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Neutrinos as a probe of CP-violation and leptogenesis

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Abstract. Establishing CP-violation in the lepton sector is one of the most challenging future tasks in neutrino physics. The lepton mixing matrix contains one Dirac phase and, if neutrinos are Majorana particles, two additional CP-violating phases. I will review the main theoretical aspects of CP-violation in the lepton sector. Then, I will present the strategies for determining the Dirac and the Majorana CP-violating phases in long-baseline and neutrinoless double beta decay experiments, respectively. Leptonic CP-violation has received recently a lot of attention as it might be at the origin of the baryon asymmetry of the Universe. Within the context of the see-saw mechanism, I will discuss the possible connection between the CP-violating phases measurable at low energy with the ones entering in leptogenesis.

Keywords. Neutrino; CP-violation; leptogenesis.

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1. Introduction

A remarkable progress in the studies of neutrino physics has been achieved. The experiments with solar [1–3], atmospheric [4,5], reactor [6] and accelerator neutrinos [7,8] have provided compelling evidence for the existence of neutrino oscillations. This implies that neutrinos have nonzero masses and that 3-neutrino mixing takes place.

From atmospheric [4,5], K2K [7] and very recent MINOS data [8], a measurement of the mass squared difference $\Delta m^2_{\rm A}$ has been obtained, in the range $\Delta m^2_{\rm A} = 2.5^{+0.20}_{-0.25} \times 10^{-3}$ eV^2 at 1σ [9]. The atmospheric mixing angle is found to be maximal or nearly maximal: $\sin^2\theta_{\rm A} \geq 0.97(0.87)$ at $1(3)\sigma$. Solar neutrino experiments [1–3] combined with KamLAND results [6] constrain in a narrow range the relevant mass squared difference $\Delta m^2_\odot = 7.9 \pm 0.3 \times 10^{-5}$ eV^2 and the solar mixing angle, $\sin^2\theta_\odot = 0.30^{+0.02}_{-0.03}$ [9] at 1σ. The fact that maximal solar mixing angle is ruled out at $\sim 6\sigma$ and that $\cos 2\theta_\odot \geq 0.36(0.28)$ at $1(2)\sigma$ has important implications for neutrinoless double beta ($(\beta\beta)_{0\nu}$-) decay. Searches of effects due to the third mixing angle θ_{13} have been so far unsuccessful. The present bound at 3σ reads

$\sin^2\theta_{13} < 0.041$ [9], from a global analysis of data from the reactor neutrino CHOOZ experiment, the solar and KamLAND experiments as well as of the constraints on the atmospheric mass squared difference from atmospheric and long-baseline neutrino data. Many experiments at present and in the future aim to a better precision in the measurement of these parameters.

We still lack important information on neutrino masses and mixing. The main goals of the future program in neutrino oscillations aim at the determination of (a) the value of θ_{13}, (b) the type of hierarchy (normal hierarchy if $m_1 < m_2 < m_3$ or inverted hierarchy if $m_3 < m_1 < m_2$), (c) the presence of CP-violation in the lepton sector.

In addition, establishing whether the massive neutrinos ν_j are Dirac fermions possessing distinct antiparticles, or are Majorana fermions, if they are identical to their antiparticles, is of fundamental importance. This information can shed light on the underlying symmetries of particle interactions and on the origin of ν-masses. Neutrinos ν_j will be Dirac fermions if particle interactions conserve some additive lepton number, e.g. the total lepton charge $L = L_{\mathrm{e}} + L_\mu + L_\tau$. Otherwise, the neutrinos ν_j will be Majorana fermions (see, e.g. ref. [10]). The crucial issue of the nature of neutrinos will be addressed in $(\beta\beta)_{0\nu}$-decay experiments.

In a 3-neutrino mixing scheme, one or three CP-violating phases are present in the lepton mixing matrix [11,12] depending on the nature of neutrinos: one phase if neutrinos are Dirac particles, three phases if they are of Majorana type. The Dirac phase δ enters in neutrino oscillation probabilities. The fact that the LMA solution for solar neutrino oscillations has been established has opened the possibility to search for CP-violating effects in long baseline appearance experiments (see, e.g. ref. [13]), exploiting the interference between the solar and atmospheric neutrino oscillation terms. I will briefly review the related theoretical and phenomenological aspects, and, in particular, the problem of degeneracies among different parameters [14–18]. If neutrinos are Majorana particles, Majorana phases are physical and enter in processes which violate the lepton number by two units. The most sensitive process of this type is $(\beta\beta)_{0\nu}$-decay. Because $\cos 2\theta_\odot \gg 0$, the predicted values of $|\langle m \rangle|$ for the inverted hierarchical ($m_3 \ll m_1 \simeq m_2$) and the quasi-degenerate ($m_1 \simeq m_2 \simeq m_3$) spectra have sizeable lower bounds and are in the range of sensitivity of present and/or future experiments [19,20]. This implies that with additional information on neutrino masses it might be possible to establish CP-violation in the lepton sector due to Majorana CP-violating phases [21,22]. This issue has been studied in detail in the past [19,20,23–27] and in a recent work [28]. I will review the main findings of these analysis in detail.

CP-violation plays a crucial role in the generation of the baryon asymmetry of the Universe [29]. Within the context of the leptogenesis mechanism [30] in see-saw Type-I models, I will discuss the possible connection between the low-energy phases measurable in future neutrino experiments and the one that enters in the lepton asymmetry (see e.g. refs [31–33]). Even if in general no unique correspondence can be found, many models allow to relate the low-energy and high-energy phases. Observing CP-violation in the lepton sector would constitute a strong indication, even if not a proof, of leptogenesis as the explanation for the observed baryon asymmetry of the Universe.

2. CP-violation in the lepton sector

The interpretation of the data from solar, atmospheric, K2K, MINOS and the KamLAND neutrino experiments in terms of neutrino oscillations requires the existence of 3-neutrino mixing. The mixing between the three left-handed flavour neutrino fields, $\nu_{l\mathrm{L}}$ and the massive eigenstates ν_j, with a mass m_j, is described by a unitary 3×3 mixing matrix U, the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix [34]:

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}. \tag{1}$$

The PMNS mixing matrix U can be parametrized by three angles, θ_{A}, $\theta_{\odot}$, and θ_{13}, and, depending on whether the massive neutrinos ν_j are Dirac or Majorana particles – by one or three CP-violating phases [11,12]. In the standard parametrization of U, the three mixing angles are denoted as θ_{12}, θ_{13} and θ_{23}:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & s_{23}c_{13}\mathrm{e}^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & c_{23}c_{13}\mathrm{e}^{i\delta} \end{pmatrix} \times \mathrm{diag}(1, \mathrm{e}^{i(\alpha_{21}/2)}, \mathrm{e}^{i(\alpha_{31}/2)}), \tag{2}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$. The phase δ is the Dirac CP-violating phase, and α_{21} and α_{31} are two Majorana CP-violating phases [11,12], which are physical only if neutrinos are Majorana particles. Otherwise, they are unphysical and can be reabsorbed in a redefinition of the fields.

For Majorana fields, the general Majorana condition is satisfied:

$$C(\bar{\nu}_j)^T = (\xi_j^*)^2\, \nu_j, \quad j = 1, 2, 3, \tag{3}$$

where ξ_j, $j = 1, 2, 3$, are three phases. Only two combinations of the six phases α_j and ξ_j represent physical Majorana CP-violating phases. All CP-violation effects associated with the Majorana nature of the massive neutrinos are generated by $\alpha_{21,31} \neq 0, \pi$. In fact, under a rephasing of the charged lepton, $l(x)$, and the neutrino, $\nu_j(x)$, fields in the weak charged lepton current, $l(x) \to \mathrm{e}^{i\eta_l} l(x)$ and $\nu_j(x) \to \mathrm{e}^{i\beta_j}\nu_j(x)$, the elements of the lepton mixing matrix and the phase factors in the Majorana condition, eq. (3), change as follows: $U_l \to U_l \mathrm{e}^{-i(\eta_l - \beta_j)}$, $\xi_j \to \xi_j \mathrm{e}^{-i\beta_j}$. As shown in refs [19,35], in the lepton sector with the mixing of three massive Majorana neutrinos there exist three rephasing invariants related to the CP-violating phases. The standard Dirac one, J, is present in the case of mixing of three massive Dirac neutrinos [35,36] (see also ref. [37]):

$$J = \mathrm{Im}(U_{\mu 2} U_{\mathrm{e}3} U_{\mu 3}^* U_{\mathrm{e}2}^*) \propto \sin\delta. \tag{4}$$

The other two, S_1 and S_2, are related to the Majorana nature of massive neutrinos ν_j [19,35]:

$$S_1 \equiv \mathrm{Im}(U_{\mathrm{e}1} U_{\mathrm{e}3}^* \xi_3^* \xi_1), \tag{5}$$

$$S_2 \equiv \mathrm{Im}(U_{\mathrm{e}2} U_{\mathrm{e}3}^* \xi_3^* \xi_2). \tag{6}$$

The two Majorana CP-violating phases α_{21} and α_{31} can be expressed in terms of the two independent rephasing invariants, $S_{1,2}$. It holds [19]:

$$\cos\alpha_{31} = 1 - 2\frac{S_1^2}{|U_{\rm e1}|^2|U_{\rm e3}|^2}, \tag{7}$$

$$\cos(\alpha_{31}-\alpha_{21}) = \cos(\alpha_3-\alpha_2) = 1 - 2\frac{S_2^2}{|U_{\rm e2}|^2|U_{\rm e3}|^2}, \tag{8}$$

$$\cos\alpha_{21} = \cos(\alpha_{31}-\alpha_{21})\cos\alpha_{31} + \sin(\alpha_{31}-\alpha_{21})\sin\alpha_{31}. \tag{9}$$

If there is CP-invariance in the lepton sector, we have, in particular, $S_1, S_2 = 0$, or $\mathrm{Re}(U_{\rm e1}U_{\rm e3}^*\xi_3^*\xi_1) = 0$, $\mathrm{Re}(U_{\rm e2}U_{\rm e3}^*\xi_3^*\xi_2) = 0$.

In all our subsequent considerations I will set for convenience (and without loss of generality) $\xi_j = 1$, $j = 1,2,3$. In this case $\alpha_{21} = \alpha_2 - \alpha_1$ and $\alpha_{31} = \alpha_3 - \alpha_1$.

The CP-invariance constraint on the elements of the lepton mixing matrix of interest reads [38] (see also [10]):

$$U_{{\rm e}j}^* = \eta_j^{\rm CP} U_{{\rm e}j}, \tag{10}$$

where $\eta_j^{\rm CP} = i\phi_j = \pm i$ is the CP-parity of the Majorana neutrino ν_j with mass $m_j > 0$.

3. Determining the Dirac CP-violating phase

The Dirac phase δ enters in neutrino oscillations in the case of 3-neutrino mixing. The probability of $\nu_{\rm i} \to \nu_{\rm f}$ transitions, $P(\nu_{\rm i} \to \nu_{\rm f}) \equiv P$, with $\nu_{\rm i}$ the initial flavour neutrino and $\nu_{\rm f}$ the final one, can be expressed in terms of a CP-conserving term and a CP-violating one:

$$\begin{aligned} P(\nu_{\rm i} \to \nu_{\rm f}) = \delta_{\rm if} &- 4\sum_{m>n} \mathrm{Re}(U_{{\rm i}m}U_{{\rm i}n}^*U_{{\rm f}m}^*U_{{\rm f}n})\sin^2\frac{\Delta m_{mn}^2 L}{4E} \\ &-2\sum_{m>n} \mathrm{Im}(U_{{\rm i}m}U_{{\rm i}n}^*U_{{\rm ff}m}^*U_{{\rm f}n})\sin\frac{\Delta m_{mn}^2 L}{2E}, \end{aligned} \tag{11}$$

where $\Delta m_{mn}^2 \equiv m_m^2 - m_n^2$, $m_{m,n}$ the neutrino masses. L indicates the baseline length and E the neutrino energy.

Notice that disappearance probabilities do not depend on the CP-violating phase δ because only the modulus of the elements of the leptonic mixing matrix enters in their expression. Information on Dirac CP-violation can be obtained considering the appearance probabilities, for eg. $P(\nu_\mu \to \nu_{\rm e})$. Leptonic CP-violation can be detected by observing the difference between the oscillation probability for neutrinos and antineutrinos [39]. It is possible to define the asymmetry $A(\nu_\mu, \nu_{\rm e})$:

$$A(\nu_\mu,\nu_{\rm e}) \equiv P(\nu_\mu \to \nu_{\rm e}) - \bar{P}(\nu_\mu \to \nu_{\rm e}), \tag{12}$$

where $\bar{P}(\nu_\mu \to \nu_{\rm e}) \equiv \bar{P}$ is the probability of oscillations for antineutrinos.

In vacuum such asymmetry takes the form:

$$A(\nu_\mu, \nu_e) = \frac{1}{2}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23} \times \left(\frac{\Delta m^2_{21} L}{2E}\right)\sin^2\frac{\Delta m^2_{31} L}{2E}\sin\delta, \tag{13}$$

where $\Delta m^2_{21} \ll |\Delta m^2_{31}|$. From eq. (13), we see that CP-violating effects are controlled by the small mixing angle θ_{13} and by the ratio of the solar versus atmospheric mass squared differences. This follows from the fact that Dirac CP-violation is present in the lepton mixing matrix only for three or more neutrino mixing. Let me tell that this does not happen for Majorana CP-violation. In this case, even for 2-neutrino mixing, Majorana CP-violating phase difference is physical and can be, in principle, observed in neutrinoless double beta decay.

CP-violation due to the δ phase can be searched for in long baseline experiments, in which neutrinos are detected after travelling through the Earth for a few hundred–thousand kilometers. Proposed future long baseline experiments include conventional beams and their upgrade to superbeams, such as T2K and NOνA, beta beams and neutrino factories. In this case, additional CP-violating effects arise due to the fact that the Earth is not CP-symmetric as it contains only electrons and not the corresponding anti-particles. The oscillation probability $P(\bar{P})$ can be approximated expanding in the small parameters θ_{13}, $\Delta m^2_{21}/\Delta m^2_{32}$, $\Delta m^2_{21}/A$ and $\Delta_{12} \equiv \Delta m^2_{21}/(2E)$ [40] (see also ref. [41]):

$$\begin{aligned} P(\bar{P})(L) \simeq\ & \sin^2\theta_{23}\,\sin^2 2\theta_{13}\left(\frac{\Delta_{13}}{A \mp \Delta_{13}}\right)^2 \sin^2\left(\frac{(A \mp \Delta_{13})L}{2}\right) \\ & + \cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin 2\theta_{12}\,\frac{\Delta_{12}}{A}\frac{\Delta_{13}}{A \mp \Delta_{13}} \\ & \times \sin\left(\frac{AL}{2}\right)\sin\left(\frac{(A \mp \Delta_{13})L}{2}\right)\cos\left(\frac{\Delta_{13} L}{2} \mp \delta\right) \\ & + \cos^2\theta_{23}\sin^2 2\theta_{12}\left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\left(\frac{AL}{2}\right), \end{aligned} \tag{14}$$

where $\Delta_{13} \equiv \Delta m^2_{13}/(2E)$. The matter effects are described by $A \equiv \sqrt{2} G_F \bar{n}_e(L)$, with $\bar{n}_e(L)$ the average electron number density, $\bar{n}_e(L) = 1/L \int_0^L n_e(L')\mathrm{d}L'$. Here $n_e(L)$ is the electron number density along the baseline.

As it can be understood from eq. (15), there are different sets of parameters (θ_{13}, θ_{23}, sgn of Δm_{31}, δ) that give the same probability for neutrino–neutrino and antineutrino–antineutrino transitions, at fixed L/E:

$$P(\theta_{13}, \theta_{23}, \mathrm{sgn}(\Delta m_{31}), \delta) = P(\theta'_{13}, \theta'_{23}, \mathrm{sgn}(\Delta m_{31})', \delta'), \tag{15}$$

$$\bar{P}(\theta_{13}, \theta_{23}, \mathrm{sgn}(\Delta m_{31}), \delta) = \bar{P}(\theta'_{13}, \theta'_{23}, \mathrm{sgn}(\Delta m_{31})', \delta'). \tag{16}$$

This is the well-known problem of degeneracies of parameters which arises in long baseline experiments [14–18].

In particular, one has:

- The (δ,θ_{13}) degeneracy [15]. Two solutions for the equations in eqs (15) and (16) can be obtained for the parameters δ and the mixing angle θ_{13}. If no independent information is available on δ, the accuracy reachable in one experiment on θ_{13} is severely worsened by this degeneracy. Conversely, the ability to establish CP-violation in the lepton sector is weakened. Due to the strong dependence on L/E of the fake solution, the latter can be excluded if information on the probability of oscillation at different L/E is available. This can be achieved by adding another detector [42,43], by combining data from different experiments, or by using the information in the even rate spectrum of a single experiment [44]. Additional constraints on θ_{13} can be obtained in reactor neutrino disappearance experiments and, if sufficiently stringent, can help in resolving this degeneracy [45].
- The ($\mathrm{sign}(\Delta m_{31}), \delta$) degeneracy [16]. The interference between intrinsic CP-violating effects due to $\delta \neq 0$ and matter-induced ones limits the ability to establish the type of neutrino mass hierarchy. For short baselines, matter effects can be subdominant and the experiments are mainly sensitive to the δ phase. This is the case for experiments with a baseline $L < 300$ km, as the CERN-Frejus beta beam and T2K. Conversely, it is possible to suppress intrinsic CP-violation effects, considering in only one experiment the probability of neutrino transition at two different baselines [46]. It has been shown that in this case the sign of Δm^2_{31} can be established independently from the value of δ.
- The ($\theta_{23}, \pi/2 - \theta_{23}$) ambiguity [14]. As disappearance experiments can measure only $\sin^2 2\theta_{23}$, the octant of θ_{23}, if θ_{23} is not maximal, cannot be established. In the appearance probabilities P and $\bar{P}$, $\sin^2\theta_{23}$ and $\cos^2\theta_{23}$ enter separately, introducing an additional source of ambiguity. The value of $\sin^2\theta_{23}$ can be, in principle, determined from atmospheric sub-GeV and multi-GeV neutrino data.

Altogether there exist an 8-fold degeneracy [17]. Given as input the values of Δm^2_{12}, $|\Delta m^2_{31}|$, θ_{12} and θ_{23}, only one experiment running in the neutrino and antineutrino modes cannot uniquely determine the value of $\sin\theta_{13}$, the type of hierarchy and the presence of CP-violation. However, considering information from different L/E experiments can break such degeneracies and allows to establish CP-violation in the lepton sector (see, e.g., [47]), the strategy will depend on the value of $\sin^2\theta_{13}$. If its value is in the reach of future superbeams as well as an optimized reactor neutrino and atmospheric neutrino experiments, the synergy between these experiments would allow to resolve the degeneracies [48]. If $\sin^2\theta_{13}$ is smaller, more sensitive machines will be required. In this case, a combination of neutrino factory with superbeams as well as of different appearance channels in a neutrino factory could be exploited [49].

4. Majorana CP-violation and neutrinoless double beta decay

One of the most important question to address in the future concerning massive neutrinos is whether they are Dirac or Majorana particles. The nature of neutrinos is directly related to the fundamental symmetries of elementary particle interactions. If massive neutrinos are Majorana fermions, processes in which the total lepton charge is not conserved and changes by two units should take place. The most sensitive process of this type is neutrinoless double beta ($(\beta\beta)_{0\nu}$-) decay in which two neutrons transform into two protons and two electrons, exchanging light virtual Majorana neutrinos.

The dependence of the half-life of $(\beta\beta)_{0\nu}$-decay, $T_{\beta\beta_{0\nu}}$, on the neutrino mass and mixing parameters factorizes in the effective Majorana mass (see, e.g. refs [10,50]), $T^{-1/2}_{\beta\beta_{0\nu}} \sim |\langle m\rangle| M$, where M is the corresponding nuclear matrix element (NME) and

$$|\langle m\rangle| = \left|m_1|U_{\rm e1}|^2 + m_2|U_{\rm e2}|^2\ {\rm e}^{i\alpha_{21}} + m_3|U_{\rm e3}|^2\ {\rm e}^{i\alpha_{31}}\right|. \tag{17}$$

Here $U_{\rm ei}$ denote the elements of the first row of the unitary PMNS mixing matrix U.

The evaluation of the NME is crucial in extracting the value of $|\langle m\rangle|$ from a measurement of or a limit on $T_{\beta\beta_{0\nu}}$. At present there are large uncertainties in their calculation [50], which amount up to a factor of 3 in the extracted value of $|\langle m\rangle|$. Encouraging results have been obtained in ref. [51]. A strong theoretical effort to solve the problem of the computation of the nuclear matrix elements is required.

Rather stringent upper bounds on $|\langle m\rangle|$ have been obtained in the ^{76}Ge experiments by the Heidelberg–Moscow Collaboration, $|\langle m\rangle| < 0.35$–$1.05$ eV (90% CL), and by the IGEX Collaboration, $|\langle m\rangle| < (0.33$–$1.35)$ eV (90% CL). A positive signal at $>3\sigma$, corresponding to $|\langle m\rangle| = (0.1$–$0.9)$ eV, at 99.73% CL, is claimed to have been observed [52]. At present NEMO3 [53] and CUORICINO [54] are taking data and will check this claim. Their first results read, at 90% CL, $|\langle m\rangle| < (0.7$–$1.2)$ eV [53] and $|\langle m\rangle| < (0.2$–$1.1)$ eV [54]. Future generation experiments CUORE, GERDA, EXO, MAJORANA, SuperNEMO, MOON, XMASS, CANDLES, aim to a sensitivity of $|\langle m\rangle| \sim (0.01$–$0.05)$ eV.

The predicted value of $|\langle m\rangle|$ depends, in the case of 3-ν mixing, on the oscillation parameters $\Delta m^2_{\rm A}$, $\theta_\odot$, $\Delta m^2_\odot$ and θ_{13} (see, e.g. ref. [19]), on the type of the neutrino mass spectrum and on the value of the lightest neutrino mass, m_0, as well as on the values of the two Majorana CP-violation phases in the PMNS matrix, α_{21} and α_{31} (see eq. (17)). For the possible types of neutrino mass spectrum: NH ($m_0 \equiv m_1 \ll m_2 \simeq \sqrt{\Delta m^2_\odot} \ll m_3 \simeq \sqrt{\Delta m^2_{\rm A}}$), IH ($m_0 \equiv m_3 \ll m_1 \simeq m_2 \simeq \sqrt{\Delta m^2_{\rm A}}$) and QD ($m_0 \equiv m_1 \simeq m_2 \simeq m_3 \gg \sqrt{\Delta m^2_\odot}, \sqrt{\Delta m^2_{\rm A}}$), we have

$$|\langle m\rangle|^{\rm NH} \simeq |\sqrt{\Delta m^2_\odot}\sin^2\theta_\odot\cos^2\theta_{13} + \sqrt{\Delta m^2_{\rm A}}\sin^2\theta_{13}{\rm e}^{i\alpha_{32}}|, \tag{18}$$

$$|\langle m\rangle|^{\rm IH} \simeq \sqrt{\Delta m^2_{\rm A}}\cos^2\theta_{13}\sqrt{1-\sin^2 2\theta_\odot\sin^2(\alpha_{21}/2)}, \tag{19}$$

$$|\langle m\rangle|^{\rm QD} \simeq m_0|(\cos^2\theta_\odot + \sin^2\theta_\odot{\rm e}^{i\alpha_{21}})\cos^2\theta_{13} + {\rm e}^{i\alpha_{31}}\sin^2\theta_{13}|, \tag{20}$$

where the notations are obvious.

Table 1. The maximal values of $|\langle m\rangle|$ (in units of meV) for the NH and IH spectra, and the minimal values of $|\langle m\rangle|$ (in units of meV) for the IH and QD spectra, at 2σ, for $\sin^2\theta_\odot = 0.25(0.30)[0.38]$, and for $\sin^2\theta_{13} = 0.0$, 0.02 and 0.04. The results for the NH and IH spectra are obtained for $m_0 = 10^{-4}$ eV, while those for the QD spectrum correspond to $m_0 = 0.2$ eV.

$\sin^2\theta_{13}$	$\lvert\langle m\rangle\rvert^{\rm NH}_{\rm max}$	$\lvert\langle m\rangle\rvert^{\rm IH}_{\rm min}$	$\lvert\langle m\rangle\rvert^{\rm IH}_{\rm max}$	$\lvert\langle m\rangle\rvert^{\rm QD}_{\rm min}$
0.0	2.7(3.2)[3.9]	18.5(16.2)[12.4]	49.5(52.0)[56.0]	77.7(67.8)[51.8]
0.02	3.7(4.1)[4.8]	18.1(15.9)[12.1]	48.5(51.0)[54.9]	72.0(62.2)[46.4]
0.04	4.7(5.1)[5.8]	17.8(15.5)[11.9]	47.5(50.0)[53.8]	66.3(56.8)[41.4]

In table 1, (i) maximal predicted value of $|\langle m\rangle|$ in the case of NH neutrino mass spectrum, (ii) the minimal and maximal values of $|\langle m\rangle|$ for the IH spectrum, and (iii) the minimal value of $|\langle m\rangle|$ for the QD spectrum are given. The indicated values of $|\langle m\rangle|$ are obtained at 2σ, by using the best-fit values for $\Delta m^2_\odot$, $\Delta m^2_{\rm A}$ (see the Introduction), three values for $\theta_\odot$, $\sin^2\theta_\odot = 0.25, 0.31$ and 0.38, and the prospective errors on the oscillation parameters, $\sigma(\Delta m^2_\odot) = 2\%$, $\sigma(\Delta m^2_{\rm A}) = 6\%$, $\sigma(\sin^2\theta_\odot) = 4\%$ and $\sigma(\sin^2\theta_{13}) = 0.004$.

As was noticed in ref. [21] (see also refs [19,25]), in the case of a *large mixing angle solution of the solar neutrino problem*, the observation of $(\beta\beta)_{0\nu}$-decay, combined with data on the neutrino masses, could provide, in principle, unique information on the CP-violation due to the Majorana CP-violating phases. Because of large but not-maximal solar mixing angle, a significant lower bound on $|\langle m\rangle|$ can be put in the case of the IH and QD spectra [19–21,25]:

$$m_{\rm max}\cos 2\theta_\odot \lesssim |\langle m\rangle| \lesssim m_{\rm max}, \tag{21}$$

where $m_{\rm max} \equiv \sqrt{\Delta m^2_{\rm A}}$ for the IH spectrum and $m_{\rm max} \equiv m_0$ for quasi-degenerate masses. The two limiting values correspond to CP-conservation: the upper (lower) bound is obtained for $\alpha_{21} = 0$ (π). This implies a significant lower bound on $|\langle m\rangle|^{\rm IH} \gtrsim 10$ meV and $|\langle m\rangle|^{\rm QD} \gtrsim 60$ meV, where the best-fit value of $\theta_\odot$ is used. If one of these spectra is realized in nature, the present and/or next future generation of $(\beta\beta)_{0\nu}$-decay experiments will find a positive signal.

In addition, for large mixing angle, there is a wide range of CP-violating values of $|\langle m\rangle|$, larger the range smaller the value of $\cos 2\theta_\odot$ and it is sizeable for the present best-fit value $\cos 2\theta_\odot = 0.4$.

In principle, the CP-violating phase α_{21} can be expressed in terms of the measurement of $|\langle m\rangle|$, $m_{\rm max}$, and $\sin^2 2\theta_\odot$:

$$\sin^2\frac{\alpha_{21}}{2} \cong \left(1 - \frac{|\langle m\rangle|^2}{m^2_{\rm max}}\right)\frac{1}{\sin^2 2\theta_\odot}. \tag{22}$$

The determination of CP-violation due to Majorana phases requires, in addition to a precise measurement of $|\langle m\rangle|$, information on the neutrino masses [19,22,26,28]. The additional input could be the measurement of neutrino mass $m_{\bar\nu_e}$ in ^{3}H β-decay experiments [55–57], or the cosmological determination of the sum of the three neutrino masses [58], $\Sigma = m_1 + m_2 + m_3$, or a derivation of a sufficiently stringent upper limit on m_0 in the IH case.

It was also pointed out in ref. [22] that the possibility of finding CP-violation 'requires quite accurate measurements' of $|\langle m \rangle|$ and, say, of $m_{\bar{\nu}_e}$, 'and holds only for a limited range of values of the relevant parameters'. It also depends crucially [28] on the measured mean value of $|\langle m \rangle|$ (and Σ), on the precision reached in the measurement of $|\langle m \rangle|$ (and Σ), and on the uncertainty in the knowledge of the value of the relevant $(\beta\beta)_{0\nu}$-decay nuclear matrix element. On the contrary, as shown in ref. [28], getting quantitative information on CP-violation from a measurement of the $(\beta\beta)_{0\nu}$-decay half-life is rather insensitive to the errors on the input neutrino oscillation parameters as long as the errors are smaller than $\sim 10\%$.

In the most favourable case of QD spectrum and assuming a precise measurement of neutrino masses obtained from cosmological observations, establishing Majorana CP-violation would require [28] for $\sin^2\theta_\odot \cong 0.31$, a $\sim$10% (or smaller) error in the measured $|\langle m \rangle|$ and Σ and knowledge of the relevant NME with an uncertainty corresponding to a factor of 1.5. For larger values of $\sin^2\theta_\odot$, e.g. $\sin^2\theta_\odot \cong 0.38$, it could be possible to obtain evidence of Majorana CP-violation at 2σ CL even for a factor of 2 in $|\langle m \rangle|$ due to the NME. If, however, $\sin^2\theta_\odot \cong 0.25$, exceedingly high precision in the measurements of $|\langle m \rangle|$ and Σ, and small NME uncertainty are required.

A detailed analysis of the possibility of establishing Majorana CP-violation as well as of the information which can be extracted from future $(\beta\beta)_{0\nu}$-decay experiments on the neutrino mass spectrum and the absolute neutrino masses, can be found in ref. [28].

5. The see-saw mechanism, leptogenesis and lepton flavour violating charged lepton decays

The origin of the matter–antimatter asymmetry is one of the most important questions in cosmology. The presently observed baryon asymmetry is [59]

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \simeq 6.1 \times 10^{-10}\,. \tag{23}$$

In 1967, A Sakharov [29] suggested that the baryon density can be explained in terms of microphysical laws. Three conditions need to be fulfilled: Baryon number (or lepton number, for the leptogenesis mechanism) violation; CP-violation, the CP symmetry being the product of charge conjugation and parity; departure from thermal equilibrium.

Several mechanisms have been proposed to understand the baryon asymmetry but many of them are disfavoured by cosmological or theoretical considerations. Leptogenesis [30] has emerged as a viable mechanism and is particularly appealing because it takes place in the context of see-saw models [60], which naturally explain the smallness of neutrino masses. Let me tell that $B-L$ is conserved in the standard theory of particle physics both at the perturbative and non-perturbative level. This implies that if one creates a net $B-L$ (e.g., a lepton number), the sphaleron processes would leave both baryon and lepton number comparable to the original $B-L$. This idea is implemented in the leptogenesis scenario [30].

The see-saw mechanism requires the existence of heavy right-handed (RH) Majorana neutrinos, completely neutral under the standard theory gauge symmetry group. Consequently, they can acquire Majorana masses that can, in principle, be much heavier than any of the known particles. Introducing a Dirac neutrino mass term and a Majorana mass term for the right-handed neutrinos via the Lagrangian

$$-\mathcal{L} = \overline{\nu_{Li}}\,(m_{\rm D})_{ij}\,N_{{\rm R}j} + \frac{1}{2}\,\overline{(N_{{\rm R}i})^c}\,(M_{\rm R})_{ij}\,N_{{\rm R}j}, \tag{24}$$

leads, for sufficiently large $M_{\rm R}$, to the well known see-saw [60] formula

$$m_\nu \simeq -m_{\rm D} M_{\rm R}^{-1} m_{\rm D}^T, \tag{25}$$

$$= U_{\rm PMNS} D_m U_{\rm PMNS}^T. \tag{26}$$

The CP-violating and out-of-equilibrium decays of RH neutrinos produce a lepton asymmetry [30,61] that can be converted into a baryon asymmetry [62,63]. The requisite CP-violating decay asymmetry is caused by the interference of the tree-level contribution and the one-loop corrections in the decay rate of the three heavy Majorana neutrinos into Higgs field, Φ, and charged leptons, ℓ. We limit our discussion to the one flavour approximation which holds for masses $M_i > 10^{12}$ GeV. Recently the issue of flavour has been addressed properly in [64]. For hierarchical heavy neutrinos one has

$$\varepsilon_i = \frac{\Gamma(N_i \to \Phi^-\,\ell^+) - \Gamma(N_i \to \Phi^+\,\ell^-)}{\Gamma(N_i \to \Phi^-\,\ell^+) + \Gamma(N_i \to \Phi^+\,\ell^-)},$$

$$\propto \sum_{j\neq i} \frac{{\rm Im}(m_{\rm D}^\dagger m_{\rm D})^2_{ij}}{(m_{\rm D}^\dagger m_{\rm D})_{ii}}\,\frac{M_i}{M_j}. \tag{27}$$

Then, the baryon asymmetry is obtained via $Y_B = a\,(\kappa/g^*)\,\varepsilon_1$, where $a \simeq -1/2$ is the fraction of the lepton asymmetry converted into a baryon asymmetry [62,63], $g^* \simeq 100$ is the number of massless degrees of freedom at the time of the decay, and κ is an efficiency factor that is obtained by solving the Boltzmann equations. Typically, one gets $Y_B \sim 6\times 10^{-10}$ when $\varepsilon_1 \sim (10^{-6} - 10^{-7})$ and $\kappa \sim (10^{-3} - 10^{-2})$. It is noted that this estimate of Y_B is valid in the supersymmetric (SUSY) theories as well.

Establishing a connection between the parameters at low energy (neutrino masses, mixing angles and CP-violating phases), measurable in principle in the present and future experiments, and at high energy (relevant in leptogenesis) has gathered a great interest in the last few years. The number of parameters in the full Lagrangian of models which implement the see-saw mechanism is larger than the ones in the low-energy sector: in the case of three light neutrinos and three heavy ones, at high energy the theory contains in the neutrino sector 18 parameters of which 12 are real ones and 6 are phases, while at low energy only 9 are accessible – three angles, three masses and three phases. The decoupling of the heavy right-handed neutrinos implies the loss of information on nine parameters. This implies that reconstructing the high-energy parameters entering in the see-saw models from the measurement of the masses, angles and CP-violating phases of m_ν is in general difficult, if not impossible, and depends on the specific model considered.

Using the weak basis in which both $M_{\rm R}$ and the charged lepton mass matrix are real and diagonal, it is useful to write the Dirac mass by the biunitary or the orthogonal parametrizations.

Biunitary parametrization. We can write the complex 3×3 Dirac mass as (see, e.g. ref. [31])

$$m_{\rm D} = U_{\rm L}^{\dagger}\, m_{\rm D}^{\rm diag}\, U_{\rm R}, \tag{28}$$

where $U_{\rm L}$ and $U_{\rm R}$ are unitary 3×3 matrices and $m_{\rm D}^{\rm diag}$ is a real diagonal matrix. All the CP-violating phases are contained in $U_{\rm L}$ and $U_{\rm R}$.

Orthogonal parametrization. By using the see-saw formula, eq. (25), we can express $m_{\rm D}$ as [32,65]:

$$m_{\rm D} = iUD_m^{1/2} R M_{\rm R}^{1/2}, \tag{29}$$

where D_m is the diagonal real matrix which contains the low-energy light neutrino masses, and R is a complex orthogonal matrix. R contains three real parameters and three phases.

The use of the two indicated parametrizations clarifies the dependence of leptogenesis on the different parameters entering in $m_{\rm D}$. In particular we have:

– for leptogenesis, the decay asymmetry ε_1 depends on the Hermitian matrix $m_{\rm D}^{\dagger} m_{\rm D}$ where

$$m_{\rm D}^{\dagger} m_{\rm D} = \begin{cases} U_{\rm R}^{\dagger}\, (m_{\rm D}^{\rm diag})^2\, U_{\rm R}, & \text{bi-unitary,} \\ M_{\rm R}^{1/2}\, R^{\dagger}\, D_m\, R\, M_{\rm R}^{1/2}, & \text{orthogonal.} \end{cases} \tag{30}$$

We can see that the PMNS unitary matrix U does not enter explicitly into the expression for the lepton asymmetry in the one flavour approximation. This conclusion does not hold if flavour is relevant and is properly taken into account. It has been shown [64,66] that in this case the baryon asymmetry depends directly on the CP-violating phases present in U and, in particular, on the δ phase and the two Majorana CP-violating phases.

– in the bi-unitary parametrization, the neutrino mass matrix m_ν can be written as

$$m_\nu = -U_{\rm L}^{\dagger} m_{\rm D}^{\rm diag} U_{\rm R} M_{\rm R}^{-1} U_{\rm R}^{T} m_{\rm D}^{\rm diag} U_{\rm L}^{*}. \tag{31}$$

This shows that the phases in U receive contributions from CP-violation both in the right-handed sector, responsible for leptogenesis, and in the left-handed one.

Let me tell that in supersymmetric models implementing the see-saw mechanism, the branching ratio of the charged lepton decays $\ell_i \to \ell_j \gamma$, $\ell_{i,j} = e, \mu, \tau$, $i > j$, get greatly enhanced with respect to the non-supersymmetric case [65,67]. They depend on a different combination of the Dirac mass. Therefore, additional information on the phases in $m_{\rm D}$ can be in principle obtained by observing such processes.

Due to the complicated way in which the high-energy phases and real parameters enter in m_ν, (eq. (31)), if there is CP-violation at high energy, as required by the leptogenesis mechanism, we can expect in general to have CP-violation at low energy, as a complete cancellation would require some fine-tuning or special forms of $m_{\rm D}$ and $M_{\rm R}$.

More specifically, from eq. (31), we see that, in general, there is not a one-to-one link between low-energy CP-violation in the lepton sector and the baryon asymmetry: a measurement of the low-energy CP-violating phases does not allow to reconstruct the leptogenesis phases, in a model independent way. However, most specific models allow for such a connection. In particular, if the number of parameters is reduced in $m_{\rm D}$, then a one-to-one correspondence between high-energy and low-energy parameters might be established. This can be achieved in models which allow for CP-violation only in the right-handed sector, that is in $U_{\rm R}$, or which reduce the number of independent parameters at high energy, for example by requiring only two right-handed neutrinos [68]. Each model of neutrino mass generation should be studied in detail separately to establish the feasibility of the leptogenesis mechanism [69].

The possible observation of $(\beta\beta)_{0\nu}$-decay would play an important role in understanding the origin of the baryon asymmetry generation as it would imply that lepton number (one of the main condition for leptogenesis) indeed is not conserved. Furthermore the Majorana nature of neutrinos would be established: the see-saw mechanism would be regarded as a reasonable explanation of neutrino mass generation. Leptogenesis naturally takes place in this scenario. Finally, the observation of CP-violation in the lepton sector, in neutrino oscillation experiments and/or $(\beta\beta)_{0\nu}$-decay, would suggest the existence of CP-violation at high energy, which might be related to the one responsible for leptogenesis.

6. Conclusions

One of the most important questions to address in the future concerning neutrino mixing is the existence of CP-violation in the lepton sector. The lepton mixing matrix contains one Dirac CP-violating phase whose effects are observable in neutrino oscillation. Future long-baseline experiments aim for its determination by comparing the neutrino and antineutrino appearance probabilities. I reviewed the main theoretical issues and the problem of degeneracies among different parameters for long-baseline neutrino experiments.

If neutrino are Majorana particles, two additional phases are physical and play a role in processes which violate the lepton number by two units. The most sensitive process of this type is the neutrinoless double beta decay. A precise measurement of $|\langle m\rangle|$, combined with information on neutrino masses and the neutrino mass spectrum, is required. In addition, establishing Majorana CP-violation might be possible only if the uncertainty in the calculation of the nuclear matrix elements is smaller than a factor 1.5–2.

Leptonic CP-violation plays an important role as the Dirac and the Majorana CP-violating phases might be at the origin of the baryon asymmetry of the Universe. Within the context of the see-saw mechanism, I discussed the possible connection between the CP-violating phases measurable at low energy with the ones which play a role in the leptogenesis mechanism.

The possible observation of $(\beta\beta)_{0\nu}$-decay, implying the violation of the global lepton number (one of the main conditions for leptogenesis), and of leptonic CP-violation in neutrino oscillations and/or neutrinoless double beta decay would be crucial in understanding the origin of the baryon asymmetry. It would be a strong indication, even if not a proof (as it is not possible to reconstruct in a model-independent way the high-energy parameters from m_ν), of leptogenesis as the explanation for the observed baryon asymmetry of the Universe.

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