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Theoretical study of two-element array of equilateral triangular patch microstrip antenna on ferrite substrate

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Abstract. The radiation characteristics of a two-element array of equilateral triangular patch microstrip antenna on a ferrite substrate are studied theoretically by considering the presence of bias magnetic field in the direction of propagation of electromagnetic waves. It is found that the natural modes of propagation in the direction of magnetic field are left- and right-circularly polarized waves and these modes have different propagation constants. In loss-less isotropic warm plasma, this array antenna geometry excites both electromagnetic (EM) and electroacoustic plasma (P) waves in addition to a nonradiating surface wave. In the absence of an external magnetic field, the EM- and P-waves can be decoupled into two independent modes, the electroacoustic mode is longitudinal while the electromagnetic mode is transverse. The far-zone EM-mode and P-mode radiation fields are derived using vector wave function techniques and pattern multiplication approaches. The results are obtained in both plasma medium and free space. Some important antenna parameters such as radiation conductance, directivity and quality factor are plotted for different values of plasma-to-source frequency.

Keywords. Microstrip array antenna; ferrite substrate; radiation properties; plasma.

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1. Introduction

In recent years, ferrite substrates have been the subject of much interest for microstrip patch antennas and their arrays. The high dielectric constant of the ferrite substrates reduces the antenna dimensions and when biased with a DC magnetic field, the antenna exhibits a number of novel properties. These include frequency tuning agility, the generation of circular polarization, reduction of surface waves and radar cross-section control. Ferrite materials are also used to generate beam scanning antennas. Microstrip antennas mounted on aerospace vehicles encounter plasma medium during their travel in space, as a result of which radiation properties are altered significantly. This change is caused due to the generation of electroacoustic (P) waves in addition to electromagnetic (EM) waves [1–5].

Table 1. Design requirement and substrate characteristics of the ferrite substrate $Ni_{1.062}Co_{0.02}Fe_{1.948}O_4$.

Design frequency (f)	1.0 GHz
Relative permittivity (ε_r)	14.78
Dieletric loss tangent (tan δ_e)	0.0005
Magnetic loss tangent (tan δ_m)	0.005
Applied DC magnetic bias field (H_0)	7.96×10^4 A/m
Saturation magnetization ($\mu_0 M_s$)	0.03 T
Patch dimension (a)	0.022 m
Substrate thickness (h)	0.0016 m
Gyromagnetic ratio (γ)	1.76×10^{11} rad /s · T

Ferrite and other magnetic materials have been extensively used in several microwave devices such as phase shifters, isolators, circulators, tunable filters, delay lines etc. Ferrite materials have a significant amount of anisotropy at microwave frequencies. This anisotropy gets induced by applying external DC magnetic field in ferrite or gyrotropic materials and brings about non-reciprocal behavior in them.

This paper describes the radiation characteristics of a two-element array of equilateral triangular patch microstrip antenna on the ferrite substrate $Ni_{1.062}Co_{0.02}Fe_{1.948}O_4$. Design requirement and substrate characteristics considered for this analysis are listed in table 1.

2. Radiation field expressions

The top and side view of the configuration of array antenna is shown in figure 1. It consists of two identical triangular microstrip patch elements of arm length a, on a ferrite substrate $Ni_{1.062}Co_{0.02}Fe_{1.948}O_4$ of thickness h and substrate permittivity ε_r. The array elements are separated by a distance d and the progressive phase excitation difference between the patches is β_1. Each patch can be excited by a microstrip transmission line connected to the edge or by a coaxial line from the back at the plane $\phi = 0$. We have considered the patch as a cavity which acts as a disc resonator. Among the various modes that may be excited in such a disc resonator, we have considered TM_{mn} mode with respect to z-axis. Here n and m are the mode numbers associated with x and y directions respectively.

The E_z component of the field inside the cavity for dominant mode is given as

$$E_z = A_{1,0,-1}[2\cos(2\pi x/\sqrt{3}a + 2\pi/3)\cos(2\pi y/3a) + \cos(4\pi y/3a)]. \quad (1)$$

When the propagation of electromagnetic waves takes place along the direction of applied magnetic bias field, two plane wave modes – the left-hand circularly polarized (LHCP) mode and the right-hand circularly polarized (RHCP) mode may exist. The magnetic properties of the ferrite substrate affect both these modes. Therefore, the effective permeability for right-hand circular polarization and left-hand circular polarization will be given as [6–8]

$$\mu_{\text{effR}} = \mu_0 \frac{1+\omega_m}{\omega_0 - \omega}, \quad (2)$$

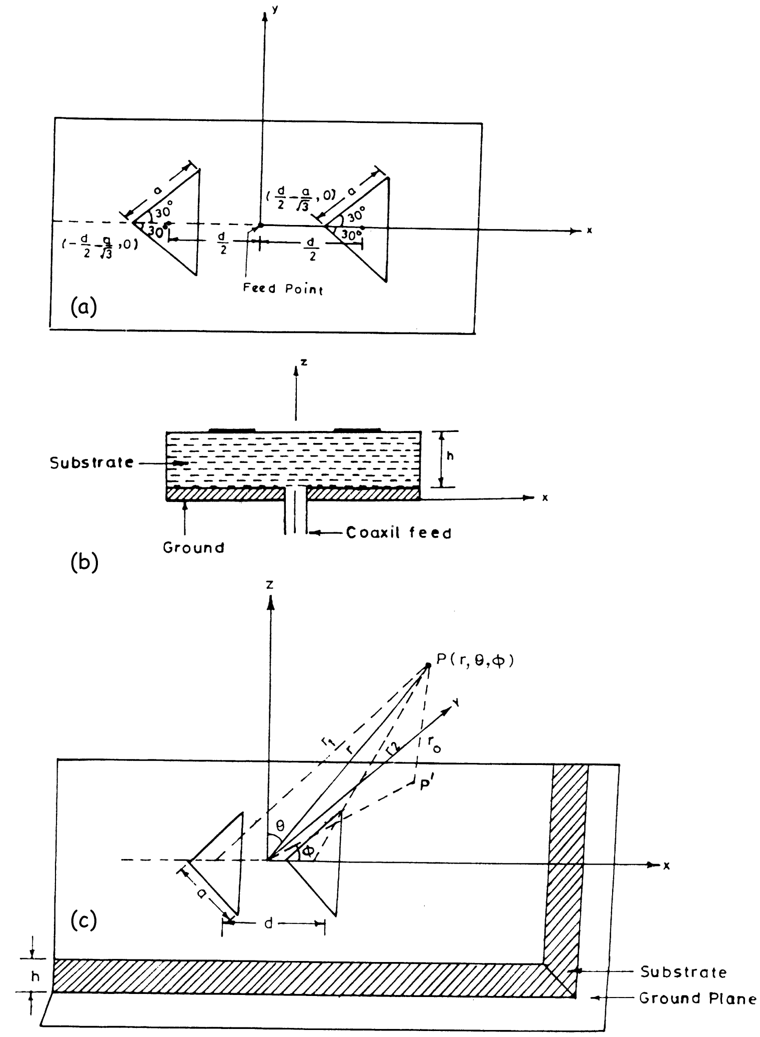

Figure 1. (**a**) 2D top view of the configuration of two-element triangular patch array microstrip antenna on ferrite substrate. (**b**) Side view of the configuration of two-element triangular patch microstrip array antenna on ferrite substrate. (**c**) 3D view of the configuration of two-element triangular patch microstrip array antenna on ferrite substrate.

$$\mu_{\mathrm{effL}} = \mu_0 \frac{1+\omega_m}{\omega_0+\omega}. \tag{3}$$

The expression of the resonant frequency of the considered equilateral triangular microstrip antenna on ferrite substrate in TM_{mn} mode is given as

$$f_r = \frac{2c\sqrt{\mu_0}}{3a\sqrt{\varepsilon_{\mathrm{r}}\mu_{\mathrm{eff}}}}. \tag{4}$$

Other field components are obtained by solving Maxwell's equations. By image theory, the ground plane may be replaced by an image of the top conductor. The magnetic currents also exist along the edges of triangular conductor and may be evaluated from

$$\mathbf{M} = 2(\mathbf{E} \times \mathbf{n}), \tag{5}$$

where $\mathbf{n}$ is a unit vector normal to the aperture.

Using linearized hydrodynamic theory of plasma, vector wave function technique and neglecting the coupling between the elements, the basic equations of far-zone fields of the two-element array of triangular microstrip patch antenna are given as [9–11]

$$E_{\theta t} = -\iota\eta_0\omega[-F_x \sin\phi + F_y \cos\phi] \left[\frac{\exp\{-\iota(\beta_e r_1 + 0.5\beta_1)\}}{r_1} + \frac{\exp\{-\iota(\beta_e r_2 - 0.5\beta_1)\}}{r_2}\right], \tag{6}$$

$$E_{\phi t} = \iota\eta_0\omega[F_x \cos\theta\cos\phi + F_y \cos\theta\sin\phi] \left[\frac{\exp\{-\iota(\beta_e r_1 + 0.5\beta_1)\}}{r_1} + \frac{\exp\{-\iota(\beta_e r_2 - 0.5\beta_1)\}}{r_2}\right]. \tag{7}$$

In the far-zone field region, the values of the radius vectors r_1, r_2 can be calculated from figure 1.

$$r_1 \approx r + d/2\cos\theta$$
$$r_2 \approx r - d/2\cos\theta, \text{ for phase variation}$$

and

$$r_1 \approx r_2 \approx r, \text{ for amplitude variation.}$$

Applying these approximations in eqs (6) and (7), we get:

For EM-mode:

$$E_{\theta t} = -\iota\eta_0\omega[-F_x \sin\phi + F_y \cos\phi] 2\cos[0.5(\beta_e d\cos\theta + \beta_1)] \frac{\mathrm{e}^{-i\beta er}}{r} \tag{8}$$

$$E_{\phi t} = \iota\eta_0\omega[F_x \cos\theta\cos\phi + F_y \cos\theta\sin\phi] 2\cos[0.5(\beta_e d\cos\theta + \beta_1) \frac{\mathrm{e}^{-i\beta er}}{r} \tag{9}$$

and for plasma-mode:

$$E_{pt} = 2h\beta_p\omega_p^2/3a\omega\varepsilon_0(\omega^2 - \omega_p^2)\exp(-\iota\beta_p r)/r \times \exp(-\iota\beta_p a\sin\theta\cos\phi/\sqrt{3}) \times 2\cos[0.5(\beta_p d\cos\theta + \beta_1) \times [E_{px} + E_{py}], \tag{10}$$

where $E_{\theta t}$, $E_{\phi t}$ are the components of total electric field vectors for EM-mode, E_{pt} is the total electric field vector for plasma-mode, F_x is the x-component of vector electric potential, F_y is the y-component of vector electric potential, E_{px} is the x-component of the electric field vector for plasma-mode, E_{py} is the y-component of electric field vector for P-mode, β_e is the phase propagation constant for EM-mode given by $2\pi A/\lambda_0$, β_p is the phase propagation constant for plasma-mode given by $\beta_e c/v$, c is the velocity of light, v is the root mean square thermal velocity of electron, A is the plasma frequency parameter given by $(1-\omega_p^2/\omega_0^2)^{1/2}$, ω_p is the angular plasma frequency, ω is the angular source frequency and β_1 is the progressive phase excitation difference between the patches.

3. Field patterns

The expression for total field pattern $R(\theta,\phi)$ is obtained as

$$R(\theta,\phi) = |E_{\theta t}|^2 + |E_{\phi t}|^2. \tag{11}$$

Then, the radiation field patterns in the E-plane ($\phi = 0$) and H-plane ($\phi = \pi/2$) are given as

$$R_e(\theta,\phi) = |E_{\theta t}|^2 + |E_{\phi t}|^2 = \eta_0^2\omega_0^2(|F_y|^2 + |F_x|^2\cos^2\theta) \quad (E\text{-plane}), \tag{12}$$

$$R_h(\theta,\phi) = |E_{\theta t}|^2 + |E_{\phi t}|^2 = \eta_0^2\omega_0^2(|F_x|^2 + |F_y|^2\cos^2\theta) \quad (H\text{-plane}). \tag{13}$$

The values of R_e and R_h are calculated for a case taking $f = 1$ GHz, $a = 2.2$ cm, $\varepsilon_r = 14.78$, applied DC magnetic bias field $(H_0) = 7.96 \times 10^4$ A/m, saturation magnetization $(\mu_0 M_s) = 0.03$ T, $d = \lambda/2$ and the phase difference $\beta_1 = \pi/2$. The results are plotted in figures 2 and 3 for two different planes ($\phi = 0$ and $\phi = \pi/2$) for $A = 1.0$, i.e. in free space and in figures 4 and 5 for two different planes ($\phi = 0$ and $\phi = \pi/2$) for $A = 0.5$, i.e. in plasma medium. The variation of effective permeability for both RHCP and LHCP modes with applied magnetic bias field (H_0) is shown in figure 6. The P-mode fields are plotted in figure 7 for $A = 0.5$ for a limited range of 10° (from 60° to 70°). The field patterns are also compared with single-element triangular patch microstrip antenna.

4. Other antenna parameters

4.1 *Radiation conductance*

The total power radiated can be calculated by employing Poynting's theorem and is given as

$$P_r = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left\{\mathrm{Re}\int\!\!\int_S (E \times H^*)\mathrm{d}s\right\}.$$

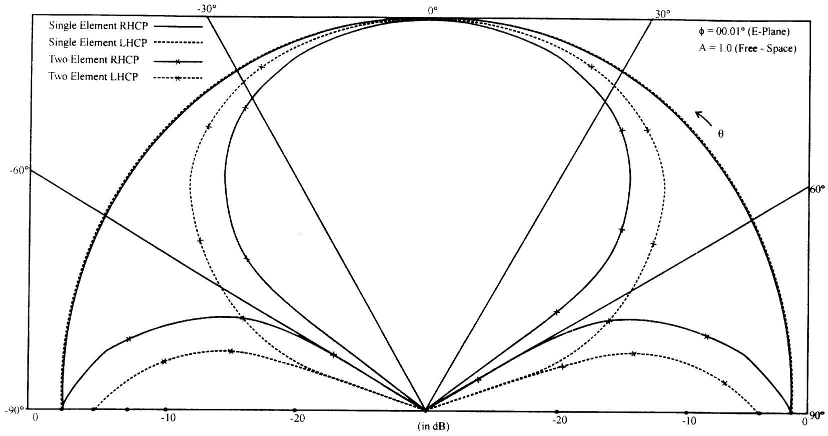

Figure 2. Variation of $R(\theta, \phi)$ for $A = 1.0$ (free space) for single-element and two-element ($\beta_1 = \pi/2$) equilateral triangular patch microstrip antenna on ferrite substrate in E-plane.

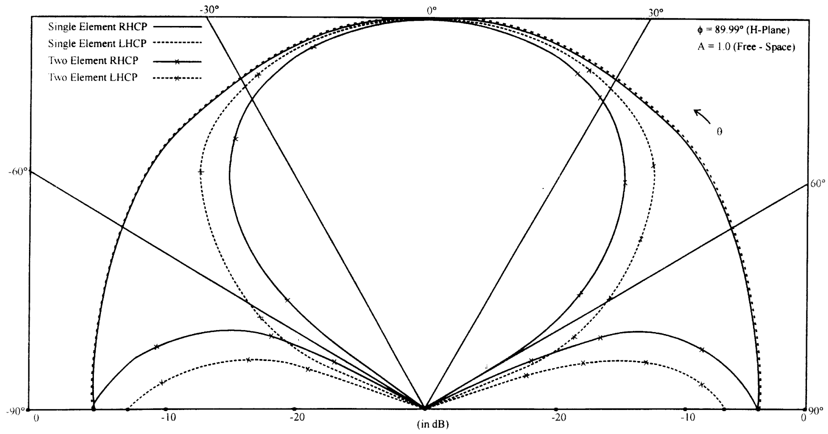

Figure 3. Variation of $R(\theta, \phi)$ for $A = 1.0$ (free space) for single-element and two-element ($\beta_1 = \pi/2$) equilateral triangular patch microstrip antenna on ferrite substrate in H-plane.

The factor $\frac{1}{2}$ is due to the fact that the power is radiated through the upper half space only and S is the total spherical surface area.

$$P_r = \left(\frac{1}{4}\right)\left\{\mathrm{Re}\int\int_S (E_\theta H_\phi^* - E_\phi H_\theta^*)\mathrm{d}s\right\}.$$

Thus, the expressions for radiated power in EM-modes is obtained using the relation [6]:

$$P_e = (A/4\eta_0)\int_0^{2\pi}\int_0^{\pi}\{|E_{\theta t}|^2 + |E_{\phi t}|^2\}r^2 \sin\theta \mathrm{d}\theta\mathrm{d}\phi. \quad (14)$$

Here, $A = (1 - \omega_p^2/\omega_0^2)^{1/2}$ and ω_p/ω_0 is the plasma-to-source frequency ratio.

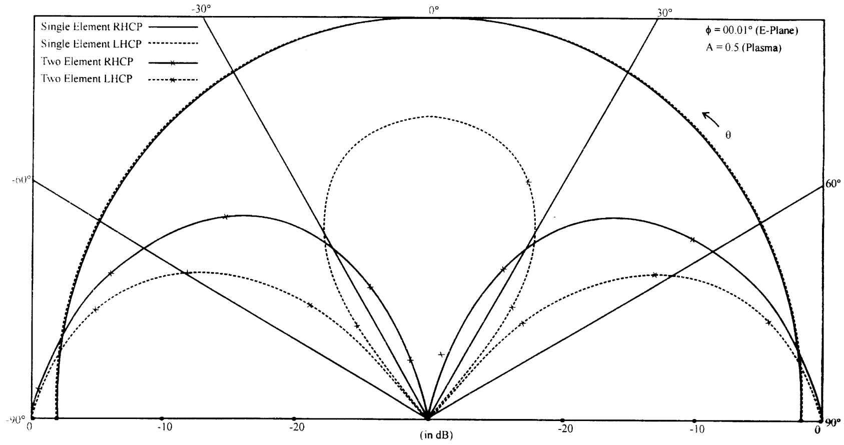

Figure 4. Variation of $R(\theta, \phi)$ for $A = 0.5$ (plasma) for single-element and two-element ($\beta_1 = \pi/2$) equilateral triangular patch microstrip antenna on ferrite substrate in E-plane.

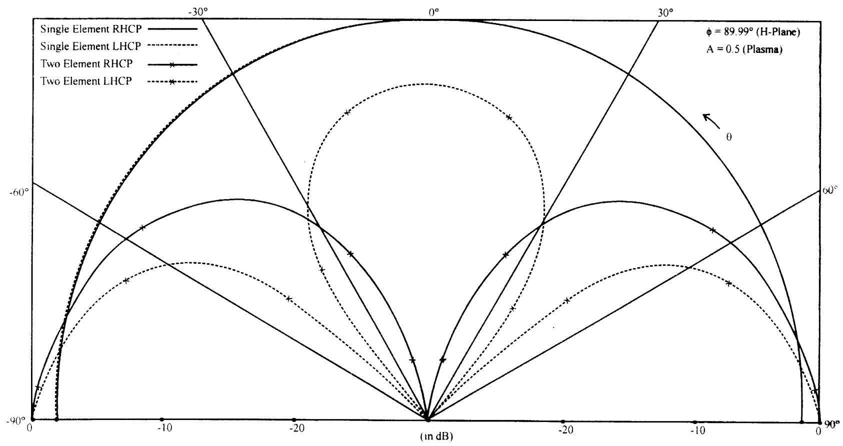

Figure 5. Variation of $R(\theta, \phi)$ for $A = 0.5$ (plasma) for single-element and two-element ($\beta_1 = \pi/2$) equilateral triangular patch microstrip antenna on ferrite substrate in H-plane.

The radiation conductance of an antenna in EM-mode can be defined as

$$G_e = 2P_e/V_0^2. \tag{15}$$

4.2 *Directivity*

The directivity of an antenna is defined as the ratio of the maxium radiation intensity (power per unit solid angle) to the average radiation intensity. It can be expressed as

$$D_e = 4\pi\{\max(|E_{\theta t}|^2 + |E_\phi|^2)\} \Big/ \left\{ \int_0^{2\pi} \int_0^{\pi} |E_{\theta t}|^2 + |E_{\phi t}|^2 r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi \right\}. \tag{16}$$

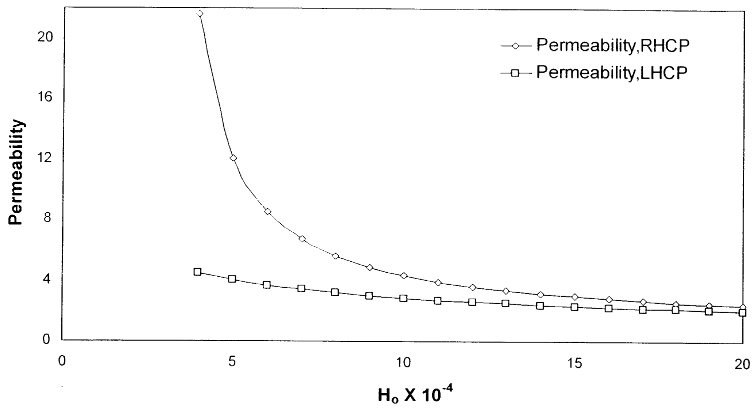

Figure 6. Variation of effective permeability of substrate with applied bias field.

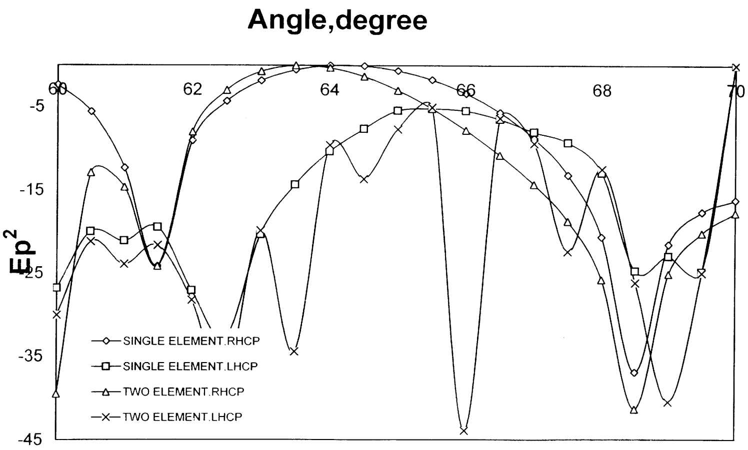

Figure 7. Variation of plasma mode field pattern $|E_{pt}|^2$ for $A = 0.5$ (plasma) for single-element and two-element ($\beta_1 = \pi/2$) equilateral triangular patch microstrip antenna on ferrite substrate.

4.3 *Quality factor*

A parameter specifying the frequency selectivity of a resonant circuit is the quality factor Q, which can be defined as the ratio between energy stored in the system and the energy lost. The total Q of a microstrip radiating element comprises contributions due to the radiation Q_r, conductor loss Q_c, and dielectric loss Q_d quality factors. So

$$1/Q_t = 1/Q_r + 1/Q_c + 1/Q_d, \tag{17}$$

where $Q_r = \omega_0 W_t/P_r$, $Q_c = \omega_0 W_t/P_c = (\pi f \mu \sigma)^{1/2} h$ and $Q_d = \omega_0 W_t/P_d = 1/\tan\delta$. Here, W_t is the energy stored in the antenna element, P_c and P_d are power

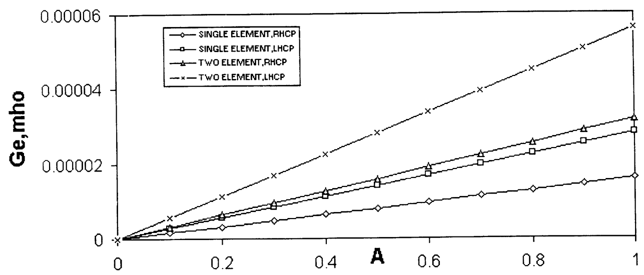

Figure 8. Variation of radiation conductance G_e for single-element and two-element equilateral triangular patch microstrip antenna on ferrite substrate for EM-mode with plasma parameter A.

loss factors due to the conductors and dielectric, respectively, σ is the conductivity of the conductors.

The energy stored in the triangular radiating element is given by

$$W_t = (\varepsilon h/2) \int\int |E_z(x,y)|^2 \mathrm{d}x\,\mathrm{d}y. \tag{18}$$

5. Conclusion

The radiation characteristics of a two-element linear array of equilateral triangular patch microstrip antenna on ferrite substrate have been studied by considering the presence of bias magnetic field in the direction of propagation of EM waves. The results of the array geometry are compared with those of single-element of equilateral triangular patch microstrip antenna. It is found that there is a significant change in the radiation characteristics of array geometry. In the case of EM-mode, the shape of the field pattern has been modified to a great extent and redistributes the field intensities considerably. It is also observed that the radiation patterns of a single-element antenna contain only one major lobe of considerably wide beam-width, while the array geometry produces a directive beam with a narrow beam-width. In the case of P-mode, the field patterns are similar to single-element microstrip antenna. It is further observed that the values of radiation conductance are considerably higher for array geometry and it increases continuously with increasing plasma parameter. The values of directivity for array geometry are also considerably higher than that of the single-element of equilateral triangular patch microstrip antenna. The values of quality factor for array geometry are lower than that of single-element of equilateral triangular patch microstrip antenna. The computed values of pattern characteristics of the present array are given in table 2. It is clear from this table that half-power beam-width of the patterns in free space is relatively small in comparison to plasma medium. Thus it reveals that the patterns are more directive in free-space than in plasma medium. Variation of radiation conductance G_e for single-element and two-element equilateral triangular patch microstrip antenna on ferrite substrate with plasma parameter A is shown in figure 8. From this figure, it is observed that the radiation conduction (G_e) of the two-element array antenna is more than that of the single-element antenna for all the plasma frequency.

Table 2. Pattern characteristics of the two-element array of triangular microstrip patch antenna on ferrite substrate.

Pattern characteristics	$\phi = 0$ plane $A = 1.0$		$\phi = 0$ plane $A = 0.5$		$\phi = \pi/2$ plane $A = 1.0$		$\phi = \pi/2$ plane $A = 0.5$	
	RHCP	LHCP	RHCP	LHCP	RHCP	LHCP	RHCP	LHCP
Half-power beam width (HPBW)	60°	80°	140°	150°	60°	80°	140°	150°
Full null beam width (FNBW)	120°	140°	360°	280°	120°	140°	360°	280°
Direction of max. radiation	0°	0°	90°	90°	0°	0°	90°	90°

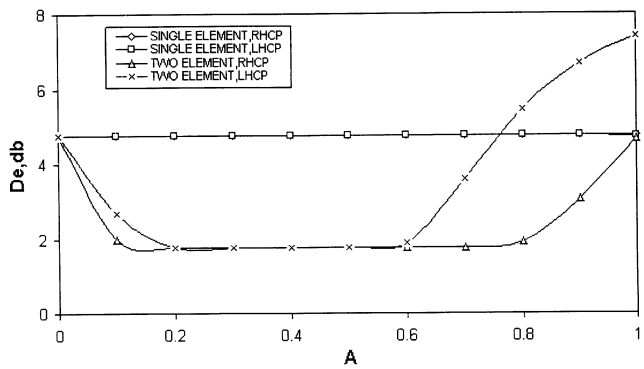

Figure 9. Variation of directivity D_e for single-element and two-element equilateral triangular patch microstrip antenna on ferrite substrate for EM–mode with plasma parameter A.

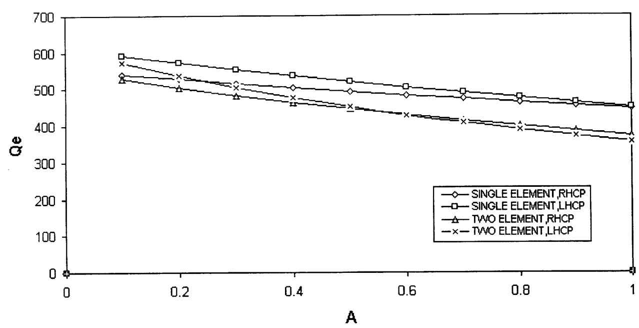

Figure 10. Variation of quality factor Q_e for single-element and two-element equilateral triangular patch microstrip antenna on ferrite substrate for EM-mode with plasma parameter A.

It is maximum in free space and decreases on increasing plasma frequency. It is also observed that the radiation conduction (G_e) with LHCP waves is higher than that with RHCP waves. Variation of radiation conductance D_e for single-

Table 3. Antenna parameters for single-element and two-element equilateral triangular patch microstrip antenna on ferrite substrate with RHCP waves.

Plasma parameter (A)	G_e for single element (mho)	G_e for two-element linear array (mho)	D_e for single element (dB)	D_e for two-element linear array (dB)	Q_e for single element	Q_e for two-element linear array
1.0	1.588×10^{-5}	$3.175{\times}10^{-5}$	4.751	4.669	444.418	371.717
0.9	1.428×10^{-5}	$2.855{\times}10^{-5}$	4.755	3.048	453.355	384.392
0.8	1.268×10^{-5}	$2.536{\times}10^{-5}$	4.758	1.927	462.635	397.928
0.7	1.109×10^{-5}	$2.218{\times}10^{-5}$	4.761	1.773	472.281	412.419
0.6	9.468×10^{-6}	$1.900{\times}10^{-5}$	4.764	1.77	482.319	427.974
0.5	7.911×10^{-6}	$1.582{\times}10^{-5}$	4.766	1.768	492.774	444.72
0.4	6.326×10^{-6}	$1.265{\times}10^{-5}$	4.768	1.766	503.678	462.803
0.3	4.743×10^{-6}	$9.486{\times}10^{-6}$	4.769	1.764	515.061	482.396
0.2	3.161×10^{-6}	$6.323{\times}10^{-6}$	4.77	1.762	526.961	503.703
0.1	1.580×10^{-6}	$3.161{\times}10^{-6}$	4.771	1.98	539.417	526.965
0.0	0.0	0.0	4.771	4.771	–	–

Table 4. Antenna parameters for single element and two-element equilateral triangular patch microstrip antenna on ferrite substrate with LHCP waves.

Plasma parameter (A)	G_e for single element (mho)	G_e for two-element linear array (mho)	D_e for single element (dB)	D_e for two-element linear array (dB)	Q_e for single element	Q_e for two-element linear array
1.0	$2.811{\times}10^{-5}$	$5.621{\times}10^{-5}$	4.736	7.395	449.784	355.889
0.9	$2.526{\times}10^{-5}$	$5.052{\times}10^{-5}$	4.742	6.667	462.144	371.617
0.8	$2.242{\times}10^{-5}$	$4.484{\times}10^{-5}$	4.748	5.45	475.146	388.724
0.7	$1.959{\times}10^{-5}$	$3.919{\times}10^{-5}$	4.754	3.585	488.847	407.407
0.6	$1.678{\times}10^{-5}$	$3.356{\times}10^{-5}$	4.758	1.896	503.311	427.904
0.5	$1.397{\times}10^{-5}$	$2.794{\times}10^{-5}$	4.762	1.772	518.612	450.503
0.4	$1.117{\times}10^{-5}$	$2.233{\times}10^{-5}$	4.765	1.769	534.831	475.559
0.3	$8.370{\times}10^{-6}$	$1.674{\times}10^{-5}$	4.768	1.766	552.062	503.507
0.2	$5.578{\times}10^{-6}$	$1.116{\times}10^{-5}$	4.77	1.763	570.413	534.897
0.1	$2.788{\times}10^{-6}$	$5.576{\times}10^{-6}$	4.771	2.688	590.006	570.422
0.0	0.0	0.0	4.771	4.77	–	–

element and two-element equilateral triangular patch microstrip antenna on ferrite substrate with plasma parameter A is shown in figure 9. From this figure, it is observed that the directivity (D_e) of this array with LHCP waves is higher than that with RHCP waves for free space and all plasma frequency. Variation of radiation conductance Q_e for single-element and two-element equilateral triangular patch microstrip antenna on ferrite substrate with plasma parameter A is shown in figure 10. From this figure, it is noticed that the low values of quality factor (Q_e) of this array antenna in free space indicate that antenna is radiating power more effectively in free space. Hence at low values of A, more energy is radiated in the form of plasma waves, which increases the quality factor of antenna in plasma medium. If we are considering the effect of LHCP and RHCP modes, then the total quality factor of this array antenna with LHCP waves is lower than that with RHCP waves.

Finally, it is concluded that two-element linear array of equilateral triangular patch microstrip antenna has unique radiation characteristics and can be employed in applications where high gain and narrow beam-width are required. The results of the present study are useful, particularly for space vehicles because such type of array antenna can be mounted on the flat surface as well as on the curved surface of the vehicles.

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