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Experimental observation of direct current voltage-induced phase synchronization

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Abstract. The dynamics of two uncoupled distinct Chua circuits driven by a common direct current voltage is explored experimentally. It was found that, with increasing current intensity, the dominant frequencies of these two Chua circuits will first vary at different speeds, approach an identical value for a certain current intensity and then separate. Techniques such as synchronization index and phase difference distribution were employed to analyze the phase coherence between these two Chua circuits.

Keywords. Chua circuit; phase synchronization; synchronization index.

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1. Introduction

Synchronization is a fundamental and ubiquitous phenomenon, first discovered in periodical oscillators by Huygens [1]. It has been an active area of research during the past decade for its essential role in nonlinear dynamics and its potential applications in engineering, physics, biology and ecology [2,3]. Various subtypes of synchronization phenomena have been defined such as complete synchronization (CS) [4,5], generalized synchronization (GS) [6–8], phase synchronization (PS) [9] and so on. PS describes an intrinsic feature between two coupled nonidentical oscillators where the mismatch between phases of two distinct oscillators is locked within 2π when the coupling strength exceeds certain level, whereas their amplitudes may remain chaotic and uncorrelated. Moreover, PS can also be established in the case of chaotic oscillators driven by an external periodic forcing either sinusoidal [10], impulsive [11] or even nonidentical chaotic oscillator [12] where the dominant frequencies of the oscillators are confined to the frequency of the external forces when the intensity of the forcing exceeds a certain level. In this paper, we investigate the dynamics of two uncoupled nonidentical Chua circuits driven by a common direct current voltage (DCV). Basically, it is equivalent to changing a parameter of the

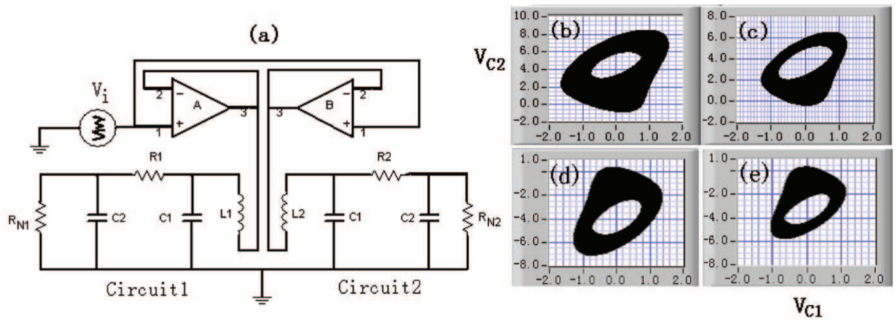

Figure 1. (**a**) Two independent Chua circuits driven by a common DCV. (**b**), (**c**) The phase portrait of the left(right) Chua circuit (V_{C1} vs. V_{C2}) respectively by setting positive initial value of V_{C2} without imposing DCV. (**d**), (**e**) The phase portrait of the left(right) Chua circuit respectively by setting negative initial value of V_{C2} without imposing DCV. Attractors of two circuits are similar but in different sizes.

circuit, where the output depends on the role of this parameter in the system. We found that the dominant frequency of one circuit decreases when the DCV intensity is increased while that of the other circuit monotonically increases. Particularly, when DCV intensity increases, their dominant frequencies approach each other, overlap at a certain level of the DCV intensity, and then separates. PS is established between the two Chua circuits when their dominant frequencies overlap. The synchronization index and phase difference distributions were calculated to analyze the PS. In §2, we introduce the experimental set-up and in §3 experimental data are analyzed. Finally, discussions and conclusions are given in the last section.

2. Experimental set-up

To explore the dynamics of the Chua circuits driven by a common DCV experimentally, we set up two nonidentical Chua circuits (denoted as circuit 1 and circuit 2) driven by a common DCV source as shown in figure 1.

C1 and C2 are the two capacitances, L1 and L2 are the inductors, R1 and R2 are the resistances that couple the two capacitors, and A, B are the operational amplifiers that keep the DCV one-way coupling to the Chua circuit. The DCV is generated by a signal generator (Agilent 33220A). R_{N1} and R_{N2} are the nonlinear resistors of three-segment piecewise linear characteristic, which are composed of two operational amplifiers (Texas Instruments type TL082C) and six resistors. The circuit equations driven by DCV are given as follows:

$$
\begin{aligned}
&C_1 V_{\mathrm{C1}} = G(V_{\mathrm{C2}} - V_{\mathrm{C1}}) - f(V_{\mathrm{C1}}),\\
&C_2 V_{\mathrm{C2}} = G(V_{\mathrm{C1}} - V_{\mathrm{C2}}) + i_L,\\
&L i_L = -V_{\mathrm{C2}} - \gamma_0 i_L + V_i,
\end{aligned}
\tag{1}
$$

where

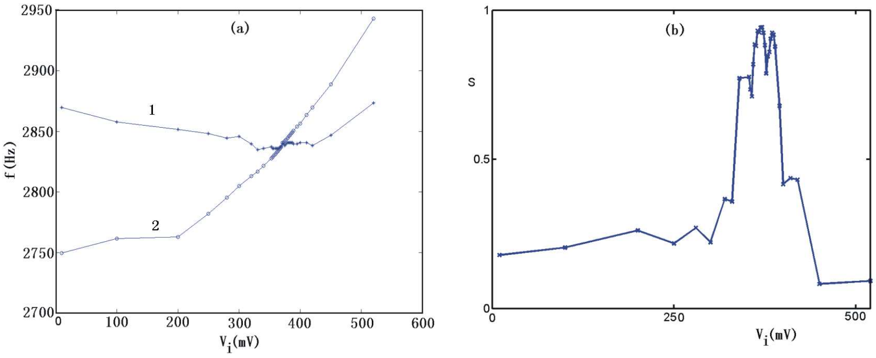

Figure 2. (**a**) The dominant frequency of two DCV-driven Chua circuits vs. DCV intensity V_i. (**b**) The index of phase coherence S vs. DCV intensity V_i.

$$f(V_{\mathrm{C1}}) = \begin{cases} G_b V_{\mathrm{C1}} + (G_b - G_a)E & \text{if } \; V_{\mathrm{C1}} < -E \\ G_a V_{\mathrm{C1}} & \text{if } \; -E \le V_{\mathrm{C1}} \le E \\ G_b V_{\mathrm{C1}} + (G_a - G_b)E & \text{if } \; V_{\mathrm{C1}} > E \end{cases} . \tag{2}$$

V_{C1} and V_{C2} are the voltages across C1 and C2 respectively and i_L is the current through L. These three variables describes the dynamical system. γ_0 is the inner resistance of the inductor, V_i is the voltage of the external force DCV. G_a and G_b are the slopes in the inner and outer regions respectively of the piecewise linear characteristic $f(V_{\mathrm{C1}})$. The parameters are set as $R_1 = 1.807$ KΩ, $R_2 = 1.862$ KΩ, $C_1 = 100$ nF, $C_2 = 10$ nF, $L_1 = 18.0$ mH, $L_2 = 18.6$ mH with tolerances of the components to be 10% for inductors, 5% for capacitors, and 1% for resistors. With the parameters presented above, both the circuits are single-scroll chaotic. If we inject a positive voltage on C2 as the initial value, the two circuits will stay in the state of figures 1b and 1c respectively, and if we inject a negative voltage on C2 as the initial value, the two circuits will stay in the state of figures 1d and 1e respectively. Data are acquired by using 6110/6111E DAQ card of National Instruments connected to a computer with a sampling rate of 6.0×10^5 p/s (samples more than 20 times each period) and a software LABVIEW is used to analyze the experimental data.

3. Data analysis

Rich dynamic states have been reported in the driven response systems for various driving sources as periodical or chaotic signals [10,12]. In ref. [10], the dynamics of the Chua circuit driven by external cosine signals were analyzed. PS between Chua circuit and the driving signals can be realized by controlling the frequency and amplitude of driving signals. The dominant frequency of the Chua circuit can be driven to that of the driving signal when the amplitude and frequency of the signal are properly adjusted. In ref. [12], rich dynamics such as point attractor, single scroll periodic, chaotic and double scroll, and two routes of transition from CS to PS have been reported in Chua circuits driven by nonidentical Chua circuits. In this paper, we are interested in the dynamics of the DCV-driven Chua circuits. The parameter values of two Chua circuits are distinct and set that make the attractors on the states as shown in figures 1b and 1c respectively. With increasing DCV intensity, the dominant frequencies of the two Chua circuits vary in dramatically different scenarios. Figure 2a shows the dominant frequency of two nonidentical DCV-driven Chua circuits vs. the DCV intensity. As the DCV intensity increases, the dominant frequency of circuit 1 first decreases in the interval of $V_i = 0$–420 mV, followed by an increase while that of circuit 2 monotonically increases in the whole interval of $V_i = 0$–520 mV. They meet and overlap at $V_i = 371$ mV. The frequency results cannot indicate the actual phase relationship between driven signal and the Chua circuit. The synchronization index S [13] was used to quantify the phase coherence between two circuits. PS was found between the two DCV-driven Chua circuits at DCV intensity $V_i = 371$ mV. The synchronization index S is defined as follows. Given two time series of signals, $x(t)$ and $y(t)$ of frequency ω, each time series can be represented by its Fourier image $F_x(\omega)$ or $F_x^*(\omega)$:

$$F_{x,y}(\omega) = |F_{x,y}(\omega)| \exp[i\theta_{x,y}(\omega)] = \int_{-\infty}^{+\infty} x(t)[y(t) \exp(-i\omega t)\mathrm{d}t. \tag{3}$$

Here $\theta_{x,y}$ are the Fourier phases of the signals at the frequency ω. The power spectra of the signals can be given by $P_{x,y}(\omega) = \langle F_{x,y}(\omega)F_{x,y}^*(\omega)\rangle$ and their cross spectrum $C_{xy}(\omega) = \langle F_x(\omega)F_y^*(\omega)\rangle$, where $\langle\,\rangle$ denotes averaging on ensembles or over time (we suppose the signals are ergodic processes). These characteristics describe the processes and their interdependence in terms of frequencies. Normalizing the cross spectrum to the power spectra $\sigma_{xy}(\omega) = |\frac{C_{xy}(\omega)}{[P_x(\omega)+P_y(\omega)]/2}|$, we get the coherence function $\sigma(\omega)$, which characterizes the phase coherence between two oscillations on the frequency ω. Particularly, $\sigma(\omega) = 1$ means that the differences between the Fourier-phase remains constant (i.e. $\theta_x(\omega) - \theta_y(\omega) = \text{const}$) at frequency ω and $\sigma \to 0$ means that the differences $\theta_x(\omega) - \theta_y(\omega)$ are random values uniformly distributed in $[-\pi, \pi]$ (the strict equality $\sigma(\omega) = 0$ achieves only when the number of data points approach infinity).

The coherence function presents an effective method to describe the interdependence of two signals in terms of frequencies. However, in order to measure the interdependencies between signals for all frequencies simultaneously, we must average the coherence function over all frequencies considering the contribution of every harmonic to the power of the signals. We may define the synchronization index (in the phase coherence sense) S between the signals x and y as the normalized average coherence magnitude:

$$S = \frac{\int_0^\infty [P_x(\omega) + P_y(\omega)]\sigma_{xy}(\omega)\mathrm{d}\omega}{\int_0^\infty [P_x(\omega) + P_y(\omega)]\mathrm{d}\omega}. \tag{4}$$

Its value is just the ratio of the power of coherent motions to the total power of signals x and y. It ranges from $S = 0$ (incoherent of signals x and y for all frequencies) to $S = 1$ (complete coherence of two signals for all frequencies).

The index S is calculated for V_{C1} of two Chua circuits under various DCV intensities. The curve of the index S vs. the DCV intensity V_i is shown in figure 2b, where S has a substantial hump in the interval of $V_i = 350$–400 mV, which

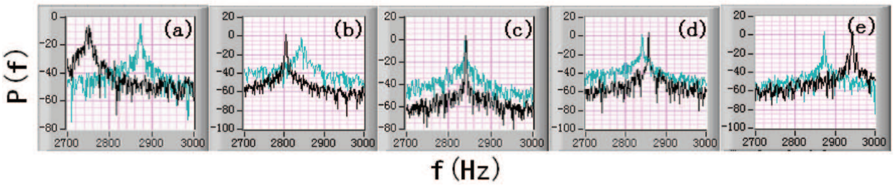

Figure 3. (**a**)–(**e**). Power spectra of the two DCV-driven Chua circuits with different DCV intensities $V_i = 10$, 280, 371, 400 and 520 mV, respectively.

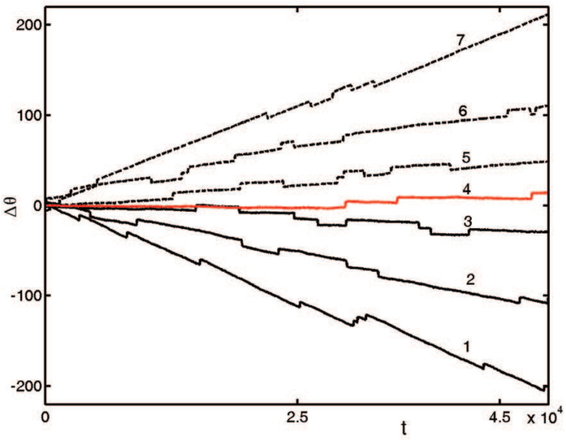

Figure 4. Time dependence of instantaneous phase difference for the two Chua circuits driven by DCV with various intensities $V_i = 100$, 300, 350, 371, 387, 400 and 420 mV marked by 1–7 respectively.

indicates strong coherence. The coherence is much weaker outside this interval. Namely, strong coherence can be achieved only for properly chosen DCV intensities in this circumstance. The power spectra of two circuits under various DCV intensities verified the result in figures 3a–3e, where the dominant frequencies of two circuits will approach each other, meet and overlap for a while, and then separate with the DCV intensity increasing from 10 to 520 mV.

A more efficient way to explore the phase coherence is to calculate the phase difference between two circuits. It is very important that the phase of the chaotic oscillator should be carefully defined. For simplicity, we use one-scroll chaotic attractor as a start. The phase can be defined by first choosing a time-window moving on a time series, for each segment in current time-window and recording the occurrence time $t_1, t_2, \ldots, t_n \ldots$ of every local maximum $V_{\mathrm{C1}}(t)$. Then $\phi(t) = 2\pi\frac{t-t_n}{t_{n+1}-t_n} + 2n\pi$, $t \in [t_n, t_{n+1}]$. The phase difference is $\Delta\phi(t) = \phi_1(t) - \phi_2(t)$. The phase coherence between two circuits driven by a common DCV is obvious with a certain DCV intensity as shown in figure 4, where the phase differences vs. time are shown in different DCV intensities V_i, whose platforms indicate the phase locking between two signals. As the DCV intensity increases, the two circuits undergo the process of weak phase coherence (lines marked 1, 2) to strong phase coherence (marked 3, 4, 5) and then the weak phase coherence (marked 6, 7). The distribution

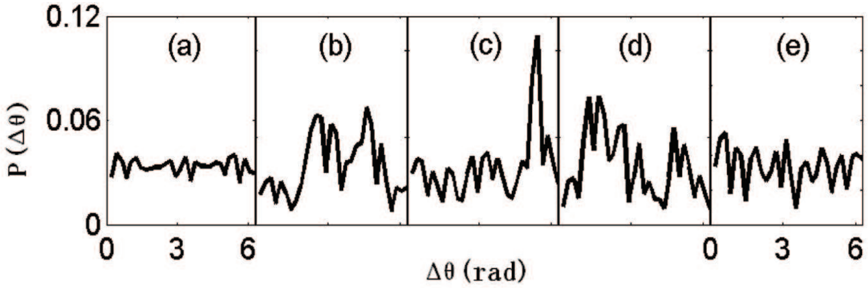

Figure 5. **(a)–(e).** The distribution of the phase differences with different DCV intensities V_i corresponding to figures 3a–e.

of the phase differences between two circuits verified the results shown in figures 5a–5e. With weak DCV intensity $V_i = 10$ mV, the two driven circuits are nearly uncorrelated and the distribution is almost uniform. As $V_i = 280$ mV, the structure of the distribution has a characteristic change as seen in 5b, and the distributions of phase differences form additional peaks away from the zero phase difference, and yield a sharp peak at $V_i = 371$ mV as seen in 5c. At larger intensities, the distribution of phase difference return to uniform distribution as seen in 5d and 5e.

However, if the attractors are initially set on the state as shown in figures 1d and 1e respectively. With increasing DCV, the attractor size tends to shrink and transit to the state as shown in figures 1b and 1c respectively as $V_i = 320$ mV. No phase locking can be observed before the transition.

4. Conclusion

In this paper, we experimentally explored dynamics of two independent nonidentical Chua circuits driven by a common DCV with various DCV intensities. The dominant frequencies of the two circuits will first approach each other, meet and overlap for a while, and then separate as DCV intensity increases. Moreover, the phase coherence undergoes a process from weak to strong and then to weak, and the phase locking is clearly observed at $V_i = 371$ mV, which is well-captured by the synchronization index S and phase difference distribution $\Delta\phi$ between two Chua circuits. This accounts for the complicated structure of the Chua chaotic attractor.

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