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The effect of instanton-induced interaction on P-wave meson spectra in constituent quark model

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Abstract. The mass spectrum of the P-wave mesons is considered in a non-relativistic constituent quark model. The full Hamiltonian used in the investigation includes the kinetic energy, the confinement potential, the one-gluon-exchange potential (OGEP) and the instanton-induced quark–antiquark interaction (III). A good description of the mass spectrum is obtained. The respective role of III and OGEP in the P-wave meson spectrum is discussed.

Keywords. Quark model; one-gluon-exchange potential; instanton-induced interaction; P-wave spectra.

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1. Introduction

Numerous papers have been devoted to the study of meson spectra in the framework of the potential models. These models are either non-relativistic quark models (NRQM) with a suitably chosen potential, or relativistic models where the interaction is treated perturbatively. The NRQM have been proven to be very successful in describing hadronic properties [1–9]. In most of these works, it is assumed that the quark interaction is dominated by a linear or quadratic confinement potential and is supplemented by a short-range potential stemming from the one-gluon exchange mechanism [1,2]. The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and a hyperfine interaction term, which has often been taken as an effective one-gluon-exchange potential (OGEP) [10]. Other types of hyperfine interaction have also been introduced in the literature. For example, the instanton-induced interaction (III), deduced by a non-relativistic reduction of the t'Hooft interaction [11,12], has already been successfully applied in several studies of the hadron spectra [5,6,13–16]. The main achievement of III in hadron spectroscopy is the resolution of the $U_{\mathrm{A}}(1)$ problem, which leads to a good prediction of the masses of η and η' mesons.

The Goldstone-boson-exchange interaction introduced by Glozman and Riska [17] furnishes another example of hyperfine interaction; it allows a good description of the baryon spectrum, and yields, in particular, a correct ordering for the positive and negative parity states. However, this model of Glozman and Riska can only be applied to study baryons and is thus unable to provide a unified description of the spectrum of hadrons. Very recently, the light cone harmonic oscillator models have been employed to study meson spectra and have been found to be very successful [18–20].

In view of the apparent success of NRQM in the description of S-wave spectra of mesons, we feel it is worthwhile to apply it to the case of orbitally excited states. This will allow much better understanding of the P-wave meson spectroscopy, where some of the $q\bar{q}$ quark model assignments of the known meson are still controversial. We hope it will also allow us a better understanding of the production properties of the P-wave mesons. In literature there are numerous attempts to understand the P-wave meson spectroscopy. The reference can be found in the review [21].

Previously, we had employed the NRQM [22] and the relativistic harmonic model (RHM) [23] along with III to investigate the ground state masses of pseudoscalar and vector mesons. In the RHM Hamiltonian of [23], we had used, in addition to III, a Lorentz scalar plus vector confinement potential along with the OGEP. The NRQM [22] had, besides the III, the usual kinetic energy term along with confinement potential and OGEP. In both the cases the results showed that the inclusion of III diminished the relative importance of OGEP for the hyperfine splitting. The aims of our earlier investigations were also to test whether quark gluon coupling constant (α_{s}) can be treated as a perturbative effect and to understand the role played by the III in meson spectra. Having met with reasonable success, in this work we have extended the NRQM to study P-wave spectra. The full Hamiltonian used in the investigation has, in addition to the central part of NRQM [22], spin-orbit and tensor terms of OGEP and III. In addition, III also has antisymmetric spin-orbit term proportional to $\vec{L}\cdot\vec{\Delta}$ where $\vec{\Delta}$ is defined in terms of the Pauli matrices as $\frac{1}{2}(\vec{\sigma}_1-\vec{\sigma}_2)$. The full discussion of the Hamiltonian is given in §2.

In general, the masses of the triplet P states of π, K and ϕ are higher than that of their singlet states. For example, in the $s\bar{s}$ sector, the mass of the singlet $h_1(1380)$ (with $N^{2S+1}L_J = 1^1P_1, J^{PC} = 1^{+-}$ following usual spectroscopic notation), is 1386±19 MeV [24] whereas masses of $f_0^*(1500)$, $f_1(1420)$ and $f_2'(1525)$ (given by spectroscopic notations $(1^3P_0, 0^{++}); (1^3P_1, 1^{++})$, and $(1^3P_2, 2^{++})$ are 1507±5, 1426.3±1.1 and 1525±5 MeV respectively. If the same set of parameters are used to reproduce the ground state and P-wave spectra only with OGEP it will not be possible to reproduce the observed spectra as the tensor and spin-orbit terms of OGEP are attractive, and hence naturally triplet states masses will be lower than the corresponding singlet states. Hence, to reproduce the full P-wave spectra it is essential to include the hyperfine interaction term of III to have a consistent description. We also attempt to examine the role of anti-symmetric spin-orbit term of III which couples ${}^1L_{J=L}$ and ${}^3L_{J=L}$ in the K-meson sector. In §2, we review briefly the NRQM and give explicit expression of the full potential of OGEP and III. We also discuss the parameters involved in our model. The results of the calculation are presented in §4 and the conclusions are given in §5.

2. The model

The full Hamiltonian is given by

$$H = K + V_{\text{conf}} + V_{\text{OGEP}} + V_{\text{III}}, \tag{2.1}$$

where

$$K = \left[\sum_{i=1}^{2} M_i + \frac{P_i^2}{2M_i}\right] - K_{\text{cm}} \tag{2.2}$$

with M_i and P_i representing the ith quark mass (see table 1) and its momentum respectively. Thus K is the sum of the kinetic energies including the rest mass minus the kinetic energy of the center-of-mass motion K_{cm} of the total system. The potential energy part consists of confinement term V_{conf} and the residual interactions, namely the one-gluon-exchange potential represented by V_{OGEP} and the instanton-induced interaction V_{III}. It may be noted that in the calculation of mass spectrum, inclusion of only two-body potentials as functions of relative position $\vec{r}_{ij}$ would suffice.

2.1 *Central parts of the potentials*

The confinement potential V_{conf} is entirely central in character and is given by

$$V_{\text{conf}}(\vec{r}_{ij}) = -a_{\text{c}} r_{ij} \lambda_i \cdot \lambda_j, \tag{2.3}$$

where a_c is the confinement strength. The term r_{ij} here and elsewhere in the paper stands for the relative distance between the two quarks and λ_i represents the generator of the color $SU(3)$ group for the ith quark. The following central part of the OGEP is usually employed:

$$V_{\text{conf}}(\vec{r}_{ij}) = -a_{\text{c}} r_{ij} \lambda_i \cdot \lambda_j, \tag{2.4}$$

$$V_{\text{OGEP}}^{\text{Cent}}(\vec{r}_{ij}) = \frac{\alpha_{\text{s}}}{4} \lambda_i \cdot \lambda_j \left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}\left(1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right)\delta(\vec{r}_{ij})\right], \tag{2.5}$$

where α_s represents the quark-gluon coupling constant. The first term of the equation represents the residual Coulomb energy and the second term the chromomagnetic interaction leading to the hyperfine splitting. The radial part of III is given by [5,6]

$$V_{\rm III}^{\rm Cent}(\vec{r}_{ij}) = \begin{cases} -8g\delta(\vec{r}_{ij})\delta_{S,0}\delta_{L,0}, & I=1, \\ -8g'\delta(\vec{r}_{ij})\delta_{S,0}\delta_{L,0}, & I=1/2, \\ 8\begin{pmatrix} g & \sqrt{2}g' \\ \sqrt{2}g' & 0 \end{pmatrix}\delta(\vec{r}_{ij})\delta_{S,0}\delta_{L,0}, & I=0. \end{cases} \tag{2.6}$$

where g and g' are coupling constants given in table 1. The Dirac delta functions are regularized by the Gaussian-like function [5,6]

$$\delta(\vec{r}_{ij}) \to \frac{1}{(\Lambda\sqrt{\pi})^3}\exp\left(-\frac{r_{ij}^2}{\Lambda^2}\right), \tag{2.7}$$

with a scale parameter Λ specified in table 1.

2.2 *Non-central part of OGEP*

The non-central part of the OGEP constitutes two terms, namely, the tensor term $V_{\rm OGEP}^{\rm T}(\vec{r}_{ij})$ and the spin-orbit interaction $V_{\rm OGEP}^{\rm SO}(\vec{r}_{ij})$. There are several versions of the tensor term in literature. We use the expression derived in [10] from the QCD Lagrangian in the non-relativistic limit and used subsequently by many authors (see [25,26]):

$$V_{\rm OGEP}^{\rm T}(\vec{r}_{ij}) = -\frac{\alpha_{\rm s}}{4}\lambda_i\cdot\lambda_j\left[\frac{1}{4M_iM_j}\frac{1}{r_{ij}^3}\right]\hat{S}_{ij}, \tag{2.8}$$

where $\hat{S}_{ij}= 3(\vec{\sigma}_i\cdot\hat{r})(\vec{\sigma}_j\cdot\hat{r})-\vec{\sigma}_i\cdot\vec{\sigma}_j$. The tensor potential is a scalar which is obtained by contracting two second rank tensors. Here, $\hat{r}=\hat{r}_i-\hat{r}_j$ is the unit vector in the direction of $\hat{r}$. In the presence of the tensor interaction, $\vec{L}$ is no longer a good quantum number.

The spin-orbit (SO) interaction of the OGEP is given by

$$\begin{aligned} V_{\rm OGEP}^{\rm SO}(\vec{r}_{ij}) &= -\frac{\alpha_{\rm s}}{4}\lambda_i\cdot\lambda_j \\ &\times\left[\frac{3}{8M_iM_j}\frac{1}{r_{ij}^3}(\vec{r}_{ij}\times\vec{P}_{ij})\cdot(\vec{\sigma}_i+\vec{\sigma}_j)\right], \end{aligned} \tag{2.9}$$

where the angular momentum is defined as usual in terms of relative position $\vec{r}_{ij}$ and the relative momentum $\vec{P}_{ij}$. Unlike the tensor force, the spin-orbit force does not mix states of different $\vec{L}$, since L^2 commutes with $\vec{L}\cdot\vec{S}$, $\vec{L}$ is still a constant of motion, but L_z is not.

2.3 *Non-central part of III*

The tensor term of III is

$$V_{\rm III}^{\rm T}(\vec{r}_{ij}) = \frac{\hat{S}_{ij}}{M_i M_j}\sum_{k=7}^{8}\kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3}. \tag{2.10}$$

The spin-orbit term of III is (see refs [5,6]) given by

$$V_{\rm III}^{\rm SO}(\vec{r}_{ij}) = V_{\rm LS}(r_{ij})\vec{L}\cdot\vec{S} + V_{\rm L\Delta}(r_{ij})\vec{L}\cdot\vec{\Delta}. \tag{2.11}$$

The first term in eq. (2.11) is the traditional symmetric spin-orbit term proportional to the operator $\vec{L}\cdot\vec{S}$. The other term is the anti-symmetric spin-orbit term proportional to $\vec{L}\cdot\vec{\Delta}$, where $\vec{\Delta} = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)$. The radial functions of eq. (2.11) are expressed as

$$V_{\rm III}^{\rm LS}(r_{ij}) = \left(\frac{1}{M_i^2} + \frac{1}{M_j^2}\right)\sum_{k=1}^{2}\kappa_k \frac{\exp(-r_{ij}^2/\eta_k^2)}{(\eta_k\sqrt{\pi})^3} + \frac{1}{M_i M_j}\sum_{k=3}^{4}\kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-2}^2)}{(\eta_{k-2}\sqrt{\pi})^3} \tag{2.12}$$

and

$$V_{\rm III}^{\rm L\Delta}(r_{ij}) = \left(\frac{1}{M_i^2} - \frac{1}{M_j^2}\right)\sum_{k=5}^{6}\kappa_k \frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3}. \tag{2.13}$$

The terms κ_i and η_i are free parameters in the theory [6]. The term $V_{\rm LS}^{\rm III}(r_{ij})$ is responsible for the splitting of the ${}^3L_J$ states with $J = L-1, L, L+1$. The term $V_{\Delta}^{\rm III}(r_{ij})$ couples states ${}^1L_{J=L}$ and ${}^3L_{J=L}$ and due to mass dependence this term is inoperative when the quarks are identical. Hence it contributes to the splitting in the $u-s$ and $d-s$ sector. In eq. (2.13), M_i corresponds to mass of the strange (s) quark and M_j corresponds to the mass of u/d quark. This term accounts partially for the splitting between ${}^3P_1$ and ${}^1P_1$ states in K-sector. The above form of expressions (2.10)–(2.13) are used by a number of authors (for e.g. ref. [6]). It is derived from the special solution of the QCD Lagrangian and is obtained from the non-relativistic reduction of the t'Hooft's interaction [5,11]. It is to be noted that the III and the OGEP have the same spin dependence except for the $V_{\rm L\Delta}$ term.

3. Fitting procedure

The purpose of the present work is to reproduce the P-wave spectra of light mesons in the framework of NRQM. In our calculation we have expressed the product of quark–antiquark oscillator wave functions in terms of oscillator wave functions corresponding to the relative and center-of-mass coordinates. The relative wave function for $0P$ state is

$$\psi_{0P}(r_{ij}) = \sqrt{\frac{8}{3\sqrt{\pi}}}\frac{r_{ij}}{b^{5/2}}\exp\left(\frac{-r_{ij}^2}{2b^2}\right), \tag{3.1}$$

Table 1. Values of the parameters used in our model.

b	0.77 fm
$M_{\mathrm{u,d}}$	371 MeV
M_{s}	512 MeV
a_{c}	23.0 MeV fm^{-1}
α_{s}	0.3
g	0.0847×10^{-4} MeV^{-2}
g'	0.0535×10^{-4} MeV^{-2}
Λ	0.35 fm
η_1	0.194 fm
η_2	0.294 fm
η_3	0.112 fm
η_4	1.501 fm
κ_1	0.213
κ_2	0.091
κ_3	-2.13
κ_4	2.651
κ_5	38.64
κ_6	40.43

where b is the oscillator size parameter. There are several papers in literature where the size parameter b is defined [27,28]. In computing the meson masses we have diagonalized the matrix $(\langle\Psi_{ip}|H|\Psi_{jp}\rangle)_{i,j=0,1,2,3,4}$ in the relative space, where we have restricted the angular momentum of center-of-mass wave function to zero.

There are nine parameters associated with the central parts of the potentials as mentioned in the previous paper [22]. These are the masses of the u, d and s quarks, the confinement strength a_{c}, the harmonic oscillator size parameter b, the strong coupling constant α_{s} and the strength parameters of III, namely, g, g' and Λ respectively (eqs (2.2)–(2.7)). To reproduce the S-wave spectra of light mesons we needed these nine parameters [22]. In the current work, these parameter values are fixed at the values chosen in [22] and are given in table 1.

Among the non-central parts of the potentials, the hyperfine terms of III has 12 additional strength and size parameters κ and η (eqs (2.10)–2.13)) respectively. These are chosen as free parameters in the present model. We are able to reproduce the light P-wave meson masses with all η and κ_1 to κ_6 parameters held fixed and by varying only the κ_7 and κ_8 parameters. The values of κ_7 and κ_8 are listed in table 2. It is further remarkable that for each category of meson nonet, κ_7 is held fixed and only the κ_8 varies. We would wish to stress that we have used the same value of α_{s} as in our previous work which is compatible with the perturbative treatment. As remarked earlier, the non-central term of OGEP is attractive, whereas the strengths of the interaction of III (i.e., κ) can have both positive and negative values [6]. We are led to the conclusion that inclusion of III in the formalism is essential. This also enables us to bring down the value of α_{s} so as to be compatible with the perturbative treatment.

Table 2. Values of κ_7 and κ_8 parameters used in our model.

$N^{2S+1}L_J$	Meson	κ_7	κ_8
1^3P_0	$a_0(980)$	−38.88	−27.968
1^3P_0	$K_0^*(1430)$	−38.88	−24.235
1^3P_0	$a_0(1450)$	−38.88	−39.952
1^3P_0	$f_0(1500)$	−38.88	−35.563
1^3P_1	$a_1(1260)$	−18.92	22.083
1^3P_1	$K_1(1270)$	−18.92	8.071
1^3P_1	$f_1(1420)$	−18.92	38.695
1^3P_2	$a_2(1320)$	−18.92	9.097
1^3P_2	$K_2^*(1430)$	−18.92	34.712
1^3P_2	$f_2'(1525)$	−18.92	72.179

Table 3. The pseudo-vector meson masses (in MeV).

Meson	$b_1(1235)$	$h_1'(1380)$	$K_{1B}(1400)$
Experiment	1229.5±3.2	1386±19	1402±7
Theory	1229.14	1388.06	1403.7

4. Results and discussions

The $q\bar{q}$ wave function for each meson is expressed in terms of oscillator wave functions corresponding to the CM and relative coordinates. The oscillator quantum number for the CM wave functions are restricted to $N_{\rm cm} = 0$. The Hilbert space of relative wave functions is truncated at radial quantum number $n = 4$. The Hamiltonian matrix is constructed for each meson separately in the basis states of $|N_{\rm cm} = 0, L_{\rm cm} = 0; N^{2S+1}L_J\rangle$ and diagonalized.

The masses of the singlet and triplet P-wave mesons after diagonalization in harmonic oscillator basis with $n_{\rm max} = 4$ are listed in tables 3 and 4 respectively. The diagonal contributions to the masses of some of the mesons by the kinetic energy and the color electric, color magnetic and confinement potential terms are listed in table 5.

We have observed that only the OGEP hyperfine interaction is not sufficient to reproduce the masses of the mesons. The important role played by the III in reproducing the masses of these mesons (as shown in table 4) can be gauged by examining table 6 where the masses of the scalar mesons calculated after switching off the III in the full Hamiltonian (in eq. (2.1)) are tabulated. This is because the tensor and spin-orbit terms of OGEP are attractive and hence bring down the masses of the triplet state. The κ parameters in the tensor and spin-orbit terms of III are treated as free (tunable) parameters, the attractive or repulsive nature of III being governed by the sign of the κ. Thus by tuning the κ parameters appropriately, we are able to reproduce the meson masses in our model.

Table 4. The triplet meson masses (in MeV).

$N^{2S+1}L_J$	Meson	Experiment	Theory
1^3P_0	$a_0(980)$	984.7±1.2	988.53
1^3P_0	$K_0^*(1430)$	1412±6	1412.53
1^3P_0	$a_0(1450)$	1474±19	1473.59
1^3P_0	$f_0^*(1500)$	1507±5	1512.28
1^3P_1	$a_1(1260)$	1230±40	1233.07
1^3P_1	$K_1(1270)$	1273±7	1274.10
1^3P_1	$f_1(1420)$	1426.3±0.9	1423.37
1^3P_2	$a_2(1320)$	1318±0.6	1347.65
1^3P_2	$K_2^*(1430)$	1425.6±1.5	1432.41
1^3P_2	$f_2'(1525)$	1525±5	1529.33

Table 5. The diagonal contributions to the masses of some mesons by the kinetic energy and the color-electric and color-magnetic terms of V_{OGEP} and confinement potential in units of MeV. The contributions for $a_0(1450)$ has only been listed.

Meson	KE	CE-OGEP	CM-OGEP	Linear confinement
a_0	1184.604	−73.051	2.71	142.106
K_0^*	1264.659	−74.17	1.96	142.106
f_0	1344.715	−74.98	1.42	142.106
b_1	1184.604	−73.051	−8.13	142.106
K_1	1264.659	−74.17	−5.89	142.106
h_1	1344.715	−74.98	−4.26	142.106

Table 6. The masses of scalar mesons (in units of MeV) after diagonalization with only V_{OGEP}.

Meson	Mass
$a_0(1450)$	1020.32
K_0^*	1162.32
f_0	1280.34

4.1 *Pseudovector meson nonet* (1^1P_1)

We have investigated three mesons of the 1^1P_1 pseudovector meson nonet with $J^{\mathrm{PC}} = 1^{+-}$, namely, $b_1(1235), h_1'(1380), K_1(1400)$ [24]. It may be pointed out here that there is no contribution from the III for the singlet states except for the $^1P_1$ state in the K-sector. In the K-sector, the singlet P state receives a significant repulsive contribution of 93 MeV from the off-diagonal matrix element (ME) of $\langle {}^3P_1|V_{L\Delta}|{}^1P_1\rangle$.

4.2 *Scalar meson nonet* (1^3P_0)

The spectrum of the scalar meson nonet is very large and the actual number of resonances in the region of 1–2 GeV far exceeds the number of states which the conventional quark models can accommodate. Several of these states, however, have been interpreted as exotic mesons. It is well-known that a $q\bar{q}$ meson with orbital angular momentum l and total spin s must have parity $P = (-1)^{l+1}$ and charge conjugation quantum number $C = (-1)^{l+s}$. On this basis, we define an exotic meson to be one which does not have the above spectroscopic configurations. Thus a resonance with $J^{\mathrm{PC}} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$ are exotic. Such states could be a gluonic excitation such as a hybrid ($q\bar{q}g$) or glue ball (2g, 3g,...) or a multi-quark state ($q\bar{q}q\bar{q}$).

The particle data group (PDG) lists isoscalar states, the $a_0(980)$ and $a_0(1450)$ [24] having masses of 984.7 ± 1.2 MeV and 1450 ± 40 MeV respectively. Theories based on chiral sigma models with three flavors [29] suggest that $a_0(980)$ would form a scalar nonet. The scalar $K_0^*(1430)$ is well-established. Several groups have claimed different isoscalar scalar structures close to 1500 MeV [30,31]. In this work, we focus our attention only on the non-exotic scalar mesons with $J^{\mathrm{PC}} = 0^{++}$ and assigned as $a_0(980)$, $a_0(1450)$, $K_0^*(1430)$, $f_0(1500)$ [24].

4.3 *Axial vector meson nonet* (1^3P_1)

In our model, for axial vector mesons, the tensor and $\vec{L} \cdot \vec{S}$ parts of OGEP and III have opposite signs. The contributions due to tensor terms are repulsive, whereas those due to $\vec{L} \cdot \vec{S}$ terms are attractive. As the OGEP has the same strength parameter for these terms, the contribution of the hyperfine interaction terms of OGEP is negligible whereas, due to the different strength parameters κ_i, the corresponding terms of III contribute differently. Besides, the contribution of III to the masses is also significant because of the different radial form of tensor and spin-orbit terms. We have treated κ_i as free parameters so as to reproduce the masses of $a_1(1260)$, $K_1(1270)$ and $f_1(1420)$. However, it should be noted that the $a_1(1260)$, with $I = 1$ has a significant width of 400 MeV and has a dominant decay channel $a_1 \to \rho\pi$. This property makes the determination of its mass difficult. The QCD sum rules [32] produce a mass of 1150 ± 40 MeV. According to Bowler [33], the a_1 mass and width are safely within the ranges $\simeq 1235 \pm 40$ MeV and 400 ± 100 MeV respectively. These values are in agreement with those currently adopted by PDG [24], i.e., mass of 1230 ± 40 MeV and width 250 MeV to 600 MeV. In the K-sector, we have fitted to $K_1(1270)$. The contribution from the ME $\langle 1^1P_1|V_{L\Delta}|1^3P_1\rangle$ has been found to be significant. PDG cite two f_1 meson states [24] with $J^{\mathrm{PC}} = 1^{++}$, namely, $f_1(1285)$ and $f_1(1420)$. There has been considerable discussion on the quark structure of these mesons [8]. We have been able to fit the masses $f_1(1420)$ as shown in table 4.

4.4 *Tensor meson nonet* (1^3P_2)

We consider some of the well-established members of the tensor meson nonets, with $J^{\mathrm{PC}} = 2^{++}$, i.e., $a_2(1320), K_2^*(1430)$ and $f_2'(1525)$. The contributions due to tensor and $\vec{L}\cdot\vec{S}$ terms of OGEP and III bear opposite signs. The tensor potential is attractive whereas $\vec{L}\cdot\vec{S}$ part is repulsive. However, the off-diagonal tensor ME $\langle 3P_2|V^{\mathrm{T}}_{\mathrm{OGEP}}|^3F_2\rangle$ is strongly repulsive. In our model the mass difference between f_1 and f_2' essentially comes from the off-diagonal ME of tensor potential of OGEP and III.

In literature some more $J^{\mathrm{PC}} = 2^{++}$ states like $f_2(1520)$, $f_2(1810)$, $f_2(2010)$, $f_2(2340)$ have been considered. Of these, $f_2(1810)$ is likely to be the 2^3P_2 state [8]. Our model prediction for $f_2(1810)$ is 1724.52 MeV.

5. Conclusions

We have shown that light meson P-wave spectra can be described in the framework of NRQM with the conventional OGEP and by including III. The quark masses, the oscillator size, α_{s} of OGEP, confinement strength a_{c} and the parameters of the central part of III (g, g' and Λ) are fixed from S-wave meson spectra [22]. Our calculations clearly point out the importance of III. The contribution of III to triplet P states is of the order of the contribution of OGEP. Also, the inclusion of III consequently diminish the relative importance of OGEP. The III also restricts α_{s} to be 0.3 and thus justifying the perturbative truncation of multi-gluon exchanges. The near mass degeneracy of the experimentally established iso-doublet states of the scalar and tensor meson nonets K_0^* and K_2^* could be accounted by the off-diagonal tensor ME of OGEP and III. The simultaneous mass degeneracy of the pseudo-vector $K_{1\mathrm{B}}$ and axial vector $K_{1\mathrm{A}}$ which mix to give physical $K_1(1270)$ and $K_1(1400)$ states observed experimentally could be accounted for by the anti-symmetric spin-orbit term $V_{L\Delta}$ of III. As we have shown, NRQM with OGEP and III provides quite a good description of the pseudo-vector, scalar, axial vector and tensor P-wave mesons with the same constituent quark masses, oscillator size, confinement strength a_{c} and OGEP strength α_{s}. Hence, we have a consistent NRQM which reasonably reproduce the S- and P-wave light meson spectra.

In our work, we have investigated the effect of the III on the P-wave masses of light mesons in the framework of NRQM. We have shown that the computation of triplet P-wave mesonic masses/mass splitting using OGEP only is inadequate. To obtain the masses of triplet P states, it is necessary to use a combination of OGEP (with a relatively smaller strength) and III potentials.

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