{2\pi\in_{11}}\frac{x_2}{(x_1 - Vt)^2 + x_2^2},$$

$$\sigma_{23} = \sigma_{32} = \frac{(\tilde{c}_{44}\alpha + \tilde{R}_3^2)(\alpha b + \tilde{R}_3 d)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_1(x_1 - Vt)}{(x_1 - Vt)^2 + \beta_1^2 x_2^2} + \frac{\tilde{R}_3(\alpha - \tilde{c}_{44})(\alpha d - \tilde{R}_3 b)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_2(x_1 - Vt)}{(x_1 - Vt)^2 + \beta_2^2 x_2^2} + \frac{e_{15}^{(1)}(\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d)}{2\pi\in_{11}}\frac{x_1 - Vt}{(x_1 - Vt)^2 + x_2^2}, \tag{25}$$

$$H_{31} = -\frac{\tilde{R}_3(\alpha + \tilde{K}_2)(\alpha b + \tilde{R}_3 d)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_1 x_2}{(x_1 - Vt)^2 + \beta_1^2 x_2^2} - \frac{(\tilde{K}_2\alpha - \tilde{R}_3^2)(\alpha d - \tilde{R}_3 b)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_2 x_2}{(x_1 - Vt)^2 + \beta_2^2 x_2^2} - \frac{e_{15}^{(2)}(\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d)}{2\pi\in_{11}}\frac{x_2}{(x_1 - Vt)^2 + x_2^2},$$

$$H_{32} = \frac{\tilde{R}_3(\alpha + \tilde{K}_2)(\alpha b + \tilde{R}_3 d)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_1(x_1 - Vt)}{(x_1 - Vt)^2 + \beta_1^2 x_2^2} + \frac{(\tilde{K}_2\alpha - \tilde{R}_3^2)(\alpha d - \tilde{R}_3 b)}{2\pi(\alpha^2 + \tilde{R}_3^2)}\frac{\beta_2(x_1 - Vt)}{(x_1 - Vt)^2 + \beta_2^2 x_2^2} + \frac{e_{15}^{(2)}(\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d)}{2\pi\in_{11}}\frac{x_1 - Vt}{(x_1 - Vt)^2 + x_2^2}, \tag{26}$$

and the electric displacements are given by

$$D_1 = \frac{\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d}{2\pi}\frac{x_2}{(x_1 - Vt)^2 + x_2^2},$$
$$D_2 = -\frac{\in_{11}\Delta\phi - e_{15}^{(1)}b - e_{15}^{(2)}d}{2\pi}\frac{x_1 - Vt}{(x_1 - Vt)^2 + x_2^2}. \tag{27}$$

It can be easily checked that when $R_3 = e_{15}^{(2)} = 0$, the results derived here can just reduce to those obtained by Wang and Zhong [17] for a screw dislocation moving in piezoelectric crystals.

2.3 *Energy of the moving piezoelectric screw dislocation*

The total energy W per unit length on the dislocation line of the moving screw dislocation is composed of the kinetic energy $W_{\rm k}$ and the potential energy $W_{\rm p}$, which are given by the following integrals:

$$W_{\rm k} = \frac{\rho}{2}\int(\dot{u}_3^2 + \dot{w}_3^2)\mathrm{d}x_1\mathrm{d}x_2, \tag{28}$$

$$W_{\rm p} = \frac{1}{2}\int(\sigma_{3j}u_{3,j} + H_{3j}w_{3,j} + D_j\phi_{,j})\mathrm{d}x_1\mathrm{d}x_2, \tag{29}$$

where the integration should be taken over the circular annulus $r_0 \le r \le R_0$, with r_0 being the radius of the screw dislocation core.

The specific expressions of $W_{\rm k}$ and $W_{\rm p}$ are finally given by

$$W_{\rm k} = \frac{k_{\rm k}}{4\pi}\ln\frac{R_0}{r_0}, \quad W_{\rm p} = \frac{k_{\rm p}}{4\pi}\ln\frac{R_0}{r_0}, \tag{30}$$

where

$$k_{\rm k} = \frac{\rho V^2}{2(\alpha^2 + \tilde{R}_3^2)}\left[\frac{(\alpha b + \tilde{R}_3 d)^2}{\beta_1} + \frac{(\alpha d - \tilde{R}_3 b)^2}{\beta_2}\right], \tag{31}$$

$$\begin{aligned} k_{\rm p} = {} & \left(\beta_1 + \frac{1}{\beta_1}\right)\frac{(\tilde{c}_{44}\alpha^2 + \tilde{K}_2\tilde{R}_3^2 + 2\alpha\tilde{R}_3^2)}{2(\alpha^2 + \tilde{R}_3^2)^2}(\alpha b + \tilde{R}_3 d)^2 \\ & + \left(\beta_2 + \frac{1}{\beta_2}\right)\frac{(\tilde{c}_{44}\tilde{R}_3^2 + \tilde{K}_2\alpha^2 - 2\alpha\tilde{R}_3^2)}{2(\alpha^2 + \tilde{R}_3^2)^2}(\alpha d - \tilde{R}_3 b)^2 \\ & - \in_{11}\left(\Delta\phi - \frac{e_{15}^{(1)}b + e_{15}^{(2)}d}{\in_{11}}\right)^2. \end{aligned} \tag{32}$$

Consequently, the total energy is given by

$$W = \frac{k_{\rm k} + k_{\rm p}}{4\pi}\ln\frac{R_0}{r_0}. \tag{33}$$

It can be observed from the above that the total energy W becomes infinite when $V \to \min\{s_1, s_2\}$. Thus $\min\{s_1, s_2\}$ is the limit of the velocity of the screw dislocation.

In addition when $V \leq \min\{s_1, s_2\}$, the total energy can be written as follows:

$$W \cong W_0 + \frac{1}{2} m_0 V^2, \tag{34}$$

where W_0 is the potential energy per unit length of a stationary screw dislocation, i.e.,

$$W_0 = \frac{(c_{44}b^2 + K_2 d^2 - \in_{11}\Delta\phi^2 + 2R_3 bd + 2e_{15}^{(1)} b\Delta\phi + 2e_{15}^{(2)} d\Delta\phi)}{4\pi} \ln \frac{R_0}{r_0}, \tag{35}$$

and $m_0 = \frac{\rho(b^2+d^2)}{4\pi} \ln \frac{R_0}{r_0}$ is the static mass of the dislocation per unit length.

3. Interaction of a straight dislocation with a semi-infinite crack in an octagonal quasicrystal

3.1 *General solution*

The generalized Hooke's law for octagonal quasicrystalline materials with point groups $8mm$, 822, $\bar{8}m2$ and $8/mmm$, whose period direction is the x_3-axis, and whose quasiperiodic plane is the (x_1, x_2)-plane, is given by [18]

$$\begin{aligned}
\sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} + R(w_{11} + w_{22}),\\
\sigma_{22} &= C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} - R(w_{11} + w_{22}),\\
\sigma_{33} &= C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33},\\
\sigma_{12} &= \sigma_{21} = (C_{11} - C_{12})\varepsilon_{12} - R(w_{12} - w_{21}),\\
\sigma_{23} &= \sigma_{32} = 2C_{44}\varepsilon_{23},\\
\sigma_{13} &= \sigma_{31} = 2C_{44}\varepsilon_{13},\\
H_{11} &= R(\varepsilon_{11} - \varepsilon_{22}) + K_1 w_{11} + K_2 w_{22},\\
H_{22} &= R(\varepsilon_{11} - \varepsilon_{22}) + K_2 w_{11} + K_1 w_{22},\\
H_{12} &= -2R\varepsilon_{12} + (K_1 + K_2 + K_3) w_{12} + K_3 w_{21},\\
H_{21} &= 2R\varepsilon_{12} + K_3 w_{12} + (K_1 + K_2 + K_3) w_{21},\\
H_{13} &= K_4 w_{13},\\
H_{23} &= K_4 w_{23}.
\end{aligned} \tag{36}$$

where σ_{ij} and H_{ij} are phonon and phason stress components; ε_{ij} and w_{ij} are phonon and phason strains; $C_{11}, C_{12}, C_{13}, C_{33}, C_{44}$ are five elastic constants in the phonon field, K_1, K_2, K_3, K_4 are four elastic constants in the phason field, R is the phonon–phason coupling constant.

The phonon and phason strains ε_{ij} and w_{ij} are related to the phonon and phason displacements u_i and w_i through the following relationship:

$$\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad w_{ij} = w_{i,j}. \tag{37}$$

In the absence of body forces, the static equilibrium equations for the quasicrystal are given by [18]

$$\sigma_{ij,j} = 0, \quad H_{ij,j} = 0. \tag{38}$$

For the plane strain problems in which the displacement components u_1, u_2, w_1, w_2 are independent of x_3, and furthermore $u_3 = 0$, the equations of motion can be expressed in terms of the displacement components u_1, u_2, w_1, w_2 as follows:

$$\begin{aligned}
&2C_{11}u_{1,11} + (C_{11} - C_{12})u_{1,22} + (C_{11} + C_{12})u_{2,12} \\
&\qquad +2R(w_{1,11} - w_{1,22} + 2w_{2,12}) = 0, \\
&(C_{11} + C_{12})u_{1,12} + (C_{11} - C_{12})u_{2,11} + 2C_{11}u_{2,22} \\
&\qquad +2R(w_{2,11} - w_{2,22} - 2w_{1,12}) = 0, \\
&K_1 w_{1,11} + (K_1 + K_2 + K_3)w_{1,22} + (K_2 + K_3)w_{2,12} \\
&\qquad +R(u_{1,11} - u_{1,22} - 2u_{2,12}) = 0, \\
&(K_1 + K_2 + K_3)w_{2,11} + K_1 w_{2,22} + (K_2 + K_3)w_{1,12} \\
&\qquad +R(u_{2,11} - u_{2,22} + 2u_{1,12}) = 0.
\end{aligned} \tag{39}$$

The above set of equations can be equivalently written in the following matrix form:

$$\mathbf{L}\begin{bmatrix} u_1 \\ u_2 \\ w_1 \\ w_2 \end{bmatrix} = \mathbf{0}_{4\times 1}, \tag{40}$$

where the components of the 4×4 symmetric differential operator $\mathbf{L}$ are given by

$$L_{11} = 2C_{11}\frac{\partial^2}{\partial x_1^2} + (C_{11} - C_{12})\frac{\partial^2}{\partial x_2^2}, \quad L_{12} = L_{21} = (C_{11} + C_{12})\frac{\partial^2}{\partial x_1 \partial x_2},$$

$$L_{13} = L_{31} = 2R\left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2}\right), \quad L_{14} = L_{41} = 4R\frac{\partial^2}{\partial x_1 \partial x_2},$$

$$L_{22} = (C_{11} - C_{12})\frac{\partial^2}{\partial x_1^2} + 2C_{11}\frac{\partial^2}{\partial x_2^2},$$

$$L_{23} = L_{32} = -4R\frac{\partial^2}{\partial x_1 \partial x_2}, \quad L_{24} = L_{42} = 2R\left(\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2}\right),$$

$$L_{33} = 2K_1\frac{\partial^2}{\partial x_1^2} + 2(K_1 + K_2 + K_3)\frac{\partial^2}{\partial x_2^2}, \quad L_{34} = L_{43} = 2(K_2 + K_3)\frac{\partial^2}{\partial x_1 \partial x_2},$$

$$L_{44} = 2(K_1 + K_2 + K_3)\frac{\partial^2}{\partial x_1^2} + 2K_1\frac{\partial^2}{\partial x_2^2}. \tag{41}$$

Now we introduce a displacement function F, which satisfies the following equation:

$$|\mathbf{L}|\, F = 0, \tag{42}$$

where $|\mathbf{L}|$ is the determinant of the differential operator matrix $\mathbf{L}$. Omitting the intermediate procedures, the displacement function F finally satisfies the following partial differential equation [14]:

$$\nabla^2\nabla^2\nabla^2\nabla^2 F - 4\varepsilon\nabla^2\nabla^2\Lambda^2\Lambda^2 F + 4\varepsilon\Lambda^2\Lambda^2\Lambda^2\Lambda^2 F = 0, \tag{43}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2},\ \Lambda^2 = \tfrac{\partial^2}{\partial x_1^2} - \tfrac{\partial^2}{\partial x_2^2},$$

and

$$\varepsilon = \frac{R^2(C_{11} + C_{12})(K_2 + K_3)}{[(C_{11} - C_{12})(K_1 + K_2 + K_3) - 2R^2]\ [C_{11}K_1 - R^2]}, \quad 0 < \varepsilon < 1. \tag{44}$$

Applying the differential operator theory, one general solution to eq. (40) can now be expressed as

$$u_1 = L_{11}^* F, \quad u_2 = L_{12}^* F, \quad w_1 = L_{13}^* F, \quad w_2 = L_{14}^* F, \tag{45}$$

where $L_{11}^*, L_{12}^*, L_{13}^*$ and L_{14}^* are the algebraic cofactors of $\mathbf{L}$. The specific expressions of $L_{11}^*, L_{12}^*, L_{13}^*$ and L_{14}^* are given below:

$$\begin{aligned} L_{11}^* &= 4K_1[(C_{11} - C_{12})(K_1 + K_2 + K_3) - 2R^2]\frac{\partial^6}{\partial x_1^6} \\ &\quad + 8[K_1(2C_{11} - C_{12})(K_1 + K_2 + K_3) \\ &\quad - 3R^2(K_1 + 3K_2 + 3K_3)]\frac{\partial^6}{\partial x_1^4 \partial x_2^2} \\ &\quad + 4\Big[K_1(5C_{11} - C_{12})(K_1 + K_2 + K_3) \\ &\quad - 6R^2(K_1 - 2K_2 - 2K_3)\Big]\frac{\partial^6}{\partial x_1^2 \partial x_2^4} \\ &\quad + 8(C_{11}K_1 - R^2)(K_1 + K_2 + K_3)\frac{\partial^6}{\partial x_2^6}, \end{aligned} \tag{46}$$

$$L_{12}^* = -4\left[K_1(C_{11}+C_{12})(K_1+K_2+K_3)+6R^2(K_2+K_3)\right] \times\left(\frac{\partial^6}{\partial x_1^5\partial x_2}+\frac{\partial^6}{\partial x_1\partial x_2^5}\right) - 8\bigg[K_1(C_{11}+C_{12})(K_1+K_2+K_3) -10R^2(K_2+K_3)\bigg]\frac{\partial^6}{\partial x_1^3\partial x_2^3}, \tag{47}$$

$$L_{13}^* = -4R\left[(C_{11}-C_{12})(K_1+K_2+K_3)-2R^2\right]\frac{\partial^6}{\partial x_1^6} -8R\left[K_1(2C_{11}+C_{12})+(C_{11}+3C_{12})(K_2+K_3)-R^2\right]\frac{\partial^6}{\partial x_1^4\partial x_2^2} -4R\left[K_1(C_{11}+3C_{12})-(7C_{11}+C_{12})(K_2+K_3)+2R^2\right]\frac{\partial^6}{\partial x_1^2\partial x_2^4} +8R(C_{11}K_1-R^2)\frac{\partial^6}{\partial x_2^6}, \tag{48}$$

$$L_{14}^* = 4R\left[K_1(3C_{12}-C_{11})+(C_{11}-C_{12})(K_2+K_3)+4R^2\right]\frac{\partial^6}{\partial x_1^5\partial x_2} +8R\left[(C_{11}+3C_{12})(K_2+K_3)-K_1(3C_{11}-C_{12})+4R^2\right]\frac{\partial^6}{\partial x_1^3\partial x_2^3} -4R\left[K_1(5C_{11}+C_{12})+(7C_{11}+C_{12})(K_2+K_3)-4R^2\right]\frac{\partial^6}{\partial x_1\partial x_2^5}. \tag{49}$$

Here it shall be mentioned that the displacement function F is derived by means of the differential operator theory. It is observed that the method given here for the derivation of F is more straightforward than that presented in [14] and [19]. In addition it is not difficult to understand the fact that the displacement function F satisfies exactly the same partial differential equation as that derived by Zhou and Fan [14].

The general solution to eq. (43) can be expressed as

$$F = \mathrm{Re}\,\{f_1(z_1)+f_2(z_2)+f_3(z_3)+f_4(z_4)\}, \tag{50}$$

where $z_j = x_1 + p_j x_2$, $\mathrm{Im}\{p_j\} > 0$ ($j = 1$–4) and p_j ($j = 1$–4) are explicitly given by

$$\begin{aligned} p_1 &= [(1+\sqrt{\varepsilon})^{1/2}+\varepsilon^{1/4}]\,[\varepsilon^{1/4}+i(1-\sqrt{\varepsilon})^{1/2}],\\ p_2 &= [(1+\sqrt{\varepsilon})^{1/2}+\varepsilon^{1/4}]\,[-\varepsilon^{1/4}+i(1-\sqrt{\varepsilon})^{1/2}],\\ p_3 &= [(1+\sqrt{\varepsilon})^{1/2}-\varepsilon^{1/4}]\,[\varepsilon^{1/4}+i(1-\sqrt{\varepsilon})^{1/2}],\\ p_4 &= [(1+\sqrt{\varepsilon})^{1/2}-\varepsilon^{1/4}]\,[-\varepsilon^{1/4}+i(1-\sqrt{\varepsilon})^{1/2}]. \end{aligned} \tag{51}$$

Consequently, the phonon and phason displacements can also be expressed in terms of $f_j(z_j)$ ($j = 1$–4) as follows:

$$u_1 = \mathrm{Re}\left\{\delta_{11}f_1^{(6)}(z_1)+\delta_{12}f_2^{(6)}(z_2)+\delta_{13}f_3^{(6)}(z_3)+\delta_{14}f_4^{(6)}(z_4)\right\},$$
$$u_2 = \mathrm{Re}\left\{\delta_{21}f_1^{(6)}(z_1)+\delta_{22}f_2^{(6)}(z_2)+\delta_{23}f_3^{(6)}(z_3)+\delta_{24}f_4^{(6)}(z_4)\right\},$$
$$w_1 = \mathrm{Re}\left\{\delta_{31}f_1^{(6)}(z_1)+\delta_{32}f_2^{(6)}(z_2)+\delta_{33}f_3^{(6)}(z_3)+\delta_{34}f_4^{(6)}(z_4)\right\},$$
$$w_2 = \mathrm{Re}\left\{\delta_{41}f_1^{(6)}(z_1)+\delta_{42}f_2^{(6)}(z_2)+\delta_{43}f_3^{(6)}(z_3)+\delta_{44}f_4^{(6)}(z_4)\right\}, \quad (52)$$

where $f^{(j)}(z) = \partial^j f/\partial z^j$, and the constants δ_{ij} are given by

$$\begin{aligned}
\delta_{1j} = {} & 4K_1\left[(C_{11}-C_{12})(K_1+K_2+K_3)-2R^2\right] \\
& +8p_j^2\left[K_1(2C_{11}-C_{12})(K_1+K_2+K_3)-3R^2(K_1+3K_2+3K_3)\right] \\
& +4p_j^4\left[K_1(5C_{11}-C_{12})(K_1+K_2+K_3)-6R^2(K_1-2K_2-2K_3)\right] \\
& +8p_j^6(C_{11}K_1-R^2)(K_1+K_2+K_3), \\
\delta_{2j} = {} & -4p_j(1+p_j^4)\left[K_1(C_{11}+C_{12})(K_1+K_2+K_3)+6R^2(K_2+K_3)\right] \\
& -8p_j^3\left[K_1(C_{11}+C_{12})(K_1+K_2+K_3)-10R^2(K_2+K_3)\right], \\
\delta_{3j} = {} & -4R\left[(C_{11}-C_{12})(K_1+K_2+K_3)-2R^2\right] \\
& -8Rp_j^2\left[K_1(2C_{11}+C_{12})+(C_{11}+3C_{12})(K_2+K_3)-R^2\right] \\
& -4Rp_j^4\left[K_1(C_{11}+3C_{12})-(7C_{11}+C_{12})(K_2+K_3)+2R^2\right] \\
& +8Rp_j^6(C_{11}K_1-R^2), \\
\delta_{4j} = {} & 4Rp_j\left[K_1(3C_{12}-C_{11})+(C_{11}-C_{12})(K_2+K_3)+4R^2\right] \\
& +8Rp_j^3\left[(C_{11}+3C_{12})(K_2+K_3)-K_1(3C_{11}-C_{12})+4R^2\right] \\
& -4Rp_j^5\left[K_1(5C_{11}+C_{12})+(7C_{11}+C_{12})(K_2+K_3)-4R^2\right] \ (j = 1\text{–}4)
\end{aligned} \quad (53)$$

The phonon and phason stress components can also be expressed in terms of the four analytic functions $f_j(z_j)$ $(j = 1\text{–}4)$ as

$$\sigma_{12} = \sigma_{21} = \mathrm{Re}\left\{\gamma_{11}f_1^{(7)}(z_1)+\gamma_{12}f_2^{(7)}(z_2)+\gamma_{13}f_3^{(7)}(z_3)+\gamma_{14}f_4^{(7)}(z_4)\right\},$$
$$\sigma_{22} = \mathrm{Re}\left\{\gamma_{21}f_1^{(7)}(z_1)+\gamma_{22}f_2^{(7)}(z_2)+\gamma_{23}f_3^{(7)}(z_3)+\gamma_{24}f_4^{(7)}(z_4)\right\},$$
$$H_{12} = \mathrm{Re}\left\{\gamma_{31}f_1^{(7)}(z_1)+\gamma_{32}f_2^{(7)}(z_2)+\gamma_{33}f_3^{(7)}(z_3)+\gamma_{34}f_4^{(7)}(z_4)\right\},$$
$$H_{22} = \mathrm{Re}\left\{\gamma_{41}f_1^{(7)}(z_1)+\gamma_{42}f_2^{(7)}(z_2)+\gamma_{43}f_3^{(7)}(z_3)+\gamma_{44}f_4^{(7)}(z_4)\right\}, \quad (54)$$

$$\sigma_{11} = \mathrm{Re}\left\{\lambda_{11}f_1^{(7)}(z_1)+\lambda_{12}f_2^{(7)}(z_2)+\lambda_{13}f_3^{(7)}(z_3)+\lambda_{14}f_4^{(7)}(z_4)\right\},$$
$$\sigma_{33} = \mathrm{Re}\left\{\lambda_{21}f_1^{(7)}(z_1)+\lambda_{22}f_2^{(7)}(z_2)+\lambda_{23}f_3^{(7)}(z_3)+\lambda_{24}f_4^{(7)}(z_4)\right\},$$
$$H_{11} = \mathrm{Re}\left\{\lambda_{31}f_1^{(7)}(z_1)+\lambda_{32}f_2^{(7)}(z_2)+\lambda_{33}f_3^{(7)}(z_3)+\lambda_{34}f_4^{(7)}(z_4)\right\},$$
$$H_{21} = \mathrm{Re}\left\{\lambda_{41}f_1^{(7)}(z_1)+\lambda_{42}f_2^{(7)}(z_2)+\lambda_{43}f_3^{(7)}(z_3)+\lambda_{44}f_4^{(7)}(z_4)\right\}, \quad (55)$$

where the constants γ_{ij} and λ_{ij} are related to δ_{ij} through

$$\begin{aligned}
\gamma_{1j} &= \frac{C_{11}-C_{12}}{2}(p_j\delta_{1j}+\delta_{2j}) - R(p_j\delta_{3j}-\delta_{4j}),\\
\gamma_{2j} &= C_{12}\delta_{1j} + C_{11}p_j\delta_{2j} - R(\delta_{3j}+p_j\delta_{4j}),\\
\gamma_{3j} &= -R(p_j\delta_{1j}+\delta_{2j}) + (K_1+K_2+K_3)p_j\delta_{3j} + K_3\delta_{4j},\\
\gamma_{4j} &= R(\delta_{1j}-p_j\delta_{2j}) + K_2\delta_{3j} + K_1p_j\delta_{4j},
\end{aligned} \tag{56}$$

$$\begin{aligned}
\lambda_{1j} &= C_{11}\delta_{1j} + C_{12}p_j\delta_{2j} + R(\delta_{3j}+p_j\delta_{4j}),\\
\lambda_{2j} &= C_{13}(\delta_{1j}+p_j\delta_{2j}),\\
\lambda_{3j} &= R(\delta_{1j}-p_j\delta_{2j}) + K_1\delta_{3j} + K_2p_j\delta_{4j},\\
\lambda_{4j} &= R(p_j\delta_{1j}+\delta_{2j}) + K_3p_j\delta_{3j} + (K_1+K_2+K_3)\delta_{4j}.
\end{aligned} \tag{57}$$

Now introduce four stress functions $\Phi_1, \Phi_2, \Psi_1, \Psi_2$, which are related to the phonon and phason stresses through the following equations:

$$\begin{aligned}
\sigma_{11} &= -\frac{\partial\Phi_1}{\partial x_2}, \quad \sigma_{12} = \frac{\partial\Phi_1}{\partial x_1},\\
\sigma_{21} &= -\frac{\partial\Phi_2}{\partial x_2}, \quad \sigma_{22} = \frac{\partial\Phi_2}{\partial x_1},\\
H_{11} &= -\frac{\partial\Psi_1}{\partial x_2}, \quad H_{12} = \frac{\partial\Psi_1}{\partial x_1},\\
H_{21} &= -\frac{\partial\Psi_2}{\partial x_2}, \quad H_{22} = \frac{\partial\Psi_2}{\partial x_1}.
\end{aligned} \tag{58}$$

Apparently, the introduced stress functions automatically satisfy the equilibrium equations (38). It follows from eqs (54) and (58) that the four stress functions $\Phi_1, \Phi_2, \Psi_1, \Psi_2$ can be expressed in terms of the four analytic functions $f_j(z_j)$ ($j = 1$–4) as

$$\begin{aligned}
\Phi_1 &= \mathrm{Re}\left\{\gamma_{11}f_1^{(6)}(z_1) + \gamma_{12}f_2^{(6)}(z_2) + \gamma_{13}f_3^{(6)}(z_3) + \gamma_{14}f_4^{(6)}(z_4)\right\},\\
\Phi_2 &= \mathrm{Re}\left\{\gamma_{21}f_1^{(6)}(z_1) + \gamma_{22}f_2^{(6)}(z_2) + \gamma_{23}f_3^{(6)}(z_3) + \gamma_{24}f_4^{(6)}(z_4)\right\},\\
\Psi_1 &= \mathrm{Re}\left\{\gamma_{31}f_1^{(6)}(z_1) + \gamma_{32}f_2^{(6)}(z_2) + \gamma_{33}f_3^{(6)}(z_3) + \gamma_{34}f_4^{(6)}(z_4)\right\},\\
\Psi_2 &= \mathrm{Re}\left\{\gamma_{41}f_1^{(6)}(z_1) + \gamma_{42}f_2^{(6)}(z_2) + \gamma_{43}f_3^{(6)}(z_3) + \gamma_{44}f_4^{(6)}(z_4)\right\}.
\end{aligned} \tag{59}$$

3.2 *Field potentials*

3.2.1. *A straight dislocation in an infinite octagonal quasicrystal.* Consider the Burgers vector of a straight dislocation, which is infinitely long in the period direction, with the core at the origin in an infinite octagonal quasicrystal, $\mathbf{b} \oplus \mathbf{d} = (b_1, b_2, 0, d_1, d_2)$ where

$$\oint \mathrm{d}u_1 = b_1, \quad \oint \mathrm{d}u_2 = b_2, \quad \oint \mathrm{d}w_1 = d_1, \quad \oint \mathrm{d}w_2 = d_2, \tag{60}$$

for any loop C surrounding the dislocation line.

We can assume that in this case the four analytic functions $f_j(z_j)$ ($j = 1$–4) take the following forms:

$$\begin{aligned} &f_1^{(6)}(z_1) = A_1 \ln z_1, \quad f_2^{(6)}(z_2) = A_2 \ln z_2, \\ &f_3^{(6)}(z_3) = A_3 \ln z_3, \quad f_4^{(6}(z_4) = A_4 \ln z_4, \end{aligned} \tag{61}$$

where A_j ($j = 1$–4) are complex constants to be determined.

In view of eq. (60), we can arrive at the following set of linear algebraic equations:

$$\begin{aligned} &\mathrm{Im}\{\delta_{11}A_1 + \delta_{12}A_2 + \delta_{13}A_3 + \delta_{14}A_4\} = -\frac{b_1}{2\pi}, \\ &\mathrm{Im}\{\delta_{21}A_1 + \delta_{22}A_2 + \delta_{23}A_3 + \delta_{24}A_4\} = -\frac{b_2}{2\pi}, \\ &\mathrm{Im}\{\delta_{31}A_1 + \delta_{32}A_2 + \delta_{33}A_3 + \delta_{34}A_4\} = -\frac{d_1}{2\pi}, \\ &\mathrm{Im}\{\delta_{41}A_1 + \delta_{42}A_2 + \delta_{43}A_3 + \delta_{44}A_4\} = -\frac{d_2}{2\pi}, \end{aligned} \tag{62a}$$

$$\begin{aligned} &\mathrm{Im}\{\gamma_{11}A_1 + \gamma_{12}A_2 + \gamma_{13}A_3 + \gamma_{14}A_4\} = 0, \\ &\mathrm{Im}\{\gamma_{21}A_1 + \gamma_{22}A_2 + \gamma_{23}A_3 + \gamma_{24}A_4\} = 0, \\ &\mathrm{Im}\{\gamma_{31}A_1 + \gamma_{32}A_2 + \gamma_{33}A_3 + \gamma_{34}A_4\} = 0, \\ &\mathrm{Im}\{\gamma_{41}A_1 + \gamma_{42}A_2 + \gamma_{43}A_3 + \gamma_{44}A_4\} = 0. \end{aligned} \tag{62b}$$

Then the four unknown constants A_j ($j = 1$–4) and their conjugates can be uniquely determined from the above set of equations.

3.2.2 *Asymptotic fields around a semi-infinite crack.* Here we consider a semi-infinite crack which lies on the negative real axis. The crack penetrates through the solid along the period direction. We assume that in this case the four analytic functions $f_j(z_j)$ ($j = 1$–4) take the following form:

$$\begin{aligned} &f_1^{(6)}(z_1) = B_1\sqrt{z_1}, \quad f_2^{(6)}(z_2) = B_2\sqrt{z_2}, \\ &f_3^{(6)}(z_3) = B_3\sqrt{z_3}, \quad f_4^{(6)}(z_4) = B_4\sqrt{z_4}, \end{aligned} \tag{63}$$

where B_j ($j = 1$–4) are complex constants to be determined.

The satisfaction of traction-free boundary conditions $\sigma_{12} = \sigma_{22} = H_{12} = H_{22} = 0$ on the crack surfaces $x_1 < 0, x_2 = 0$ will result in

$$\begin{aligned} &\mathrm{Im}\{\gamma_{11}B_1 + \gamma_{12}B_2 + \gamma_{13}B_3 + \gamma_{14}B_4\} = 0, \\ &\mathrm{Im}\{\gamma_{21}B_1 + \gamma_{22}B_2 + \gamma_{23}B_3 + \gamma_{24}B_4\} = 0, \\ &\mathrm{Im}\{\gamma_{31}B_1 + \gamma_{32}B_2 + \gamma_{33}B_3 + \gamma_{34}B_4\} = 0, \\ &\mathrm{Im}\{\gamma_{41}B_1 + \gamma_{42}B_2 + \gamma_{43}B_3 + \gamma_{44}B_4\} = 0. \end{aligned} \tag{64}$$

According to the following stress intensity factors on the crack tip [6],

$$K_{\mathrm{I}} = \lim_{x_1\to 0, x_2\to 0} \sqrt{2\pi x_1}\sigma_{22}, \quad K_{\mathrm{II}} = \lim_{x_1\to 0, x_2\to 0} \sqrt{2\pi x_1}\sigma_{12},$$
$$T_{\mathrm{I}} = \lim_{x_1\to 0, x_2\to 0} \sqrt{2\pi x_1}H_{22}, \quad T_{\mathrm{II}} = \lim_{x_1\to 0, x_2\to 0} \sqrt{2\pi x_1}H_{12}. \tag{65}$$

Then we arrive at another set of linear algebraic equations

$$\mathrm{Re}\{\gamma_{11}B_1 + \gamma_{12}B_2 + \gamma_{13}B_3 + \gamma_{14}B_4\} = \frac{K_{\mathrm{II}}}{\sqrt{2\pi}},$$
$$\mathrm{Re}\{\gamma_{21}B_1 + \gamma_{22}B_2 + \gamma_{23}B_3 + \gamma_{24}B_4\} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}},$$
$$\mathrm{Re}\{\gamma_{31}B_1 + \gamma_{32}B_2 + \gamma_{33}B_3 + \gamma_{34}B_4\} = \frac{T_{\mathrm{II}}}{\sqrt{2\pi}},$$
$$\mathrm{Re}\{\gamma_{41}B_1 + \gamma_{42}B_2 + \gamma_{43}B_3 + \gamma_{44}B_4\} = \frac{T_{\mathrm{I}}}{\sqrt{2\pi}}. \tag{66}$$

Consequently, the four unknowns B_j ($j = 1$–4) can be uniquely determined from eqs (64) and (66) as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \frac{1}{\sqrt{2\pi}} \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{bmatrix}^{-1} \begin{bmatrix} K_{\mathrm{II}} \\ K_{\mathrm{I}} \\ T_{\mathrm{II}} \\ T_{\mathrm{I}} \end{bmatrix}. \tag{67}$$

Substituting eq. (63) into eqs (54) and (55), we can obtain the asymptotic phonon and phason stress fields around the semi-infinite crack in an octagonal quasicrystal. Apparently all the phonon and phason stress components σ_{ij} and H_{ij} exhibit the classical inverse square root singularity near the crack tip.

3.2.3 *A straight dislocation near a semi-infinite crack in an octagonal quasicrystal.* As shown in figure 1, we consider an octagonal quasicrystal containing a straight dislocation near a semi-infinite crack. The semi-infinite crack also lies on the negative real axis, and the dislocation is located at $x_1 = x_{1o}, x_2 = x_{2o}$.

The satisfaction of continuity condition of tractions across the total real axis $-\infty < x_1 < +\infty, x_2 = 0$ will result in the following equations:

$$\sum_{j=1}^{4} \overline{\gamma_{1j}} \bar{f}_j^{(7)}(z) = \sum_{j=1}^{4} \gamma_{1j} f_j^{(7)}(z) - \sum_{j=1}^{4} \left(\frac{\gamma_{1j}A_j}{z - z_{jo}} - \frac{\overline{\gamma_{1j}}\overline{A_j}}{z - \overline{z_{jo}}} \right),$$
$$\sum_{j=1}^{4} \overline{\gamma_{2j}} \bar{f}_j^{(7)}(z) = \sum_{j=1}^{4} \gamma_{2j} f_j^{(7)}(z) - \sum_{j=1}^{4} \left(\frac{\gamma_{2j}A_j}{z - z_{jo}} - \frac{\overline{\gamma_{2j}}\overline{A_j}}{z - \overline{z_{jo}}} \right),$$
$$\sum_{j=1}^{4} \overline{\gamma_{3j}} \bar{f}_j^{(7)}(z) = \sum_{j=1}^{4} \gamma_{3j} f_j^{(7)}(z) - \sum_{j=1}^{4} \left(\frac{\gamma_{3j}A_j}{z - z_{jo}} - \frac{\overline{\gamma_{3j}}\overline{A_j}}{z - \overline{z_{jo}}} \right),$$
$$\sum_{j=1}^{4} \overline{\gamma_{4j}} \bar{f}_j^{(7)}(z) = \sum_{j=1}^{4} \gamma_{4j} f_j^{(7)}(z) - \sum_{j=1}^{4} \left(\frac{\gamma_{4j}A_j}{z - z_{jo}} - \frac{\overline{\gamma_{4j}}\overline{A_j}}{z - \overline{z_{jo}}} \right), \tag{68}$$

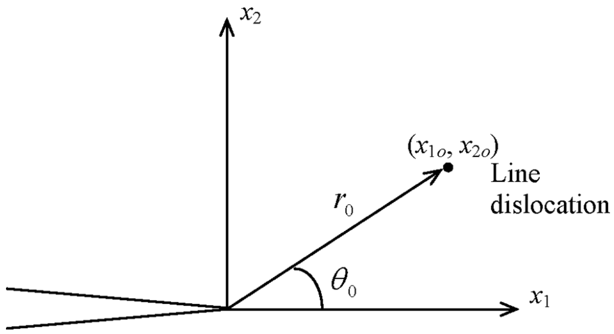

Figure 1. A line dislocation near a semi-infinite crack in an octagonal quasicrystal.

where $z_{jo} = x_{1o} + p_j x_{2o}$ ($j = 1$–4) and A_j ($j = 1$–4) are determined from eqs (62a) and (62b). Here we have replaced the complex variables z_j ($j = 1$–4) by the common complex variable $z = x_1 + \mathrm{i}x_2$ due to the fact that $z_1 = z_2 = z_3 = z_4 = z$ on the real axis [20,21]. When the analysis is finished, the complex variable $z = x_1 + \mathrm{i}x_2$ shall be changed back to the corresponding variables z_j ($j = 1$–4).

The traction-free boundary conditions $\sigma_{12} = \sigma_{22} = H_{12} = H_{22} = 0$ on the surfaces of the semi-infinite crack $x_1 < 0, x_2 = 0$ can be expressed as

$$\begin{aligned}
&\sum_{j=1}^{4} \gamma_{1j} f_j^{(7)}(x_1^+) + \sum_{j=1}^{4} \overline{\gamma_{1j}} \bar{f}_j^{(7)}(x_1^-) = 0,\\
&\sum_{j=1}^{4} \gamma_{2j} f_j^{(7)}(x_1^+) + \sum_{j=1}^{4} \overline{\gamma_{2j}} \bar{f}_j^{(7)}(x_1^-) = 0,\\
&\sum_{j=1}^{4} \gamma_{3j} f_j^{(7)}(x_1^+) + \sum_{j=1}^{4} \overline{\gamma_{3j}} \bar{f}_j^{(7)}(x_1^-) = 0,\\
&\sum_{j=1}^{4} \gamma_{4j} f_j^{(7)}(x_1^+) + \sum_{j=1}^{4} \overline{\gamma_{4j}} \bar{f}_j^{(7)}(x_1^-) = 0 \quad (x_1 < 0).
\end{aligned} \tag{69}$$

Substitution of eq. (68) into eq. (69) will result in

$$\begin{aligned}
&\sum_{j=1}^{4} \gamma_{1j} \left[f_j^{(7)}(x_1^+) + f_j^{(7)}(x_1^-) \right] = \sum_{j=1}^{4} \left(\frac{\gamma_{1j} A_j}{x_1 - z_{jo}} - \frac{\overline{\gamma_{1j}}\,\overline{A_j}}{x_1 - \overline{z_{jo}}} \right),\\
&\sum_{j=1}^{4} \gamma_{2j} \left[f_j^{(7)}(x_1^+) + f_j^{(7)}(x_1^-) \right] = \sum_{j=1}^{4} \left(\frac{\gamma_{2j} A_j}{x_1 - z_{jo}} - \frac{\overline{\gamma_{2j}}\,\overline{A_j}}{x_1 - \overline{z_{jo}}} \right),\\
&\sum_{j=1}^{4} \gamma_{3j} \left[f_j^{(7)}(x_1^+) + f_j^{(7)}(x_1^-) \right] = \sum_{j=1}^{4} \left(\frac{\gamma_{3j} A_j}{x_1 - z_{jo}} - \frac{\overline{\gamma_{3j}}\,\overline{A_j}}{x_1 - \overline{z_{jo}}} \right),
\end{aligned}$$

$$\sum_{j=1}^{4}\gamma_{4j}\left[f_j^{(7)}(x_1^+)+f_j^{(7)}(x_1^-)\right]=\sum_{j=1}^{4}\left(\frac{\gamma_{4j}A_j}{x_1-z_{jo}}-\frac{\overline{\gamma_{4j}}\overline{A_j}}{x_1-\overline{z_{jo}}}\right)\quad (x_1<0). \tag{70}$$

In order to simplify the analysis, we introduce four new analytic functions defined by

$$\begin{aligned} h_1(z)&=f_1^{(7)}(z)-\frac{A_1}{z-z_{1o}},\quad h_2(z)=f_2^{(7)}(z)-\frac{A_2}{z-z_{2o}},\\ h_3(z)&=f_3^{(7)}(z)-\frac{A_3}{z-z_{3o}},\quad h_4(z)=f_4^{(7)}(z)-\frac{A_4}{z-z_{4o}}. \end{aligned} \tag{71}$$

Consequently, eq. (70) can be rewritten as

$$\begin{aligned} \sum_{j=1}^{4}\gamma_{1j}\left[h_j(x_1^+)+h_j(x_1^-)\right]&=-\sum_{j=1}^{4}\left(\frac{\gamma_{1j}A_j}{x_1-z_{jo}}+\frac{\overline{\gamma_{1j}}\overline{A_j}}{x_1-\overline{z_{jo}}}\right),\\ \sum_{j=1}^{4}\gamma_{2j}\left[h_j(x_1^+)+h_j(x_1^-)\right]&=-\sum_{j=1}^{4}\left(\frac{\gamma_{2j}A_j}{x_1-z_{jo}}+\frac{\overline{\gamma_{2j}}\overline{A_j}}{x_1-\overline{z_{jo}}}\right),\\ \sum_{j=1}^{4}\gamma_{3j}\left[h_j(x_1^+)+h_j(x_1^-)\right]&=-\sum_{j=1}^{4}\left(\frac{\gamma_{3j}A_j}{x_1-z_{jo}}+\frac{\overline{\gamma_{3j}}\overline{A_j}}{x_1-\overline{z_{jo}}}\right),\\ \sum_{j=1}^{4}\gamma_{4j}\left[h_j(x_1^+)+h_j(x_1^-)\right]&=-\sum_{j=1}^{4}\left(\frac{\gamma_{4j}A_j}{x_1-z_{jo}}+\frac{\overline{\gamma_{4j}}\overline{A_j}}{x_1-\overline{z_{jo}}}\right)\quad (x_1<0). \end{aligned} \tag{72}$$

or equivalently

$$\begin{aligned} h_1(x_1^+)+h_1(x_1^-)&=-\frac{A_1}{x_1-z_{1o}}+\sum_{j=1}^{4}\frac{\beta_{1j}}{x_1-\overline{z_{jo}}},\\ h_2(x_1^+)+h_2(x_1^-)&=-\frac{A_2}{x_1-z_{2o}}+\sum_{j=1}^{4}\frac{\beta_{2j}}{x_1-\overline{z_{jo}}},\\ h_3(x_1^+)+h_3(x_1^-)&=-\frac{A_3}{x_1-z_{3o}}+\sum_{j=1}^{4}\frac{\beta_{3j}}{x_1-\overline{z_{jo}}},\\ h_4(x_1^+)+h_4(x_1^-)&=-\frac{A_4}{x_1-z_{4o}}+\sum_{j=1}^{4}\frac{\beta_{4j}}{x_1-\overline{z_{jo}}}\quad (x_1<0), \end{aligned} \tag{73}$$

where

$$\begin{bmatrix}\beta_{1j}\\ \beta_{2j}\\ \beta_{3j}\\ \beta_{4j}\end{bmatrix}=-\overline{A_j}\begin{bmatrix}\gamma_{11}&\gamma_{12}&\gamma_{13}&\gamma_{14}\\ \gamma_{21}&\gamma_{22}&\gamma_{23}&\gamma_{24}\\ \gamma_{31}&\gamma_{32}&\gamma_{33}&\gamma_{34}\\ \gamma_{41}&\gamma_{42}&\gamma_{43}&\gamma_{44}\end{bmatrix}^{-1}\begin{bmatrix}\overline{\gamma_{1j}}\\ \overline{\gamma_{2j}}\\ \overline{\gamma_{3j}}\\ \overline{\gamma_{4j}}\end{bmatrix}(j=1\text{–}4). \tag{74}$$

The Riemann–Hilbert problems (eq. (73)) can be easily solved as

$$
\begin{aligned}
h_1(z) &= -\frac{A_1}{2(z-z_{1o})} + \sum_{j=1}^{4} \frac{\beta_{1j}}{2(z-\overline{z_{jo}})} \\
&\quad + \frac{1}{2\sqrt{z}} \left[B_1 + \frac{A_1\sqrt{z_{1o}}}{z-z_{1o}} - \sum_{j=1}^{4} \frac{\beta_{1j}\sqrt{\overline{z_{jo}}}}{z-\overline{z_{jo}}} \right], \\
h_2(z) &= -\frac{A_2}{2(z-z_{2o})} + \sum_{j=1}^{4} \frac{\beta_{2j}}{2(z-\overline{z_{2o}})} \\
&\quad + \frac{1}{2\sqrt{z}} \left[B_2 + \frac{A_2\sqrt{z_{2o}}}{z-z_{2o}} - \sum_{j=1}^{4} \frac{\beta_{2j}\sqrt{\overline{z_{jo}}}}{z-\overline{z_{jo}}} \right], \\
h_3(z) &= -\frac{A_3}{2(z-z_{3o})} + \sum_{j=1}^{4} \frac{\beta_{3j}}{2(z-\overline{z_{3o}})} \\
&\quad + \frac{1}{2\sqrt{z}} \left[B_3 + \frac{A_3\sqrt{z_{3o}}}{z-z_{3o}} - \sum_{j=1}^{4} \frac{\beta_{3j}\sqrt{\overline{z_{jo}}}}{z-\overline{z_{jo}}} \right], \\
h_4(z) &= -\frac{A_4}{2(z-z_{4o})} + \sum_{j=1}^{4} \frac{\beta_{4j}}{2(z-\overline{z_{4o}})} \\
&\quad + \frac{1}{2\sqrt{z}} \left[B_4 + \frac{A_4\sqrt{z_{4o}}}{z-z_{4o}} - \sum_{j=1}^{4} \frac{\beta_{4j}\sqrt{\overline{z_{jo}}}}{z-\overline{z_{jo}}} \right],
\end{aligned} \tag{75}
$$

where B_j $(j = 1\text{–}4)$ are given by eq. (67).

In view of eqs (71) and (75), the explicit expressions for the four analytic functions $f_j(z_j)$ $(j = 1\text{–}4)$ are given by

$$
\begin{aligned}
f_1^{(7)}(z_1) &= \frac{A_1}{2(z_1-z_{1o})} + \sum_{j=1}^{4} \frac{\beta_{1j}}{2(z_1-\overline{z_{jo}})} \\
&\quad + \frac{1}{2\sqrt{z_1}} \left[B_1 + \frac{A_1\sqrt{z_{1o}}}{z_1-z_{1o}} - \sum_{j=1}^{4} \frac{\beta_{1j}\sqrt{\overline{z_{jo}}}}{z_1-\overline{z_{jo}}} \right], \\
f_2^{(7)}(z_2) &= \frac{A_2}{2(z_2-z_{2o})} + \sum_{j=1}^{4} \frac{\beta_{2j}}{2(z_2-\overline{z_{2o}})} \\
&\quad + \frac{1}{2\sqrt{z_2}} \left[B_2 + \frac{A_2\sqrt{z_{2o}}}{z_2-z_{2o}} - \sum_{j=1}^{4} \frac{\beta_{2j}\sqrt{\overline{z_{jo}}}}{z_2-\overline{z_{jo}}} \right], \\
f_3^{(7)}(z_3) &= \frac{A_3}{2(z_3-z_{3o})} + \sum_{j=1}^{4} \frac{\beta_{3j}}{2(z_3-\overline{z_{3o}})}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2\sqrt{z_3}}\left[B_3+\frac{A_3\sqrt{z_{3o}}}{z_3-z_{3o}}-\sum_{j=1}^{4}\frac{\beta_{3j}\sqrt{\overline{z_{jo}}}}{z_3-\overline{z_{jo}}}\right],\\
f_4^{(7)}(z_4)={}&\frac{A_4}{2(z_4-z_{4o})}+\sum_{j=1}^{4}\frac{\beta_{4j}}{2(z_4-\overline{z_{4o}})}\\
&+\frac{1}{2\sqrt{z}}\left[B_4+\frac{A_4\sqrt{z_{4o}}}{z_4-z_{4o}}-\sum_{j=1}^{4}\frac{\beta_{4j}\sqrt{\overline{z_{jo}}}}{z_4-\overline{z_{jo}}}\right]. \qquad (76)
\end{aligned}
$$

The local phonon and phason stress intensity factors induced by the straight dislocation and remote loads are determined by

$$
\begin{aligned}
k_{\mathrm{I}}&=\lim_{x_1\to0,x_2\to0}\sqrt{2\pi x_1}\sigma_{22}, \quad k_{\mathrm{II}}=\lim_{x_1\to0,x_2\to0}\sqrt{2\pi x_1}\sigma_{12},\\
t_{\mathrm{I}}&=\lim_{x_1\to0,x_2\to0}\sqrt{2\pi x_1}H_{22}, \quad t_{\mathrm{II}}=\lim_{x_1\to0,x_2\to0}\sqrt{2\pi x_1}H_{12}. \qquad (77)
\end{aligned}
$$

The local phonon and phason stress intensity factors induced by the straight dislocation and remote loads can be finally derived to be

$$
\begin{aligned}
k_{\mathrm{II}}&=K_{\mathrm{II}}-2\sqrt{2\pi}\mathrm{Re}\left\{\sum_{j=1}^{4}\frac{\gamma_{1j}A_j}{\sqrt{z_{jo}}}\right\},\\
k_{\mathrm{I}}&=K_{\mathrm{I}}-2\sqrt{2\pi}\mathrm{Re}\left\{\sum_{j=1}^{4}\frac{\gamma_{2j}A_j}{\sqrt{z_{jo}}}\right\},\\
t_{\mathrm{II}}&=T_{\mathrm{II}}-2\sqrt{2\pi}\mathrm{Re}\left\{\sum_{j=1}^{4}\frac{\gamma_{3j}A_j}{\sqrt{z_{jo}}}\right\},\\
t_{\mathrm{I}}&=T_{\mathrm{I}}-2\sqrt{2\pi}\mathrm{Re}\left\{\sum_{j=1}^{4}\frac{\gamma_{4j}A_j}{\sqrt{z_{jo}}}\right\}. \qquad (78)
\end{aligned}
$$

Particularly when the dislocation lies on the positive real axis, i.e., $x_{1o}>0$ and $y_{1o}=0$, then the stress intensity factors induced by dislocation are given below in view of eqs (62b) and (78):

$$
\begin{aligned}
k_{\mathrm{II}}&=K_{\mathrm{II}}-2\sqrt{\frac{2\pi}{x_{1o}}}\sum_{j=1}^{4}\gamma_{1j}A_j, \quad k_{\mathrm{I}}=K_{\mathrm{I}}-2\sqrt{\frac{2\pi}{x_{1o}}}\sum_{j=1}^{4}\gamma_{2j}A_j,\\
t_{\mathrm{II}}&=T_{\mathrm{II}}-2\sqrt{\frac{2\pi}{x_{1o}}}\sum_{j=1}^{4}\gamma_{3j}A_j, \quad t_{\mathrm{I}}=T_{\mathrm{I}}-2\sqrt{\frac{2\pi}{x_{1o}}}\sum_{j=1}^{4}\gamma_{4j}A_j. \qquad (79)
\end{aligned}
$$

4. Conclusions

In §2 a moving screw dislocation in a 1D hexagonal piezoelectric quasicrystal with point group $6mm$ is analyzed in detail within the framework of Landau theory. We first present a general solution in terms of two functions φ_1, φ_2, which satisfy wave equations, and another harmonic function φ_3. Based on the obtained general solution, analytic expressions for all the field variables such as displacements, electric potential, strains, stresses, electric fields and electric displacements are found. We also present the total energy of the moving screw dislocation. The obtained solutions are verified by comparison with existing ones. It shall be pointed out that the derived solutions are valid when $V < \min\{s_1, s_2\}$. The obtained general solution can also be conveniently utilized to analyze a Yoffe-type moving crack in a 1D hexagonal piezoelectric quasicrystal with point group $6mm$.

In §3 we have investigated the interaction of a straight dislocation with a semi-infinite crack in an octagonal quasicrystalline solid. The shielding or anti-shielding effect on the crack tip due to the neighboring straight dislocation can be easily observed from the expressions (78) and (79) for the local stress intensity factors induced by the straight dislocation.

Acknowledgements

The authors are indebted to the referee for his/her very helpful comments and suggestions. This work is supported in part by AFRL/ARL.

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