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Application of inertia-induced excitation theory for nonlinear acoustic modes in colloidal plasma equilibrium flow

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Abstract. Application of inertia-induced acoustic excitation theory offers a new resonant excitation source channel of acoustic turbulence in the transonic domain of plasma flow. In bi-ion plasmas like colloidal plasma, two well-defined transonic points exist corresponding to the parent ion and the dust grain-associated acoustic modes. As usual, the modified ion acoustic mode (also known as dust ion-acoustic (DIA) wave) dynamics associated with parent ion inertia is excitable for both nanoscale- and micronscale-sized dust grains. It is found that the so-called (ion) acoustic mode (also known as dust-acoustic (DA) wave) associated with nanoscale dust grain inertia is indeed resonantly excitable through the active role of weak but finite parent ion inertia. It is interestingly conjectured that the same excitation physics, as in the case of normal plasma sound mode, operates through the active inertial role of plasma thermal species. Details of the nonlinear acoustic mode analyses of current interest in transonic domains of such impure plasmas in hydrodynamic flow are presented.

Keywords. Dust-acoustic (DA) wave; dust ion-acoustic (DIA) wave; constant dust charge model; *d*-KdV equation; colloidal plasma fluids; dusty plasma; complex plasma; transonic plasma; fluid acoustic modes; acoustic turbulence; acoustic fluctuations.

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1. Introduction

Acoustic mode in plasmas is actually a pressure-driven longitudinal wave like the ordinary sound mode in neutral gas. In normal two-component plasmas, the electron thermal pressure drives the collective ion oscillations to propagate as the ion sound (acoustic) wave. Here the electron thermal pressure provides the restoring force to allow the collective ion dynamics in the form of ionic compression and rarefaction to move in the plasma background. Thermal plasma species (like electrons) are free to carry out screening of the potential. In the absence of any dissipative mechanism, the ion sound wave moves with constant amplitude. For mathematical description of the ion sound kinetics, the electrons are normally treated inertialess and the ions with full inertial dynamics. However, recent discovery of ion sound wave excitation in transonic plasma condition [1,2] of hydrodynamic equilibrium offers a new physical scope of acoustic turbulence due to weak but finite electron inertial delay effect. Qualitative and quantitative modifications are introduced into its nonlinear counterpart [3] as well.

The transonic transition of the plasma flow motion occurs quite naturally in the neighborhood of the boundary wall surface of laboratory plasmas, self-similar expansion of plasmas into vacuum, in solar wind plasmas, astrophysical plasmas, etc. The self-similar expansion plasma model predicts supersonic motion of plasma flow into vacuum. This model is widely used to describe the motion of intense ion plasma jets produced by short time pulse laser interaction with solid target ([4] and references therein). Recently, self-similar plasma expansion into vacuum is modeled by the appropriate consideration of space charge separation effect on the expanding front [4].

If there is multispecies ionic composition in a plasma system, varieties of plasma sound waves are likely to exist depending on, in principle, the number of ion species. In plasmas containing dust or fine suspended particles, two distinct kinds of natural plasma sound modes are possible [5,6]. Such plasmas, termed as the colloidal plasmas [7], have become the subject of intensive study in various fields of physics and engineering such as space, astrophysics [8,9], plasma physics [10], plasma-aided manufacturing [11,12] and lastly, fusion technology [13,14]. The dust grains or the solid fine particles suspended in low-temperature gaseous plasmas are usually negatively charged. It is also observed that plasmas including microscale sized and nanoscale sized suspended particles exist in many natural conditions of technological values. Such plasmas have been generated in laboratories with a view to investigate the dust grain charging physics, plasma wave physics as well as instability phenomena.

The two distinct sound modes, however, in colloidal plasma are well-separated in space and time-scales due to a wide range of variations of mass scaling of the normal ions and the charged dust grains and free electrons' populations. The charged dust grains are termed as the dust grain-like impurity ions [6] to distinguish from the normal impurity ions. The present contribution applies the inertia-induced acoustic excitation theory to nonlinear description of plasma sound modes [1–3] in colloidal plasma. Two separate cases of ion flow motion and dust grain motions are considered. It is indeed found that the modified ion acoustic wave (m-IAW) [5,6] or dust ion-acoustic (DIA) wave [15,16] and the so-called (ion) acoustic wave (s-IAW) [5,6] or dust acoustic (DA) wave [15,16] both become nonlinearly unstable due to the active role of weak but finite inertial correction of the respective plasma thermal species. Proper mass domain of the dust grains for instability to occur is estimated.

By these calculations, a generalized statement is reported that all possible sound modes in multi-species plasmas with drift motions could be destabilized by the inertial delay effect of the corresponding plasma thermal species that carry out thermal screening of acoustic potential. Of course, threshold values may differ depending on the choice of the plasma sound mode under consideration. Section 2 includes general physical discussions of the plasma model and equilibrium. Section 3 considers the nonlinear wave propagation behavior of the m-IAW. Similarly, §4 includes the nonlinear wave propagation dynamics of the s-IAW. Section 5 contains results and discussions. Distinctive features of our acoustic excitation mechanism are presented in §6. Lastly, conclusions are presented in §7.

2. Plasma model

A. General physical outlook

A simplified field-free model of a dust grain laden gaseous mixture of free ions, electrons and dust grain-like impurity ions is considered. This is a classical system of solid phase distribution of dust grains in the gaseous phase of plasma background and is popularly known as the colloidal plasma [7]. This is also called by different nomenclatures like, dusty or complex plasma [15,16]. However, 'colloidal plasma' is believed to be the more suitable nomenclature over the others because of the intermixed state of solid–gas phases. The unique quality of colloidal plasma is associated with the dust grain charge as an additional physical variable of relevance. We use constant dust grain charge model under the spherical symmetry of individual dust grain charging. Thus a simple normal bi-ion plasma model could be applied to study the bulk acoustic mode behaviors in colloidal plasma system.

Traditionally, the role of weak but finite inertia of respective plasma thermal species is ignored under the asymptotic mass limits of $m_e/m_i \to 0$ and $m_i/m_I \to 0$. This, in fact, is inadequate and impractical for a proper discussion of the acoustic modes in transonic plasma flow condition of the corresponding inertial species. This is to note that the basic set of dynamical equations of the plasma particles' fluid will depend on the choice of the acoustic mode under consideration.

Interestingly, for dust grain-associated acoustic mode, the electrodynamical role of thermal electrons in perturbations could be ignored if $\lambda_{\mathrm{De}} \gg \lambda_{\mathrm{Di}}$. This is again justified under the background that the collisional process of thermal loss of plasma electrons charges dust grains negatively and hence, electron density is reduced. In this limit, a scope exists for ion inertia-induced excitation of the so-called (ion) acoustic mode (also known as dust acoustic wave) in the presence of nanoscale sized particles. The following estimation of $m_e/m_I \approx \varepsilon^2$ and $m_i/m_I \approx m_e/m_i = \varepsilon$ for nanosize particles justifies this. For micron size particles $m_e/m_I \approx \varepsilon^5$ and $m_i/m_I \approx \varepsilon^4$ and hence, only modified ion acoustic wave (also known as dust ion-acoustic (DIA) wave) is excitable for both micro and nanosize particles.

B. Equilibrium description

A simplified situation is considered where the electrons, ions and dust grains have acquired thermodynamic equilibrium. Now there are two kinds of collective scale equilibrium states of plasmas: exact hydrostatic type and quasi-hydrostatic (hydrodynamic) type. The latter has a global flow motion with Bernoulli kind of equilibrium condition such that the total energy is an integral of motion. Mathematically, the hydrostatic kind of equilibrium is described by an exact balancing of the global forces acting on the plasma. But even this simple equilibrium has produced a quite rich scope of basic understanding of the wave dynamics in plasmas.

However, in reality, the plasmas in space as well as in laboratory conditions are always associated with finite global flow motions. Here for clarity, let us define the global scale as any scale length much larger than the plasma electron Debye length (λ_{De}). A simple form of this kind of equilibrium is mathematically describable by the Bernoulli-type of equation. Here the exact force balancing does not exist between the mean flow kinetic energy and potential energy and hence, sustains the desired steady state of the global flow motion of the plasma. Now the question arises whether the quasi-neutral plasma flow motions will carry any current or not. For this, one would have to look into the following form of the current equation:

$$\nabla \cdot J + \frac{\partial}{\partial t}(\nabla \cdot E) = 0. \tag{1}$$

The net conduction current density in our defined colloidal plasma system is defined and given by $J = J_e + J_i + J_I = -en_e v_e + q_i n_i v_i - q_I n_I v_I$. In one-dimensional case of electrostatic field approximation, eq. (1) can be rewritten as

$$\frac{\partial}{\partial x}\left(J + \frac{\partial}{\partial t}E\right) = 0. \tag{2}$$

This implies that $J + \frac{\partial}{\partial t}E =$ constant or a solenoidal vector field (divergence-free) and hence, space-invariant conduction current is supposed to flow in hydrodynamic equilibrium. The notations used are generic in nature. Here E represents electric field vector. To ensure exact quasi-neutral equilibrium flow ($\nabla \cdot E = 0$), divergence-free current ($\nabla \cdot J = 0$) has to be sustained. Now the expression for the total conduction current density (J_0) for quasi-neutral colloidal plasma flow in hydrodynamic equilibrium can be written as

$$J_0 = -en_{e0}\left(v_{e0} - v_{I0}\right) + q_i n_{i0}\left(v_{i0} - v_{I0}\right). \tag{3}$$

Now two different current regimes of flow motions will be considered. In one regime of colloidal plasma flow motions as defined by ($v_0 \sim v_{e0} \sim v_{i0} \sim c_{sm} \gg v_{I0} \sim c_{ss}$), $J_0 \approx -en_{e0}(v_{e0}) + q_i n_{i0}(v_{i0})$. This may be termed as the parent plasma current dominated flow regime. In this regime of plasma flow motion, the modified ion acoustic wave (m-IAW) is more likely to be excited by electron inertia-induced acoustic excitation theory. In this regime, it looks as if the parent plasma as a whole is moving through an approximately static background of dust grains with supersonic speed with respect to m-IAW speed.

In the second regime of colloidal plasma flow motions as defined by ($v_0 \sim v_{I0} \approx c_{ss}$), it is seen that $J_0 \approx -q_I n_{I0}(v_{I0})$. In this regime of dust grain dominated current, the so-called (ion) acoustic wave (s-IAW) is more likely to be excited by parent ion inertia-induced excitation physics. Such physical situations are practically realizable in laboratory conditions [17]. For uniform plasma density and flow motions, the divergence-free current approximation could be justified. Under these simplified situations of equilibrium colloidal plasma flow motion, linear behaviors of two distinct eigenmodes of acoustic spectrum in transonic plasma conditions of ion and dust grain flow motions will be investigated and discussed.

3. Modified ion acoustic wave

In the presence of any additional heavier ion inertial species like dust grain-like impurity ions, the given background (parent) plasma system may become highly deficient of free electrons and hence the parent plasma becomes electrically non-neutral. The loss of free electrons appears in the form of negative charging of the dust grains and hence, the overall quasi-neutrality of the global colloidal plasma volume holds well. Both frequency and phase speed of the normal ion acoustic wave of the parent plasma system are scaled up in the presence of dust grains. These two become the function of the charge state and charge density ratio of parent plasma ions with respect to plasma electrons.

For parent ion inertial time-scale, the dust grain dynamics could be ignored. This may be justified for micron size and the nanosize colloidal particles as well, as discussed in §2.

A. Basic equations

As discussed earlier, a more suitable form of electron continuity equation for the description of m-IAW in colloidal plasma with drifting ions in the immobile dust grain background is given as

$$\frac{\partial \phi}{\partial t} + v_e \frac{\partial \phi}{\partial x} + \frac{T_e}{e} \frac{\partial v_e}{\partial x} = 0. \tag{4}$$

Here ϕ is the electrostatic potential due to local charge imbalance, v_e is the electron velocity and T_e is the electron temperature in energy units. This is to remind the readers that eq. (4) has been obtained by using the zeroth order electron density expression of $n_e = n_{e0}\ \exp(e\phi/T_e)$ in the usual form of electron continuity equation. Electron momentum equation is given as usual.

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = \frac{e}{m_e} \frac{\partial \phi}{\partial x} - \frac{T_e}{m_e n_e} \frac{\partial n_e}{\partial x}. \tag{5}$$

A few comments deserve the attention of readers: (i) The particular set of electron dynamic equations (4) and (5) is justifiably useful only for low-frequency plasma sound mode excitation in transonic plasma equilibrium as clarified earlier [1–3], (ii) it is well-known that for low-frequency wave fluctuations, electron continuity equation becomes redundant under the asymptotic mass limit of $m_e/m_i \to 0$. In this limit, the zeroth order solution of eq. (5) yields Boltzmann electron density distribution to describe the dynamics of ion sound wave motion, and lastly, (iii) a weak but finite inertial correction is made in the dynamics of parent plasma thermal electrons by incorporating leading order solution (in the absence of inertia) of eq. (5) onto an inertially perturbed solution (in the presence of weak inertia). The same set of electron dynamic equations will be used for the m-IAW description in transonic-like equilibrium of colloidal plasma hydrodynamic flow motion.

The usual form of ion continuity equation and ion momentum equation as given below describes ion dynamics by eqs (6) and (7) respectively.

$$\frac{\partial n_i}{\partial t} + v_i \frac{\partial n_i}{\partial x} + n_i \frac{\partial v_i}{\partial x} = 0, \tag{6}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{q_i}{m_i} \frac{\partial \phi}{\partial x}. \tag{7}$$

Since the dust grain dynamics is negligible according to the defined colloidal plasma model, the Poisson's equation (8) as given below closes the required set of basic dynamical equations for m-IAW description. It thus couples the dynamics of all the ionic species in the colloidal plasma system together.

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi(e n_e + q_I n_I - q_i n_i). \tag{8}$$

B. Nonlinear wave dynamics of m-IAW

For nonlinear normal mode description of the m-IAW, we first normalize the basic set of dynamical equations (4)–(8). The plasma potential (ϕ) is normalized by the plasma electron thermal potential (T_e/e), ion fluid speed by the modified ion acoustic speed (c_{sm}), density of the parent electrons and ions by the equilibrium bulk plasma ion density (n_{i0}), time by the parent ion plasma oscillation time-scale (ω_{pi}^{-1}) and distance by plasma electron Debye screening length (λ_{De}). The normalized forms of eqs (4) and (5) electron electrodynamic description are given as follows:

$$\frac{\partial \phi}{\partial t} + \frac{\partial v_e}{\partial x} + v_e \frac{\partial \phi}{\partial x} = 0, \tag{9}$$

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = \frac{\varepsilon_m}{\varepsilon_n} \left[\frac{\partial \phi}{\partial x} - \frac{1}{n_e} \frac{\partial n_e}{\partial x} \right]. \tag{10}$$

The normalized forms of eqs (5) and (6) are given as follows:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{11}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{z_i}{\varepsilon_n} \frac{\partial \phi}{\partial x}. \tag{12}$$

The normalized form of Poisson's equation (8) for finite electrostatic potential distribution due to local charge imbalance is given by

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{n_{i0}}{n_{e0}}(n_e + z_I n_I - z_i n_i), \tag{13}$$

where $\varepsilon_n = n_{i0} q_i^2 / n_{e0} e^2, \varepsilon_m = m_i/m_e, z_I = q_I/e$ and $z_i = q_i/e$. Now, under reductive perturbation method, we expand the physical variables around the defined equilibrium of colloidal plasma system as follows:

$$
\begin{aligned}
n_e &= \frac{n_{e0}}{n_{i0}} + \varepsilon n_e^{(1)} + \varepsilon^2 n_e^{(2)} + \cdots, \\
n_i &= 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \cdots, \\
n_I &= \frac{n_{I0}}{n_{i0}} + 0, \\
v_e &= \varepsilon v_e^{(1)} + \varepsilon^2 v_e^{(2)} + \cdots, \\
v_i &= v_{i0} + \varepsilon v_i^{(1)} + \cdots.
\end{aligned}
$$

Using the standard procedure of the well-known reductive perturbation technique, we carry out order-by-order analysis of eqs (9)–(13) in a stretched coordinate system defined by $\xi = \varepsilon^{1/2}(x - Mt)$ and $\tau = \varepsilon^{3/2} t$. We arrive at the following linear order solution of the m-IAW in the form of dispersion relation as given below:

$$
\left(1 + \frac{\varepsilon_n}{\varepsilon_m} M^2\right) = \frac{1}{(M - v_{i0})^2}. \tag{14}
$$

Without going into lengthy course of the higher-order calculations, the required Korteweg–de Vries (KdV) equation for the m-IAW can be derived and written as

$$
\begin{aligned}
&\left[\frac{\varepsilon_n}{\varepsilon_m} M + \frac{1}{(M - v_{i0})^3}\right] \frac{\partial \phi^{(1)}}{\partial \tau} + \frac{1}{2}\left[\frac{3}{(M - v_{i0})^4} - \left(1 + \frac{\varepsilon_n^2}{\varepsilon_m^2} M^4\right)\right] \phi^{(1)} \\
&\frac{\partial \phi^{(1)}}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0.
\end{aligned} \tag{15}
$$

By eq. (15), it is obvious that the Mach number (M) is a complex quantity that makes the coefficients of the KdV equation complex. Now following the procedure as laid down in our earlier publication [3], we can transform the complex coefficients into real ones by global phase transformation method (GPTM) into a circular geometry. It leads thereby to the following form of a source-driven KdV (d-KdV) equation:

$$
K_{0m} \frac{\partial \phi}{\partial \tau} + M_{0m} \phi \frac{\partial \phi}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \phi}{\partial \xi^3} = \gamma_m' K_{0m} \phi. \tag{16}
$$

The complex response coefficients (CRC) K_{0m}, M_{0m} are defined as given below:

$$
K_{0m} = \sqrt{A_m^2 + B_m^2},
$$

where

$$
A_m = \frac{M_r \varepsilon_n}{\varepsilon_m} + \frac{(M_{\mathrm{Dr}})^3 - 3 M_{\mathrm{Dr}} M_i^2}{(M_{\mathrm{Dr}}^2 + M_i^2)^3}
$$

$$
B_m = \frac{M_i \varepsilon_n}{\varepsilon_m} + \frac{(M_i)^3 - 3 M_{\mathrm{Dr}}^2 M_i}{(M_{\mathrm{Dr}}^2 + M_i^2)^3}
$$

and

$$M_{0m} = \sqrt{C_m^2 + D_m^2},$$

where

$$C_m = \frac{1}{2}\left[\frac{3\{(M_{\mathrm{Dr}}^2 - M_i^2)^2 - 4M_{\mathrm{Dr}}^2 M_i^2\}}{(M_{\mathrm{Dr}}^2 + M_i^2)^4} - \frac{\{(M_r^2 - M_i^2)^2 - 4M_r^2 M_i^2\}\varepsilon_n^2}{\varepsilon_m^2} - 1\right],$$

$$D_m = -\frac{1}{2}\left[\frac{12\left(M_{\mathrm{Dr}}^2 - M_i^2\right) M_{\mathrm{Dr}} M_i}{(M_{\mathrm{Dr}}^2 + M_i^2)^4} + \frac{\left\{4\left(M_r^2 - M_i^2\right) M_r M_i\right\}\varepsilon_n^2}{\varepsilon_m^2}\right],$$

$$M_{\mathrm{Dr}} = |v_{i0}|, \quad M_r = 1 - |v_{i0}| \quad \text{and}$$

$$M_{\mathrm{Di}} = M_i = \sqrt{2\left(\frac{\varepsilon_m}{\varepsilon_n}\right)}|(1 - |v_{i0}|)^{1/2}|.$$

Here γ'_m is the normalized (by parent ion plasma oscillation time-scale) growth rate of the m-IAW as given below:

$$\gamma'_m = \sqrt{2\left(\frac{m_i}{m_e}\right)}(k\lambda_{\mathrm{De}})|(1 - v_{i0}/c_{sm})|^{1/2}. \tag{17}$$

Equation (17) defines the coefficient of the self-consistent driving source-term appearing due to inertia of the parent plasma thermal electrons in the d-KdV equation (16) that describes the nonlinear wave kinetics of m-IAW. Again transforming the dispersion relation (14) into the corresponding unnormalized form and applying the hydrodynamical condition of ion fluid acoustic mode, one can deduce the phase speed (c_{sm}) of the m-IAW as follows:

$$c_{sm} = \left[\left(\frac{n_{i0}q_i^2}{n_{e0}e^2}\right)\right]^{1/2} c_s. \tag{18}$$

Let us consider that dust grains-like impurity ions are embedded into a background parent quasi-neutral hydrogen plasma with density $n \sim 10^{20}$ m^{-3}. Charge state of such micron size dust grains ($r \sim 1$ μm) can be estimated to yield $z_I \sim 10^3$ for electron temperature $T_e \sim 1$ eV. Again, a nanoscale sized particle ($r \sim 1$ nm) under the same parent plasma environment will carry a charge state $z_I \sim 1$. Similarly, the charge states corresponding to 10 nm and 100 nm particles can be estimated to be $z_I \sim 6$ and $z_I \sim 64$ respectively at the cost of collisional loss of parent plasma electrons. Thus, it can arbitrarily be assumed that $\varepsilon_n = (n_{i0}q_i^2/n_{e0}e^2) \sim 10^5$. Then the phase speed of m-IAW can be quantitatively determined from eq. (18) as $c_{sm} = 9.78 \times 10^8$ m/s.

4. So-called (ion) acoustic wave

Let us consider this in isolation of the m-IAW description so as to clarify the nature of used basic governing equations. Otherwise, confusion may arise with those used for m-IAW descriptions. The s-IAW is the normal acoustic mode on dust grain inertial time scale that can be distinguished by collective electrodynamical behaviors of the parent plasma particles [18]. There are experimental situations, wherein the background flows in the parent plasma particles are minimized to ensure the directed motion of dust grains to be measurable [17]. This is to further clarify that the abbreviation ‘s’ already used stands for the term ‘so-called’. In addition, the word ‘ion’ has been added ahead of the ‘acoustic’ to tailor the nomenclature of this sound mode to make its abbreviation (s-IAW) in symmetry with m-IAW.

A. Basic equations

Now for the s-IAW description in transonic flow regime of dust drift motion, the role of electrons in acoustic perturbations is ignored if $\lambda_{\mathrm{De}} \gg \lambda_{\mathrm{Di}}$. This can be justified for the non-isothermal parent plasma system containing cold dust grains. The ions are then supposed to perform the job of potential screening created by dust grain associated acoustic perturbations termed as the s-IAW [15–18]. The ion continuity equation reads as

$$\frac{\partial \phi}{\partial t} + v_i \frac{\partial \phi}{\partial x} - \frac{T_i}{q_i} \frac{\partial v_i}{\partial x} = 0. \tag{19}$$

The modified form of ion continuity equation (19) is derived by substituting the zeroth order ion density distribution $n_i = n_{i0} \exp(-q_i \phi / T_i)$. This is obtainable from the ion momentum equation (20) in the asymptotic mass limit of $m_i/m_I \to 0$. Ion momentum equation reads as follows:

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{q_i}{m_i} \frac{\partial \phi}{\partial x} - \frac{T_i}{m_i n_i} \frac{\partial n_i}{\partial x}. \tag{20}$$

Physical justification of the use of ion dynamic eqs (19) and (20) for s-IAW description is based on the basic logic discussed earlier [3] for the case of electron dynamic eqs (4) and (5) for IAW description. The distinctive roles of ion dynamics for m-IAW (eqs (6) and (7)) and s-IAW (eqs (19) and (20)) arise due to separate treatment of both the plasma sound waves in mutual isolation to one another. This is justifiable because the two distinct plasma sound wave space and time-scales differ by orders of magnitude. A weak but finite inertial correction is made to the dynamics of parent plasma thermal ions by incorporating leading order solution (in the absence of inertia) of eq. (20) onto an inertially perturbed solution (in the presence of weak inertia). Of course, a coupled mode analysis of m-IAW and s-IAW may be more appropriate and interesting for a unified description of plasma sound waves in transonic colloidal plasma equilibrium. It can form an interesting and challenging problem of future research work.

Moreover, the full inertial dynamics of dust grain-like impurity ions is described by the following equations of continuity (21) and momentum (22) as given below:

$$\frac{\partial n_I}{\partial t} + v_I \frac{\partial n_I}{\partial x} + n_I \frac{\partial v_I}{\partial x} = 0, \tag{21}$$

$$\frac{\partial v_I}{\partial t} + v_I \frac{\partial v_I}{\partial x} = -\frac{q_I}{m_I} \frac{\partial \phi}{\partial x}. \tag{22}$$

As usual, Poisson's equation (23) given below closes the basic set of dynamical equations for s-IAW description thereby coupling all the dynamics of the plasma ionic species together.

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi(en_e + q_I n_I - q_i n_i). \tag{23}$$

B. Nonlinear wave dynamics of s-IAW

To derive the nonlinear wave equation of the s-IAW, we first normalize the relevant physical variables. The plasma potential (ϕ) is normalized by the parent ion thermal potential (T_i/q_i), all velocities by the s-IAW speed (c_{ss}), all densities by the equilibrium parent ion bulk plasma density (n_{i0}) and time coordinate by the dust grain oscillation time-scale (ω_{pI}^{-1}) and space coordinate by the parent ion Debye length (λ_{Di}). The normalized forms of the basic eqs (19)–(23) are given below:

$$\frac{\partial \phi}{\partial t} - \frac{\partial v_i}{\partial x} + v_i \frac{\partial \phi}{\partial x} = 0, \tag{24}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{\varepsilon_M}{\varepsilon_\nu} \left[-\frac{\partial \phi}{\partial x} - \frac{1}{n_i} \frac{\partial n_i}{\partial x} \right], \tag{25}$$

$$\frac{\partial n_I}{\partial t} + \frac{\partial}{\partial x} (n_I v_I) = 0, \tag{26}$$

$$\frac{\partial v_I}{\partial t} + v_I \frac{\partial v_I}{\partial x} = \frac{z_I}{z_i \varepsilon_\nu} \frac{\partial \phi}{\partial x}, \tag{27}$$

$$\frac{\partial^2 \phi}{\partial x^2} = z_i^{-1} \left(n_e + z_I n_I - n_i z_i \right), \tag{28}$$

where $\varepsilon_\nu = n_{I0} q_I^2 / n_{i0} q_i^2$, $\varepsilon_M = m_I/m_i$. Let us now expand the physical variables around the pre-defined equilibrium according to the standard reductive perturbation method.

$$
\begin{aligned}
n_e &= \frac{n_{e0}}{n_{i0}} + 0,\\
n_i &= 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \cdots,\\
n_I &= \frac{n_{I0}}{n_{i0}} + \varepsilon n_i^{(1)} + \varepsilon^2 v_i^{(2)} + \cdots,\\
v_I &= v_{I0} + \varepsilon v_I^{(1)} + \varepsilon^2 v_I^{(2)} + \cdots,\\
v_i &= \varepsilon v_i^{(1)} + \varepsilon^2 v_i^{(2)} + \cdots,\\
\phi &= 0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \cdots.
\end{aligned}
$$

Applying the standard procedure of the reductive perturbation technique and carrying out order-by-order analysis of eqs (24)–(28) under the stretched coordinate system defined by $\xi = \varepsilon^{1/2}(x - Mt)$ and $\tau = \varepsilon^{3/2}t$, we get the linear order dispersion relation of the s-IAW as given below:

$$
\left(1 + \frac{\varepsilon_\nu}{\varepsilon_M} M^2\right) = \frac{1}{(M - v_{I0})^2}. \tag{29}
$$

Without going into lengthy calculations, higher-order perturbation equations obtained from eqs (24)–(28) can be solved to yield the following form of KdV equation for the s-IAW dynamical description:

$$
\begin{aligned}
&\left[\frac{\varepsilon_\nu}{\varepsilon_M} M + \frac{1}{(M - v_{I0})^3}\right] \frac{\partial \phi^{(1)}}{\partial \tau} + \frac{1}{2}\left[\frac{3}{(M - v_{I0})^4} - \left(1 + \frac{\varepsilon_\nu^2}{\varepsilon_M^2} M^4\right)\right]\\
&\times \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0.
\end{aligned} \tag{30}
$$

It is again obvious from eq. (29) that the Mach number (M) is complex under the resonant excitation condition of the s-IAW, which makes the coefficients of eq. (30) complex. Again, following the procedure as laid down earlier [3], one can transform the complex coefficients of KdV equation (30) into real ones by global phase transformation method (GPTM) into a circular geometry. As a result, the following form of d-KdV equation can be derived and written as

$$
K_{0s}\frac{\partial \phi}{\partial \tau} + M_{0s}\phi \frac{\partial \phi}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi}{\partial \xi^3} = \gamma_s' K_{0s}\phi. \tag{31}
$$

The complex response coefficients (CRC) for s-IAW description denoted by K_{0s} and M_{0s} are defined below:

$$
K_{0s} = \sqrt{A_s^2 + B_s^2},
$$

where

$$
A_s = \frac{M_r \varepsilon \nu}{\varepsilon_M} + \frac{(M_{\mathrm{Dr}})^3 - 3M_{\mathrm{Dr}}M_i^2}{(M_{\mathrm{Dr}}^2 + M_i^2)^3},
$$

$$
B_s = \frac{M_i \varepsilon_\nu}{\varepsilon_M} + \frac{(M_i)^3 - 3M_{\mathrm{Dr}}^2 M_i}{(M_{\mathrm{Dr}}^2 + M_i^2)^3},
$$

$$M_{0s} = \sqrt{C_s^2 + D_s^2},$$

where

$$C_s = \frac{1}{2}\left[\frac{3\{(M_{\mathrm{Dr}}^2 - M_i^2)^2 - 4M_{\mathrm{Dr}}^2 M_i^2\}}{(M_{\mathrm{Dr}}^2 + M_i^2)^4} - \frac{\{(M_r^2 - M_i^2)^2 - 4M_r^2 M_i^2\}\varepsilon_\nu^2}{\varepsilon_M^2} - 1\right],$$

$$D_s = -\frac{1}{2}\left[\frac{12(M_{\mathrm{Dr}}^2 - M_i^2)M_{\mathrm{Dr}} M_i}{(M_{\mathrm{Dr}}^2 + M_i^2)^4} + \frac{\{4M_r^2 - M_i^2)M_r M_i\}\varepsilon_\nu^2}{\varepsilon_M^2}\right],$$

$$M_{\mathrm{Dr}} = |v_{I0}|, \quad M_r = 1 - |v_{I0}| \quad \text{and} \quad M_{\mathrm{Di}} = M_i = \sqrt{2\left(\frac{\varepsilon_M}{\varepsilon_\nu}\right)}|(1 - |v_{I0}|)^{1/2}|.$$

Here γ_s' is the normalized (by dust grain plasma oscillation time-scale) growth rate of the s-IAW as given below:

$$\gamma_s' = \sqrt{2\left(\frac{m_I}{m_i}\right)\left(\frac{1}{\varepsilon_\nu}\right)}(k\lambda_{\mathrm{Di}})\,|(1 - v_{I0}/c_{ss})|^{1/2}. \tag{32}$$

Again, eq. (32) defines the coefficient of the self-consistent linear driving source-term appearing due to inertia of the parent plasma thermal ions in the d-KdV equation (31) that describes the nonlinear wave kinetics of s-IAW. Again, transforming the dispersion relation (29) into the corresponding unnormalized form and applying the hydrodynamical condition of dust fluid acoustic mode, one can deduce the phase speed (c_{ss}) of s-IAW as follows:

$$c_{ss} = \left[\left(\frac{n_{I0} q_I^2}{n_{i0} q_i^2}\right)\left(\frac{T_i}{T_e}\right)\left(\frac{m_i}{m_I}\right)\right]^{1/2} c_s. \tag{33}$$

For given parameters [17] of r (size of the spherical silica dust grain) = 2 μm and mass density $\rho = 2.239$ g/cm^3, $m_I \sim 64 \times 10^{-12}$ g. Charge state of such micron size dust grains can be estimated to yield $z_I \sim 10^3$ for electron temperature $T_e \sim 1$ eV. If we consider $q_I n_{I0}/q_i n_{i0} \sim 1$ and $T_i \sim 0.01$ eV, the phase speed of the s-IAW from equation (33) could be estimated as $c_{ss} \sim 1$ cm/s under hydrogenic parent ion plasma. The measured values of dust grain flow speed (v_{I0}) in experimental condition [17] comes out to be 12 mm/s at $t = 1.4$ s and 10 mm/s at $t = 1.8$ s after the dust grains are displaced from outside. Efforts are made to minimize the background parent plasma flow. Such situations are quite realistic, practical and useful for our hydrodynamic equilibrium flow consideration of dust grain dynamics as assumed to carry out the nonlinear wave dynamics of the s-IAW.

5. Results and discussions

The point (static) charge approximation of the dust grains has been applied through entire calculations. This demands that the electron and ion Debye lengths must be quite larger than the dust grain size. This ensures the collective behavior of the dust grains. Furthermore, the long wavelength conditions demand that $\lambda_m \gg \lambda_{\mathrm{De}}$ for modified ion acoustic wavelength and $\lambda_s \gg \lambda_{\mathrm{De}}(\lambda_{\mathrm{Di}}/\lambda_{\mathrm{De}})$ for the so-called (ion) acoustic wavelength. This is to note that even the case of $\lambda_s \approx \lambda_{\mathrm{De}}$ satisfies the long wavelength approximation provided that the electron and ion Debye lengths are quite separated, i.e., $(\lambda_{\mathrm{Di}}/\lambda_{\mathrm{De}}) \ll 1$. Fluid mode approximation of the m-IAW demands the following inequalities to hold good:

$$\sqrt{(T_i/T_e)} \ll \sqrt{\varepsilon_n}(1 - v_{i0}/c_{sm}) \ll \sqrt{(m_i/m_e)}. \tag{34}$$

Similar condition is demanded for the s-IAW.

$$\sqrt{(T_I/T_i)} \ll \sqrt{\varepsilon_\nu}(1 - v_{I0}/c_{ss}) \ll \sqrt{(m_I/m_i)},\ \sqrt{(T_e/T_i))(m_I/m_e)}. \tag{35}$$

This is to note that $\varepsilon_\nu = ((\varepsilon_n - z_i)/\varepsilon_n)z_I$ provides the relation of dust-plasma parameters derivable from equilibrium quasi-neutrality condition. Let us now compare the growth rates of the acoustic waves due to inertia-induced excitation process (γ_m, γ_s) of the current concern and parametric process (γ_{p}) [6], with respect to γ_{most} of the most unstable [19] acoustic wave. Expressions for the parametric growth rate (γ_{p}) and the most unstable acoustic growth rate (γ_{most}) in the present notations are given below:

$$\gamma_{\mathrm{p}} = \left(\frac{\omega_{as}}{2}\right)\left|\left(1 - \frac{\omega_{as}^2}{16\omega_0^2}\right)^{-1/2}\right|$$

for $\omega_{as} \le \omega_0/4$, where ω_0 denotes the dust charge oscillator frequency [6] and $\omega_{as} = kc_{ss}$.

$$\gamma_{\mathrm{most}} = \left(\frac{m_e}{m_i}\right)^{1/2}\omega_{pi}.$$

For the ordering of the resonant terms as defined below,

$$|(1 - \omega_{as}^2/16\omega_0^2)^{1/2}| \sim |(1 - v_{i0}/c_{sm})^{1/2}| \sim |(1 - v_{I0}/c_{ss})^{1/2}| = \mu. \tag{36}$$

We finally compare the growth rates of all the modes under consideration as follows:

$$\begin{aligned}&\gamma_p : \gamma_m : \gamma_s : \gamma_{\mathrm{most}} :: \sqrt{\varepsilon_\nu(T_i/T_e)(m_i/m_I)} : 2\sqrt{2(m_i/m_e)}\mu^2 \\ &: 2\sqrt{2(T_i/T_e)}\mu^2 : 2\sqrt{\varepsilon_n(m_e/m_i)}\mu/(k\lambda_{\mathrm{De}}).\end{aligned} \tag{37}$$

For very close values of resonance $\mu \sim \varepsilon$, the fluid condition may be satisfied for asymptotic limit of T_i and $T_I \to 0$. In this case, γ_{most} emerges as the largest growth. However, for $\mu \sim 1$, one may find that the fluid conditions are well-satisfied even for finite values of T_i and T_I; and γ_m emerges as the largest growth rate. This is to clarify that same notations are used for Mach number values in both the cases of m-IAW and s-IAW. But they differ in the sense that the flow velocities of respective inertial species are normalized by the respective plasma sound speeds.

6. Distinction of the applied excitation mechanism

The unique distinct quality of the present excitation mechanism is the weak but active inertial role of plasma thermal species against thermal screening of wave-induced potential. It may, however, seem that the parent plasma electrons for m-IAW analysis and ions for s-IAW analysis are treated as inertialess or to have inertia in an *ad hoc* manner. This, in fact, is not so. The physics behind this, as already properly discussed in [1–3], is as follows. The role of weak but finite inertia of thermal species is introduced by the physical justification [1–3] of weak inertial perturbations (dictated by eqs (19) and (20)) over leading order non-inertial dynamics (dictated by eqs (4) and (14)) of the respective thermal species in plasmas. It is noteworthy that if weak but finite inertial corrections are made, the thermal species are neither inertialess nor fully inertial in electrodynamical responses. This mathematical technique [1–3] must be applied to describe acoustic wave propagations through transonic plasma: an unstable zone of hydrodynamic equilibrium of quasineutral plasma gas flow motion and rich in wide range acoustic spectral components of ion acoustic wave fluctuations. The most important point behind this is that the traditional concept [1–3] of Boltzmann approximation of thermal species density distributions alone becomes deficient, inadequate and impractical to describe the acoustic wave dynamics in transonic plasma condition. The main deficiency is that it hides the physics of resonantly unstable behaviors of acoustic signals through transonic equilibria.

It is observed that in the hot plasma regime of the injected electron beam velocity, the monoergic electron beam can directly couple with the ions in plasma [20]. For a very weak electron beam, the ion sound wave becomes unstable when the sound wave phase speed is comparable to the electron beam velocity. Physically, a weak beam means that the beam plasma oscillation frequency is much smaller than the ion plasma oscillation frequency. In this limit of weak beam, the excitation physics is discussed in terms of the negative dielectric constant mechanism [20] based on the assumed frozen charge density perturbations on the electron beam. It is termed as the beam-charge density bunching which is valid only for weak beam. There are other cases of plasma-sound wave excitation like cross field current-driven ion sound wave instability [21], hot electron/ion streams-driven plasma sound wave instabilities in dusty plasma under Vlasov plasma model approach [22]. In Vlasov model, the basic process of the driving mechanism is governed by the wave-particle kinetic interactions under drifted Maxwellian velocity distributions [23].

Physical description of our proposed inertia-induced excitation mechanism of ion sound wave [1] operates through hydrodynamic tailoring of the thermal electron density distribution due to the mathematical consideration of weak but finite electron inertia in electron fluid response term. This eventually creates the active role of weak but finite perturbed electron fluid compressibility to allow for the plasma sound mode to couple with ion beam mode and grow in the presence of ion flow motion. This is basically a mode–mode coupled linear resonant instability and arises through an intermixed coupling of the positive and negative energy modes [2]. This differs from other known excitation mechanisms [24–28] of plasma sound wave, like bunching mechanism [20], wave-particle resonant interaction mechanism [22], etc. These mechanisms are ruled out to operate in our study of plasma sound wave excitation in colloidal plasma with drifting inertial species. Bunching mechanism is ruled out because the condition of weak beam is not obeyed and the question of resonant wave-particle interaction mechanism does not arise.

Let us have a glimpse of how the weak but finite role of electron inertia can arise under Vlasov model of kinetic approach [22,23]. For example, the appropriate approximation to the velocity integral for Maxwellian electrons for ion sound mode description is considered as [23]

$$\begin{aligned}\oint_C \frac{\partial f_{e0}/\partial v}{\omega_r/k - v}\mathrm{d}v &= -\oint_C \frac{1}{v}\left[1 + \frac{\omega}{kv} + \frac{\omega^2}{k^2v^2} + \cdots\right]\frac{\partial f_{e0}}{\partial v}\mathrm{d}v \\ &\approx -\int_{-\infty}^{\infty}\left[2\left(\frac{\partial f_{e0}}{\partial v^2}\right) + 3\frac{\omega}{k}\left(\frac{\partial f_{e0}}{\partial v^3}\right) + 4\frac{\omega^2}{k^2}\left(\frac{\partial f_{e0}}{\partial v^4}\right) + \cdots\right]\mathrm{d}v \\ &= -\frac{1}{v_{te}^2} - \frac{\omega^2}{k^2v_{te}^4} - \cdots. \end{aligned} \tag{38}$$

Real part of linear dispersion relation of ion sound wave for Maxwellian electrons is usually derived by retaining only leading order term in the asymptotic expansion (38). For Maxwellian ions, under appropriate approximation of power series, expansion of ion integral term leads to the following form of real part of the dispersion function.

$$D_r(\omega,k) = 1 + \frac{1}{k^2\lambda_{\mathrm{De}}^2}\left[1 + \frac{\omega^2}{k^2v_{te}^4} + \cdots\right] - \frac{\omega_{pi}^2}{\omega^2} - \cdots. \tag{39}$$

It is obvious that the second term introduces the role of electron inertia as a perturbative correction through Vlasov model treatment of the problem. This term originates from the binomial expansion of the electron inertial term in fluid dispersion relation of ion sound mode expressed as

$$1 + \frac{\omega_{pe}^2}{k^2v_{te}^2}\left(1 - \frac{\omega^2}{k^2v_{te}^2}\right)^{-1} - \frac{\omega_{pi}^2}{\omega^2} = 0. \tag{40}$$

As discussed earlier [1], in the presence of ion streaming, the weak but finite electron inertial effect in eq. (40) appears in the form of a resonantly unstable ion acoustic wave perturbation at down-shifted Doppler frequency. Moreover, the growth rate in our case is proportional to the square root of the relative motion between the ion stream and the ion sound wave perturbation. Contrary to this, the growth rate of the instability of plasma sound modes reported to occur in dusty plasmas [22] is directly proportional to relative motion between the hot electron/ion stream and plasma sound wave under consideration.

7. Conclusions

The conclusion derived from the above nonlinear calculations is that the inertia-induced acoustic excitation physics [1–3] can drive both m-IAW and s-IAW resonantly unstable in transonic domain. These two cases of instabilities, as described in the text are for isolated cases of directed motions of parent ions and dust grains respectively. The s-IAW is excitable by parent ion inertia-induced excitation theory for nanoparticles. The m-IAW is excitable for both nano- and micronsize particles due to weak but finite electron inertia dynamics. Nonlinear analyses of these two modes are also performed that produce the *d*-KdV equations as expected. As per our earlier numerical results [3], it is argued that the derived *d*-KdV equations (16) and (31) are expected to generate the usual soliton solutions as well as the oscillatory shock-like solutions. It is found that our own derived *d*-KdV equation [3] is a completely integrable classical dynamical evolutionary conservative system, the analytical solutions with suitable graphical discussions of which have already appeared elsewhere [26].

The basic dust grain charging physics of isolated dust grain is modeled as the formation of a non-neutral space charge layer over the Debye sheath scale ($\sim \lambda_{\mathrm{De}}$) around each dust grain. The capacitor model of spherical symmetry is generally used in the dust grain charge estimation. Floating condition ($J_e = J_i$) determines the surface potential of the dust grain. Usual Bohm current models the ion current. As a result, the surface potential (ϕ_s) is given by $\phi_s = (T_e/e)\log(\sqrt{m_e/m_i})$ which is used to calculate the dust grain charge under spherical capacitor model. This is to note that plane sheath approximation is used to derive the expression of the dust grain surface potential.

The following remarks may be worth mentioning for basic understanding of dust grain charging. In reality, in the presence of dust grains, the sound velocity is increased and hence if one replaces the Bohm current by modified sound speed, the dust grain surface potential is rederived into the form as $\phi_s = (T_e/e)\log(\sqrt{(m_e/m_i)(n_{i0}q_i/n_{e0}e)z_i})$. It dramatizes the accuracy of charge estimation of charged colloidal particles under plain sheath approximation! It necessitates arguing that every dust grain is surrounded by a non-neutral space charge cloud of ion domination sustained by localized region of radially in ion flow motion with transonic termination towards the bulk plasma. Could one then discuss the dust–dust interaction physics in terms of the mutual exchange of the m-IAW in the parent plasma background?

In technological application, one can argue that the proposed theoretical model for inertia-induced acoustic instability may be utilized to make a plasma device for acoustic amplifier. The amplified acoustic signals could be detected, received and analyzed for the diagnosis and characterization of hydrodynamic flow of plasmas with embedded dust contaminations. Study of the ambient acoustic spectrum associated with plasma flow motion can be termed as 'acoustic spectroscopy' of equilibrium plasma flows. This may be useful for expanding background plasmas, stellar wind plasmas and also in space plasmas through which aerodynamic and space vehicles' motions occur. Basic principles of the acoustic spectroscopy have concern to the ion acoustic wave turbulence theory and properties of the transonic plasma equilibrium.

Lastly, it is summarily concluded that the plasma sound modes in general, are excitable by the inertia-induced ion acoustic excitation theory in colloidal plasmas with uniformly directed supersonic motion of inertial species that undergo compression and rarefaction to produce acoustic perturbations. Of course, the required excitation threshold value must be fulfilled. These calculations may be useful to understand the plasma flow-driven acoustic turbulence in transonic equilibrium of the colloidal plasma flow motion. As illustrated in the text, the inertia-induced acoustic excitation physics by virtue of its distinctness and uniqueness, is a new addition to the list of known mechanisms of plasma sound wave excitation.

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