PRAMANA
— journal of
physics

Vol. 64, No. 2
February 2005
pp. 239–247

A new approach of binary addition and subtraction by non-linear material based switching technique

ARCHAN KUMAR DAS, PARTHA PRATIMA DAS and
SOURANGSHU MUKHOPADHYAY
Department of Physics and Technophysics, Vidyasagar University, Midnapore 721 102, India
E-mail: das_archan2@rediffmail.com; r_kuladeep@rediffmail.com;
dppratima2002@yahoo.co.in

MS received 21 June 2003; revised 8 August 2004; accepted 13 September 2004

Abstract. Here, we refer a new proposal of binary addition as well as subtraction in all-optical domain by exploitation of proper non-linear material-based switching technique. In this communication, the authors extend this technique for both adder and subtractor accommodating the spatial input encoding system.

Keywords. Non-linear material; optical computing; binary arithmetic; optical logic operations.

PACS Nos 42.65.-k; 42.70.-a; 42.79.Ta; 42.82.Gw

1. Introduction

All-optical parallel computation uses the parallelism of optics with all its possibilities to overcome the limitations and restrictions for arithmetic and logic operations in optical domain. Here, the authors propose a new technique of binary addition and subtraction scheme by proper all-optical switching system. This technique can be extended for the performance of some other arithmetic operations also. In connection to the addition and subtraction scheme, some related works proposed earlier by different scientists can be referred [1–11]. Here the whole scheme accommodates both the spatial input encoding and non-linear switching mechanism.

2. Coding technique of two input variables

The spatial encoding technique is established as a very successful candidate for implementation of various arithmetic and logic operations. In this process, the inputs become encoded by the combination of the presence and absence of light signal in a spatially represented coded mask.

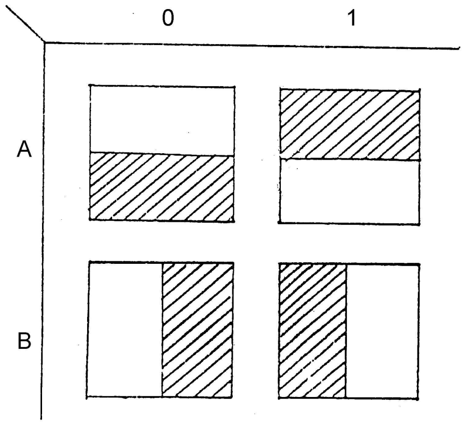

Figure 1. Coded pattern of two input variables by spatial variation of light intensity.

The coding technique for two input variables (A and B) is shown in figure 1. Here each rectangular half of the input variables is spatially coded. The input variable A takes 1, when the lower rectangular half is transparent and the upper rectangular half is opaque. It becomes 0, when the upper half is transparent and the lower half is opaque. In the same way, input B is also coded. When the input variable B takes 1, the right rectangular half is transparent and the left half is opaque. B becomes 0, when the left half is transparent and the right half is opaque. The superimposed structure of the input variables (A and B) is shown in figure 2. When both A and B take 0 value, the upper left portion of the superimposition structure is transparent. Similarly, when $A = 0$ and $B = 1$, only the upper right portion is transparent. When $A = 1$ and $B = 0$, only the lower left portion is transparent and when $A = 1$ and $B = 1$, only the lower right portion is opaque. This type of coding is very much useful in constructing an all-optical half adder.

3. Coding technique of three input variables

The coding patterns of three input variables (A, B and C) are shown in figure 3. The coding pattern of input variables A and B are the same as discussed before. Only the coding pattern of input variable C is different from the other two variables A and B. For input variable C, the square cell is divided into four vertical sections. Input variable C takes 1, when the second and the fourth vertical sections are transparent and the other two remain opaque. When the first and the third are transparent and the other two remain opaque, C becomes 0. Here also the three structures of the coded variables can be overlapped to form a superimposed structure having eight subcells. Figure 4 shows the superimposed structure for $A = 1, B = 1$ and $C = 0$. Here, only the subcell marked 6 is transparent and the other five subcells remain opaque.

The above coding procedure is successfully used in constructing an all-optical full adder and an all-optical full subtractor.

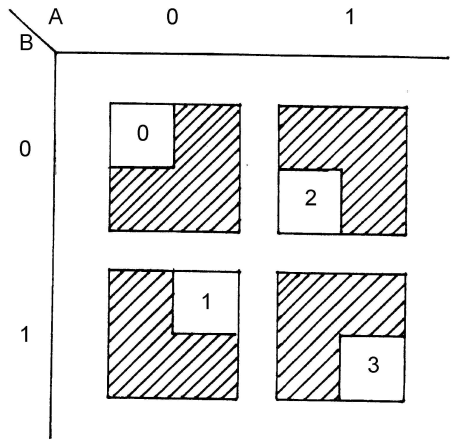

Figure 2. Superimposed pattern of two input variables (A and B) coded by spatial variation of light intensity.

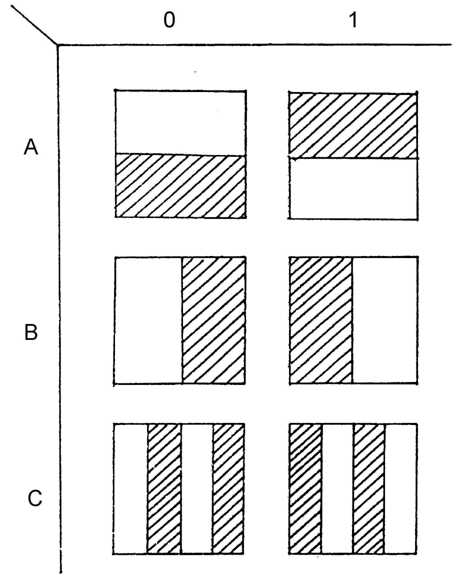

Figure 3. Coded pattern of three input variables (A, B and C) by spatial variation of light intensity.

4. All-optical half addition scheme

A half adder consists of two inputs A and B representing the bits to be added and two outputs which are designated as sum(So) output and carry(Co) output. Here, we describe the all-optical half addition scheme. In figure 5, we have shown a half addition scheme. Here a proper optical non-linear material or a photorefractive material is taken [12–16]. We keep the two coded inputs on one side and B on the other side of the optical non-linear material (NLM). The coding pattern of A and B are already shown in figure 1. Now laser beams are made incident on the NLM through the above inputs. Figure 5 gives a schematic diagram of a non-linear material-based switching system. An optical non-linear material is taken at the center and the two read laser beams are falling from opposite sides of the NLM. The probe beam is falling at the point where the input read beams are interacting on the NLM. Now output reflected light will be received through the same line of journey of the probe beam but in opposite direction.

To exploit such switching operations from a non-linear material we should first be careful about the coherency of the read and probe beams falling on the non-linear material. All the three beams should essentially be coherent laser beams that are originated from a single source. As this type of switching is an example of four-wave mixing, the proper phase-matching condition should be obeyed by the beams positively to enhance the switching operation as described in figure 5. Therefore to satisfy the above phase-matching condition the direction of all the input beams (read and probe beams) should be properly selected. The phase-matching condition

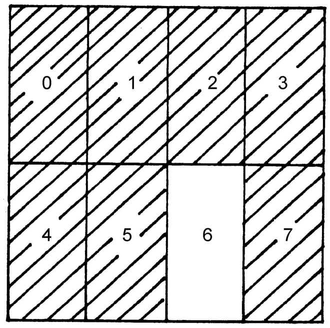

Figure 4. Superimposed pattern of input variables (A, B and C) coded by spatial variation of light intensity when $A = 1$, $B = 1$ and $C = 0$.

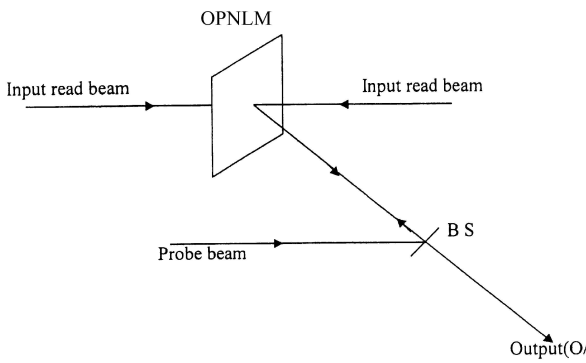

Figure 5. An optical non-linear material (OPNLM)-based switching system (BS indicates beam splitter).

of the switching operation is given as

$$\bar{K}_1 + \bar{K}_2 + \bar{K}_3 = \bar{K}_4,$$

where $\bar{K}_1$ and $\bar{K}_2$ are the wave vectors of the input read beams respectively (Here $\bar{K}_1 = -\bar{K}_2$, as the two read beams are in opposite directions.) $\bar{K}_3$ and $\bar{K}_4$ are the wave vectors of the probe beam and output (reflected) beam respectively. Here also $\bar{K}_3 = -\bar{K}_4$, as the probe beam and the output beam are in opposite directions. In this operation of four-wave mixing at any instant of time three input beams (two read beams and one probe beam) and one output beam are participating. In optical parallel computation, a lot of work has already been reported where this type of switching operation is used [12–16]. The physics of such switching mechanism is already established. In the absence of four-wave mixing (i.e. when any of the read beams or both the read beams are absent), we get no beam in the output channel. Here no four-wave mixing is happening at that time and there is no question of phase matching. Due to phase mismatch and due to the four-wave mixing not happening, we cannot expect light beam in the prescribed output channel of the system (figure 5). The switching action of such a non-linear material is used in many proposals of all-optical circuits. An example of such a non-linear material

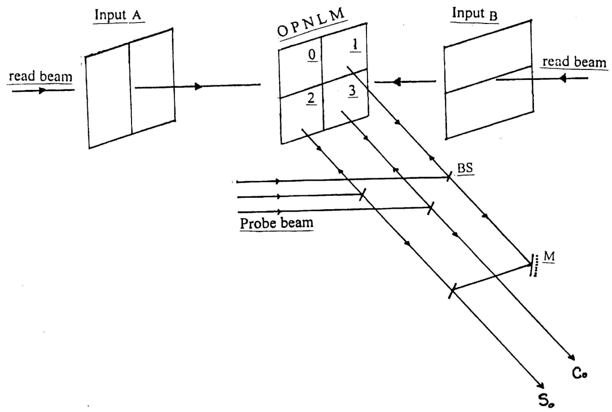

Figure 6. All-optical half addition scheme. M – mirror, BS – beam splitter, So and Co represent 'sum' output and 'carry' output.

is the photorefractive $BaTiO_3$ where such a mixing can be established. For such materials the switching mechanism can be exploited as follows. When both the read beams are present, reflected probe beam appears at the output. If any of the two or both the laser beam is absent, light will not be received at the output. This is shown in figure 5 [14]. The principle of switching follows the basic logic of four-wave mixing in non-linear material (NLM) where the directions of K_1, K_2, K_3 and K_4 are to be exactly maintained to support the phase-matching condition of such mixing. In $BaTiO_3$, the laser used for such a mixing is picosecond Nd : YAG laser.

For constructing all-optical half adder (shown in figure 6), we use the same principle of non-linear switching. Here, two read beams are coded following the coding principle described in figure 1. The beam reflected from the subcell marked 3 of the non-linear material (NLM) is called 'carry' or 'Co' output of the proposed half adder and that coming from the addition of the subcells marked 1 and 2 is called 'sum' or 'So' output of the same half adder. Here, we see that the three parallel probe beams are allowed to fall in three sections marked 1, 2 and 3 of the non-linear material (NLM) respectively. Then the reflected output beams from the subsections marked 1 and 2 are coupled together to get the single sum(So) output. If the output beam is present the 'sum' is considered as 1 and when it is absent, the sum is 0. The reflected beam coming from the subsection marked by 3 is regarded as 'carry'. When output beam is present in the carry channel, the 'carry' is considered as 1 and when it is absent, the carry is 0. Here, the input read beams are coded by spatial input encoding technique but the 'sum' and 'carry' outputs are defined by the presence and the absence of the reflected probe beam. For $A = 0$ and $B = 0$, according to the coding norm, only at the subcell marked 0 of the NLM, laser beams are incident on both sides of the NLM, i.e. only the subcell marked 0 in the NLM will be excited. So there will be no light signal at the sum(So) and carry(Co) channels, i.e. 'sum' = 0 and 'carry' = 0. For $A = 1$ and $B = 0$, only the subcell marked 2 of the NLM will be excited, i.e. this subcell will get two read laser beams from the two sides and the reflected probe beam will appear at the sum(So) output

Table 1. Truth table of a half adder.

A	B	sum(So) (EX-OR logic)	carry(Co) (AND logic)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

but not at the carry(Co) output. Here sum = 1 and carry = 0. In the same manner for $A = 0$ and $B = 1$, we get sum = 1 and carry = 0 as only the subcell marked 1 in the NLM is active to reflect the first probe beam. For $A = 1$ and $B = 1$, only the fourth subcell marked 3 in the NLM will be excited. So there will be no light at the sum(So) output but light will be present at the carry(Co) stage. Thus the result in this case is sum = 0 and carry = 1. In figure 6 the sum(So) operation is basically an EX-OR operation where the carry(Co) operation is basically an AND operation. In the above discussion we can see that the scheme discussed in figure 6 satisfies the all-optical AND operation in the 'carry' part and all-optical EX-OR operation in the 'sum' part. Here, the all-optical half addition scheme obeys the truth table of a binary half adder which is shown in table 1.

5. All-optical full addition scheme

A full adder consists of three inputs and two outputs; one output gives the 'sum' and other gives the 'carry'. The inputs A and B represent the bits to be added and C represents the 'carry'. Here we discuss the all-optical full addition scheme (figure 7). The main switching mechanism is same as the all-optical half addition scheme. Here we take a non-linear material at the center, and in one side of it, we keep the coded input A and at the other side we keep the other two coded inputs B and C. In the NLM we have eight subsections marked 0, 1, 2, 3, 4, 5, 6 and 7. Seven probe beams are allowed to be incident on the seven respective subsections of NLM except the subcell marked 0. The reflected output beams from subcells marked 1, 2, 4 and 7 are united to get a single 'sum' output and output beams coming from 3, 5, 6 and 7 are united to get a single 'carry' output. Here, the reflected output beam coming from the subcell marked 7 is coupled with both the output channels sum(So) and carry(Co). To illustrate the whole scheme we have some particular values for A, B and C. Let $A = 1, B = 1$ and $C = 0$, then only the subcell marked 6 will be excited as light beams will be incident on both sides of 6 only. So, the light signal will be present only in subcell marked 6 and hence 'carry' will be 1 (as light signal is present at the carry channel) and 'sum' will be 0 (as light signal is absent at the sum channel). For $A = 1, B = 1$ and $C = 1$ the subcell marked 7 will be excited and light signal will be present at both 'sum' output and 'carry' output, i.e. sum = 1 and carry = 1. Proceeding in the track, we can show that the above-discussed all-optical full adder follows the following truth table given in table 2 for developing an all-optical binary full adder system.

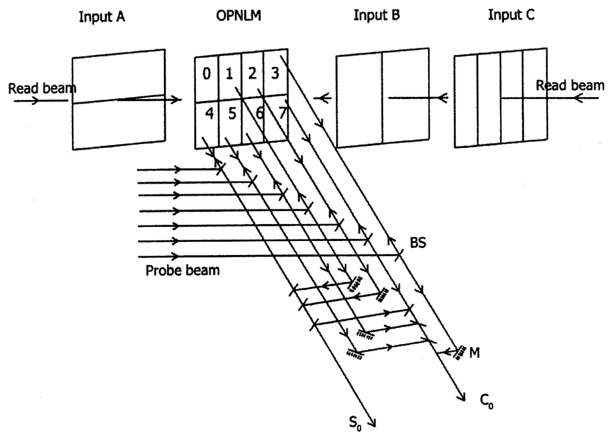

Figure 7. All-optical full addition scheme. M – mirror, BS – beam splitter, So and Co represent 'sum' output and 'carry' output.

Table 2. Truth table of a full adder.

A	B	C	sum(So)	carry(Co)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
1	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

6. All-optical full subtraction scheme

Like an all-optical adder an all-optical full subtractor will be required for data subtraction. This type of subtractor (figure 8) consists of two output channels. The two outputs are 'difference' (Do) and 'borrow' (Bo). Here, the input C is subtracting from A and the input B is the borrow. The construction of an all-optical subtractor is almost the same as the all-optical adder. Here probe beams are incident at subcells marked 1, 2, 3, 4 and 7. The reflected output beams from the subcells marked 1, 2, 4 and 7 are coupled to get 'difference' output and the reflected beam from the subcells 1, 2, 3 and 7 are coupled to get 'borrow' output. The reflected light beam coming from subcells 1, 2, 3 and 7 are coupled with both output channels (i.e. with 'difference' and 'borrow'). Suppose, $A = 1, B = 0$ and $C = 0$, then only the region marked 4 will be excited, i.e. this subcell gets light from both sides of the NLM. Then, we get light signal at the 'difference' channel and no light signal at the other output 'borrow', i.e. 'difference' = 1 and 'borrow' = 0. Similarly, we can show that the above all-optical full subtraction scheme depicted in figure 8 obeys the truth table for subtraction given in table 3.

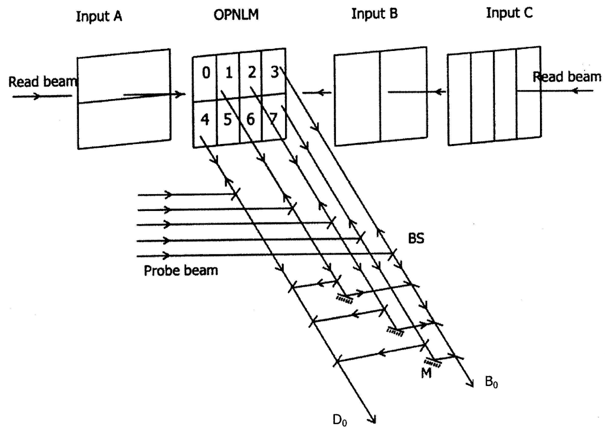

Figure 8. All-optical full subtraction scheme. M – mirror, BS – beam splitter, Do and Bo represent 'difference' output and 'borrow' output.

Table 3. Truth table of a full subtractor.

A	B	C	Difference	Borrow
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

7. Conclusion

The all-optical scheme for developing adder and subtractor exploits the switching character of a non-linear material. So a suitable non-linear material along with a suitable laser beam should be found to run the system properly in its operational mode. The full adder and the full subtractor are the essential requirement for addition and subtraction of three bits. So in the case of addition of two strings of data having number of bits as well as in case of subtraction of two strings of data having number of bits, the above systems can be used successfully to get the proper result of addition as well as subtraction. As the whole operation is all-optical, the real-time optical speed is expected from the above scheme. However for practical realization using the probe beams (read beams) from a suitable laser (which may be picosecond Nd : YAG laser) we can excite different pixels of the matrix generated on a non-linear material [17]. This material may be $BaTiO_3$ or some other material. So the systems described in figures 5–8 may give us a picosecond response time in connection to its speed of operation.

References

[1] P Ghosh, P P Das and S Mukhopadhyay, *Proc. Int. Conf. SPIE (ISPA-2001) Singapore* (October, 2001)
[2] K Huang and A Louri, *Opt. Eng.* **28**, 364 (1989)
[3] A K Datta, A Basuray and S Mukhopadhyay, *Opt. Lett.* **14**, 426 (1989)
[4] J Tandina and Y Tchikok, *J. Opt. Soc. Am.* **A73**, 800 (1983)
[5] T Yatagai, *Opt. Lett.* **11**, 260 (1986)
[6] J N Roy and S Mukhopadhyay, *Opt. Commun.* **119**, 499 (1995)
[7] S Mukhopadhyay, J N Roy and S K Bera, *Opt. Commun.* **99**, 31 (1993)
[8] S Mukhopadhyay, *Opt. Eng.* **31**, 1284 (1992)
[9] S Mukhopadhyay and J N Roy, *Proc. SPIE* **1812**, 280 (1992)
[10] J Tanida and Y Ichioka, *Appl. Opt.* **25**, 1565 (1986)
[11] S Mukhopadhyay, A Basuray and A K Datta, *Opt. Commun.* **64**, 18,189 (1988)
[12] A K Datta, S Mukhopadhyay and A Basuray, *Opt. Commun.* **88(23)**, 87 (1992)
[13] Y Fainman, C C Guest and S H Lee, *Appl. Opt.* **25(10)**, 1598 (1986)
[14] Mohammad A Karim and Abdul A S Awwal, *Optical computing: An introduction* (John Wiley & Sons Ltd., 1992) ch. 10
[15] A K Das and S Mukhopadhyay, *Opt. Eng.* **43(1)**, 126 (2004)
[16] H G Winful and J H Marburger, *Appl. Phys. Lett.* **36**, 631 (1980)
[17] Souransu Mukhopadhyay, *Optical computation and parallel processing*, Classique Books, ISBN 81-87616-01-6 (2000) ch. 5