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Contributed report: Flavor anarchy for Majorana neutrinos

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Abstract. We argue that neutrino flavor parameters may exhibit features that are very different from those of quarks and charged leptons. Specifically, within the Froggatt–Nielsen (FN) framework, charged fermion parameters depend on the ratio between two scales, while for neutrinos a third scale – that of lepton number breaking – is involved. Consequently, the selection rules for neutrinos may be different. In particular, if the scale of lepton number breaking is similar to the scale of horizontal symmetry breaking, neutrinos may become flavor-blind even if they carry different horizontal charges. This provides an attractive mechanism for neutrino flavor anarchy.

Keywords. Neutrino masses; flavor symmetries; Majorana.

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1. Introduction

With three active neutrinos that have Majorana-type masses, there are nine new flavor parameters related to the neutrino sector: three neutrino masses, three lepton mixing angles, and three phases in the mixing matrix. One may hope that measuring these parameters will shed light on the flavor puzzle, that is the question of why the charged fermion flavor parameters exhibit hierarchy and smallness.

Various experiments have provided relevant information on four parameters, which can be summarized as follows (see e.g. [1]):

$$\begin{aligned}&|U_{\mu 3}U_{\tau 3}| \sim 0.47\text{–}0.50,\\ &|U_{e1}U_{e2}| \sim 0.42\text{–}0.49,\\ &|U_{e3}| \leq 0.23,\\ &\Delta m^2_{21}/|\Delta m^2_{32}| \sim 0.02\text{–}0.04.\end{aligned} \tag{1}$$

Here, ν_1, ν_2, ν_3 are the three neutrino mass eigenstates, with the convention that ν_3 is the one with relatively large mass splitting, $|\Delta m^2_{3i}| \gg \Delta m^2_{21}$, and ν_2 is heavier than ν_1, $\Delta m^2_{21} > 0$. Note that ν_3 could be heavier (normal hierarchy) or lighter (inverted hierarchy) than ν_1 and ν_2.

The measured neutrino flavor parameters are neither manifestly small (apart from the overall mass scale) nor manifestly hierarchical. The two measured mixing angles are $\mathcal{O}(1)$ and the measured mass ratio is $\mathcal{O}(0.2)$ or larger. With the upper bound on the third mixing angle of $\mathcal{O}(0.2)$, and with no information on the remaining mass ratio and CP-violating phases, it could well be that *all* neutrino flavor parameters are non-hierarchical, that is, anarchical [2] (see however [3]).

The features of the neutrino flavor parameters should be compared to what we know about the charged fermion parameters. We formulate the latter in terms of the order of magnitude of the nine Yukawa couplings ($Y_f \sim m_f/174$ GeV), three CKM mixing angles and the KM phase:

$$\begin{aligned}
&Y_t \sim 1, \quad Y_c \sim 10^{-2}, \quad Y_u \sim 10^{-5},\\
&Y_b \sim 10^{-2}, \quad Y_s \sim 10^{-3}, \quad Y_d \sim 10^{-4},\\
&Y_\tau \sim 10^{-2}, \quad Y_\mu \sim 10^{-3}, \quad Y_e \sim 10^{-6},\\
&|V_{us}| \sim 0.2, \quad |V_{cb}| \sim 0.04, \quad |V_{ub}| \sim 0.004, \quad \delta_{\mathrm{KM}} \sim 1.
\end{aligned}$$

There is a sharp contrast between the neutrino and the charged fermion flavor parameters. Of the latter, only two parameters – the top Yukawa and the KM phase – are $\mathcal{O}(1)$, while all other eleven parameters – eight masses and three mixing angles – are small and hierarchical.

It is of course possible that yet-unmeasured neutrino parameters (θ_{13} and/or m_1/m_2) are small, and there is hierarchy in all sectors. It is also possible that the neutrino masses are quasi-degenerate ($\Delta m^2_{ij} \ll m^2_3$) which also implies that there is a special structure in the neutrino sector. We assume here that this is not the case. Then, it is interesting to understand the reason for the difference between the flavor structure of neutral and charged fermions. This difference could be accidental. For example, one could imagine that the flavor structure is a result of an approximate symmetry, and it just so happens that all lepton doublets carry the same charge under this symmetry (see, for example, [4]). In other words, each of the sectors – up, down, charged lepton and neutrino – could equally well be hierarchical or accidentally anarchical. However, a far more intriguing possibility is that the difference is due to the fact that, of all the standard model fermions, only neutrinos are Majorana fermions. Then the measured parameters reflect the interplay between flavor physics and lepton number violation. It is this interplay that we wish to explore [5].

In order to relate the flavor structure and the Majorana/Dirac nature of fermions, one must work within a framework that explains the flavor hierarchy of quarks and charged leptons. One of the most attractive such frameworks is the Froggatt–Nielsen (FN) mechanism [6]. One assumes an Abelian horizontal symmetry that is broken by a small parameter near some high 'flavor scale', M_{F}. This implies various selection rules for the flavor parameters of the standard model. We assume that the smallness of the overall scale of neutrino masses, is *not* a result of the FN selection rules but rather of the see-saw mechanism [7–9]. Neutrino masses are thus universally small because the mass of singlet Majorana neutrinos or, equivalently, the scale of lepton number violation, M_{L}, is very high. We will show that the existence of the scale M_{L}, on top of the FN scale M_{F}, has a crucial impact on neutrino flavor parameters.

2. The supersymmetric Froggatt–Nielsen framework

We consider supersymmetric Froggatt–Nielsen models [10,10a]. We assume the following symmetries:

$$G_{\rm SM} \times U(1)_{\rm H} \times U(1)_{\rm L}. \tag{2}$$

Here $G_{\rm SM}$ is the SM gauge group, spontaneously broken by two Higgs doublets, $\phi_u(1,2)_{+1/2}$ and $\phi_d(1,2)_{-1/2}$. Supersymmetry is softly broken, but since its breaking is irrelevant to our investigation, we do not specify the breaking mechanism here. The $U(1)_{\rm H}$ factor is the horizontal symmetry, which we take to be $U(1)$ for simplicity. To avoid the issue of global symmetry breaking by strong gravity effects, as well as Goldstone bosons, we could choose the horizontal symmetry to be a (gauged) discrete symmetry. We assume that it is broken by the VEV of a *single* scalar field $S_{\rm H}$ (more accurately, $S_{\rm H}$ is the scalar component in a chiral supermultiplet) that is a singlet of $G_{\rm SM} \times U(1)_{\rm L}$ and carries charge -1 under $U(1)_{\rm H}$. This choice just sets the overall normalization of H-charges. The $U(1)_{\rm L}$ symmetry is lepton number. We assume that it is broken by the VEVs of two scalar fields $S_{\rm L}$ and $\bar{S}_{\rm L}$ that are singlets of $G_{\rm SM} \times U(1)_{\rm H}$ and carry charges $+2$ and -2, respectively, under $U(1)_{\rm L}$. The two VEVs are equal in magnitude [10b].

The symmetries (2) forbid neutrino masses, and, for appropriate choices of the quark and lepton horizontal charges, most of the charged fermion masses. These masses and couplings are generated however when integrating out new heavy fields. These heavy 'FN fields' have charges similar to those of the SM quarks and leptons (that is, $\pm 2/3$, $\mp 1/3$, ∓ 1 and 0), but appear in vector representations of $G_{\rm SM} \times U(1)_{\rm H}$. If the FN fields are vector-like also under $U(1)_{\rm L}$ – as is always the case for the charged fields – they have masses at a high scale $M_{\rm F}$ (possibly the Planck scale). Heavy singlet neutrinos may, however, be chiral under $U(1)_{\rm L}$. In that case, they acquire masses at the scale of lepton number breaking, $M_{\rm L} \lesssim M_{\rm F}$. Thus there are four relevant mass scales in our framework:

1. $\langle \phi_{u,d} \rangle$, the electroweak breaking scale;
2. $M_{\rm L} \equiv \langle S_{\rm L} \rangle = \langle \bar{S}_{\rm L} \rangle$, the lepton number breaking scale;
3. $M_{\rm H} \equiv \langle S_{\rm H} \rangle$, the horizontal symmetry breaking scale;
4. $M_{\rm F}$, the mass scale of Froggatt–Nielsen vector-like quarks and leptons.

We assume the following hierarchies:

$$\langle \phi_{u,d} \rangle \ll M_{\rm L},\ M_{\rm H},\ M_{\rm F}, \quad M_{\rm L} \lesssim M_{\rm F}, \tag{3}$$

$$\lambda_{\rm H} \equiv \frac{M_{\rm H}}{M_{\rm F}} \ll 1. \tag{4}$$

For concreteness, we often use $\lambda_{\rm H} \sim 0.2$, inspired by the value of the Cabibbo angle which one may attempt to explain as being suppressed by a single power of the ratio $\langle S_{\rm H} \rangle / M_{\rm F}$. The precise numerical value is, however, irrelevant for our conclusions.

Note that we do not specify the relative sizes of the lepton-number breaking scale, $M_{\rm L}$, and the horizontal symmetry breaking scale $M_{\rm H}$. In the following, we will explore the impact of different hierarchies between these scales on neutrino parameters.

3. Charged fermion parameters

To understand the resulting quark flavor structure, it is sufficient to consider a low energy effective theory that includes only the MSSM fields. The theory has a $U(1)_{\rm H}$ symmetry which is *explicitly* broken by the spurion $\lambda_{\rm H}\sim 0.2$ of $U(1)_{\rm H}$-charge -1. This leads to the following selection rules:

1. Superpotential terms of integer H-charge $n\geq 0$ are suppressed by $\lambda_{\rm H}^n$.
2. Superpotential terms of negative or non-integer H-charge vanish.

These selection rules are sufficient in order to find the parametric suppression (that is, the $\lambda_{\rm H}$ dependence) of the flavor parameters. In particular, if holomorphic zeros play no role, the mixing angles and mass ratios are (with $i<j;\ q=u,d$):

$$V_{ij}\sim\lambda_{\rm H}^{H(Q_i)-H(Q_j)},\quad m_i/m_j\sim\lambda_{\rm H}^{H(Q_i)-H(Q_j)+H(\bar q_i)-H(\bar q_j)}\ . \tag{5}$$

For example, the quark parameters quoted above are often accounted for by the following set of H-charges:

$$\begin{aligned}&\phi_u(0),\ \phi_d(0),\quad Q_1(3),\ Q_2(2),\ Q_3(0),\\ &\bar u_1(5),\ \bar u_2(2),\ \bar u_3(0),\quad \bar d_1(3),\ \bar d_2(2),\ \bar d_3(2),\end{aligned} \tag{6}$$

which imply

$$\begin{aligned}&V_{us}\sim\lambda_{\rm H},\quad V_{cb}\sim\lambda_{\rm H}^2,\quad V_{ub}\sim\lambda_{\rm H}^3,\\ &m_u/m_c\sim\lambda_{\rm H}^4,\quad m_c/m_t\sim\lambda_{\rm H}^4,\quad m_t/\langle\phi_u\rangle\sim 1,\\ &m_d/m_s\sim\lambda_{\rm H}^2,\quad m_s/m_b\sim\lambda_{\rm H}^2,\quad m_b/\langle\phi_d\rangle\sim\lambda_{\rm H}^2,\end{aligned} \tag{7}$$

consistent (for $\tan\beta\equiv\langle\phi_u\rangle/\langle\phi_d\rangle\sim 1$) with the experimental values.

Let us see how this low energy effective theory arises in a full high energy FN model. As an example, we focus on the (c,t) sector. We add the following FN fields:

$$\bar U_{-2}+U_{+2},\quad \bar U_{-1}+U_{+1},\quad \bar U_0+U_0,\quad \bar U_{+1}+U_{-1}\ . \tag{8}$$

Here $U_h(\bar U_h)$ is an $SU(2)$-singlet quark (antiquark) of horizontal charge h. The mass matrix for rows corresponding to $(Q_2,Q_3,U_{+2},U_{+1},U_0,U_{-1})$ and columns to $(\bar u_2,\bar u_3,\bar U_{-2},\bar U_{-1},\bar U_0,\bar U_{+1})$ is given by (up to $\mathcal{O}(1)$-coefficients)

$$\begin{pmatrix}0&0&\phi_u&0&0&0\\0&\phi_u&0&0&0&0\\0&0&M_{\rm F}&S_{\rm H}&0&0\\0&S&0&M_{\rm F}&S_{\rm H}&0\\0&0&0&0&M_{\rm F}&S_{\rm H}\\S_{\rm H}&0&0&0&0&M_{\rm F}\end{pmatrix}. \tag{9}$$

When the four heavy FN fields with masses of $\mathcal{O}(M_{\rm F})$ are integrated out, we obtain

$$M_u^{(c,t)}\sim\langle\phi_u\rangle\begin{pmatrix}\lambda_{\rm H}^4&\lambda_{\rm H}^2\\ \lambda_{\rm H}^2&1\end{pmatrix}, \tag{10}$$

consistent with $m_c/m_t\sim\lambda_{\rm H}^4$ and $|V_{cb}|\sim\lambda_{\rm H}^2$.

4. Neutrino parameters

We assume that neutrino masses arise from the see-saw mechanism, that is superpotential terms of the form

$$\frac{Z_{ij}}{M_{\rm L}}\phi_u\phi_u L_i L_j. \tag{11}$$

Z is a 3×3 matrix of dimensionless Yukawa couplings. We aim to find the selection rules that apply to it and see if they are fundamentally different from those of charged fermions.

Indeed, even the most naive selection rules [13] have two special features:

1. The matrix is symmetric, $Z_{ij} = Z_{ji}$. Thus, in contrast to the charged fermion case, pairs of entries are related, and we can get a (quasi-)degeneracy.
2. Terms in (11) that carry a negative H charge, $n < 0$, might be *enhanced* by $\lambda_{\rm H}^n$ rather than vanish.

4.1 *Naive selection rules*

With selection rules similar to those of charged fermions, with the above two exceptions, the analog of (5) for the neutrinos would be [13]

$$V_{ij}\sim\lambda_{\rm H}^{H(L_i)-H(L_j)},\quad m_i/m_j\sim\lambda_{\rm H}^{2[H(L_i)-H(L_j)]}\ . \tag{12}$$

Then the neutrino data may pose a problem, but much depends on how the data is interpreted. To explain this statement, let us rewrite eq. (1) as follows:

$$\sin\theta_{23}\sim 1,\quad \sin\theta_{12}\sim 1,\quad \sin\theta_{13}<0.2,\quad m_2/m_3\gtrsim 0.15. \tag{13}$$

The simplest interpretation would be

$$\sin\theta_{23}\sim 1,\quad \sin\theta_{12}\sim 1,\quad \sin\theta_{13}\ll 1,\quad m_2/m_3\ll 1, \tag{14}$$

where '$\ll 1$' means that we believe that it should be suppressed by powers of $\lambda_{\rm H}$. Comparing (14) to (12), we learn that the simplest models of Abelian horizontal symmetry are excluded. The data could, however, be interpreted differently. First, one could have

$$\sin\theta_{23}\sim 1,\quad \sin\theta_{12}\ll 1,\quad \sin\theta_{13}\ll 1,\quad m_2/m_3\sim 1. \tag{15}$$

This set of parameters can be explained by the following H-charges:

$$L_1(1),\quad L_2(0),\quad L_3(0). \tag{16}$$

Order-one coefficients in the neutrino mass matrix should accidentally enhance $\sin\theta_{12}$ and suppress m_2/m_3 by a factor of a few. These enhancement and suppression factors are quite mild. Moreover, there is really only one 'accident' that is required. Once the lighter eigenvalue of the 2–3 block in the light neutrino mass matrix is accidentally suppressed by $\mathcal{O}(\lambda_{\rm H})$, both the suppression of m_2/m_3 and the enhancement of $\sin\theta_{12}$ are provided. Another possible interpretation of the data, mentioned already in the introduction, is the anarchical one:

$$\sin\theta_{23} \sim 1, \quad \sin\theta_{12} \sim 1, \quad \sin\theta_{13} \sim 1, \quad m_2/m_3 \sim 1. \tag{17}$$

Here, the order-one coefficients should accidentally suppress m_2/m_3 and $\sin\theta_{13}$ by a factor of a few. This interpretation allows a set of H-charges that is even consistent with GUTs. Denoting the 10 and $\bar{5}$ representations of $SU(5)$ by, respectively, T_i and F_i, and taking the small FN parameter to be $\lambda_{\rm H} \sim 0.05$, we get a reasonable fit to the flavor parameters with the following choice of H charges [14]:

$$T_1(2), \quad T_2(1), \quad T_3(0), \quad F_1(0), \quad F_2(0), \quad F_3(0). \tag{18}$$

4.2 *Lepton number breaking and the selection rules*

The lepton number breaking parameters have, however, an even more profound effect on the selection rules. Specifically, they introduce an additional parameter, on top of $\lambda_{\rm H}$ of eq. (4), that breaks the horizontal symmetry and conserves lepton number:

$$\lambda_{\rm L}^2 \equiv \frac{\langle S_{\rm H}\rangle^2}{\langle S_{\rm L}\rangle\langle \bar{S}_{\rm L}\rangle} = \frac{M_{\rm H}^2}{M_{\rm L}^2}. \tag{19}$$

The crucial point is that $\lambda_{\rm H}^2/\lambda_{\rm L}^2$ is neutral under all the symmetries (2) and therefore can affect the physical observables in a way that depends sensitively on the details of the full high-energy theory. Furthermore, the numerical value of $\lambda_{\rm L}$ depends on the hierarchy of scales $M_{\rm H}$ and $M_{\rm L}$. We can have $\lambda_{\rm L} \sim \lambda_{\rm H}$ or $\lambda_{\rm L} > \lambda_{\rm H}$ and even $\lambda_{\rm L} \gtrsim 1$.

Only in the special case that $\lambda_{\rm L} \sim \lambda_{\rm H}$, that is, $M_{\rm L} \sim M_{\rm F}$, we expect that neutrinos will have a flavor hierarchy that is related to the one in the charged fermion sectors. Generically, however, the structure of the neutrino flavor parameters depends, in addition to $\lambda_{\rm H}$, on $\lambda_{\rm L}$, and can be very different from that of quarks and charged lepton masses. In the next section we give several examples that demonstrate these statements.

5. Explicit examples

We consider a simplified framework of two light active neutrinos. As an explicit example, we take the two lepton doublets to be L_{+2} and L_0, where the sub-index denotes the H-charge. We present three different full high-energy models. The various models exhibit several interesting features that may arise in the neutrino sector and demonstrate the sensitivity of low-energy observables to the full high-energy theory.

Each of the models is defined by a set of $G_{\rm SM}$-singlet fields. To obtain the light-neutrino mass matrix, we start from the full renormalizable superpotential allowed by the symmetries. As above, we omit dimensionless $\mathcal{O}(1)$ coefficients and, for the light-neutrino mass matrices, contributions that are subleading in $\lambda_{\rm H}/\lambda_{\rm L}$. Leptons (antileptons) of H-charge h are denoted by N_h ($\bar{N}_h$).

Model I has the following (anti)lepton fields:

$$L_{+2},\ L_0,\ \bar{N}_{-2},\ N_{+2},\ \bar{N}_{-1},\ N_{+1},\ \bar{N}_0,\ \bar{N}_0. \tag{20}$$

The mass matrix in this basis is:

$$\begin{pmatrix} 0 & 0 & \phi_u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_u & \phi_u \\ \phi_u & 0 & 0 & M_{\rm F} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{\rm F} & 0 & S_{\rm H} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{\rm H} & 0 & M_{\rm F} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\rm F} & 0 & S_{\rm H} & S_{\rm H} \\ 0 & \phi_u & 0 & 0 & 0 & S_{\rm H} & \bar{S}_{\rm L} & 0 \\ 0 & \phi_u & 0 & 0 & 0 & S_{\rm H} & 0 & \bar{S}_{\rm L} \end{pmatrix}. \tag{21}$$

The light neutrino mass matrix is given by

$$M_{\rm l} \sim \frac{\langle\phi_u\rangle^2}{M_{\rm L}} \begin{pmatrix} \lambda_{\rm H}^4 & \lambda_{\rm H}^2 \\ \lambda_{\rm H}^2 & 1 \end{pmatrix}. \tag{22}$$

It leads to the 'naive' flavor structure, namely the flavor structure that would follow if the selection rules were similar to those of charged fermions [13,15]:

$$m_1/m_2 \sim \lambda_{\rm H}^4, \quad \sin\theta \sim \lambda_{\rm H}^2. \tag{23}$$

Model II has the following (anti)lepton fields:

$$L_{+2},\ L_0,\ \bar{N}_{-2},\ N_{+2},\ \bar{N}_{-1},\ \bar{N}_{+1},\ N_0,\ \bar{N}_0. \tag{24}$$

The mass matrix in this basis is:

$$\begin{pmatrix} 0 & 0 & \phi_u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_u \\ \phi_u & 0 & 0 & M_{\rm F} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{\rm F} & 0 & S_{\rm H} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{\rm H} & 0 & \bar{S}_{\rm L} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{S}_{\rm L} & 0 & S_{\rm H} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{\rm H} & S_{\rm L} & M_{\rm F} \\ 0 & \phi_u & 0 & 0 & 0 & 0 & M_{\rm F} & \bar{S}_{\rm L} \end{pmatrix}. \tag{25}$$

The light neutrino mass matrix is given by

$$M_{\rm l} \sim \frac{\langle\phi_u\rangle^2}{M_{\rm L}} \frac{\lambda_{\rm H}^2}{\lambda_{\rm L}^2} \begin{pmatrix} \lambda_{\rm H}^2\lambda_{\rm L}^2 & \lambda_{\rm L}^2 \\ \lambda_{\rm L}^2 & 1 \end{pmatrix}. \tag{26}$$

This mass matrix has the following interesting features:

1. For $\lambda_{\rm L}^2 \sim \lambda_{\rm H}^2$, the mixing and hierarchy assume their naive values, as in (23).

2. For $\lambda_{\rm H}^2 < \lambda_{\rm L}^2 < 1$, the mixing is larger, $\sin\theta \sim \lambda_{\rm L}^2$, and the hierarchy is weaker, $m_1/m_2 \sim \lambda_{\rm L}^4$, than the naive estimates.
3. For $\lambda_{\rm L}^2 > 1$, we have a pseudo-Dirac state.

Since the two spurions, $\lambda_{\rm H}$ and $\lambda_{\rm L}$, appear in the light neutrino mass matrix, the naive selection rules do not necessarily apply, and a flavor structure unique to neutrinos, such as a pseudo-Dirac state, may arise.

Model III has the following (anti)lepton fields:

$$L_{+2},\ L_0,\ \bar{N}_{-2},\ \bar{N}_{+2},\ N_{-1},\ N_{+1},\ \bar{N}_0,\ \bar{N}_0. \tag{27}$$

The mass matrix in this basis is:

$$\begin{pmatrix} 0 & 0 & \phi_u & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_u & \phi_u \\ \phi_u & 0 & 0 & \bar{S}_{\rm L} & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{S}_{\rm L} & 0 & S_{\rm H} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{\rm H} & 0 & S_{\rm L} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{\rm L} & 0 & S_{\rm H} & S_{\rm H} \\ 0 & \phi_u & 0 & 0 & 0 & S_{\rm H} & \bar{S}_{\rm L} & 0 \\ 0 & \phi_u & 0 & 0 & 0 & S_{\rm H} & 0 & \bar{S}_{\rm L} \end{pmatrix}. \tag{28}$$

The light neutrino mass matrix is given by

$$M_{\rm l} \sim \frac{\phi_u^2}{M_{\rm L}} \begin{pmatrix} \lambda_{\rm L}^4 & \lambda_{\rm L}^2 \\ \lambda_{\rm L}^2 & 1 \end{pmatrix}. \tag{29}$$

The following two ranges for $\lambda_{\rm L}$ are particularly interesting:

1. For $\lambda_{\rm L} > 1$, we obtain inverted hierarchy: the state with the highest FN-charge is the heaviest, in contrast to charged fermions.
2. For $\lambda_{\rm L} \sim 1$, there is no hierarchy in the masses and mixing angle, i.e. we have neutrino flavor anarchy.

We learn that if $U(1)_{\rm H}$ and $U(1)_{\rm L}$ are broken at the same scale, it is quite possible that neutrinos will have no special flavor structure, even if they come from lepton doublets that carry different H-charges.

6. Conclusions

The special flavor features (smallness and hierarchy) of quark and charged lepton masses and CKM mixing angles can be explained by a spontaneously broken horizontal symmetry, if the breaking scale is lower than the scale where the breaking is communicated to the light quarks and leptons.

If the light active neutrinos are Majorana fermions and derive their masses through a see-saw mechanism, an additional scale plays a role in their flavor structure, that is the scale of lepton number breaking or, equivalently, the Majorana mass scale of the heavy singlet neutrinos. This fact may have significant effects on the neutrino sector. Its flavor parameters may have a hierarchy that is very different from the charged fermions. Intriguing features, such as inverted hierarchy or a pseudo-Dirac state, can appear in the neutrino sector.

In particular, the neutrino flavor parameters may have no special structure at all. While there is no inherent motivation for neutrino anarchy in the framework that we investigated, it does arise naturally if the horizontal symmetry and the lepton number symmetry are broken at the same scale.

Thus, if future measurements of neutrino parameters strengthen the case for flavor anarchy ($|U_{e3}|$ close to the present upper bound and no quasi-degeneracy among the masses), models that relate the two scales will be favored.

The ideas presented in this work can be extended in a straightforward way to realistic, three-generation models. It would also be interesting to explore whether, in other mechanisms that explain the hierarchy in the charged fermion parameters, the Majorana nature of neutrinos introduces significant modifications that are particular to this sector.

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[10a] Supersymmetry affects our study in three ways: (i) Superpotential flavor parameters are governed by holomorphy in addition to the horizontal symmetry; (ii) the Higgs sector consists of two doublets; (iii) both the see-saw and FN mechanisms introduce new heavy fields with Yukawa couplings to the Higgs field. In the absence of supersymmetry, severe fine-tuning problems would arise

[10b] A single $\langle S_{\rm H}\rangle$ would arise naturally for a pseudo-anomalous $U(1)_{\rm H}$ [11]. Equal VEVs $\langle S_{\rm L}\rangle = \langle\bar{S}_{\rm L}\rangle$ would be necessary to preserve supersymmetry with a gauged, non-anomalous $U(1)_{\rm L}$. Similarly, two spurions of equal VEVs and opposite $U(1)_{\rm H}$ charges would be necessary if $U(1)_{\rm H}$ is a gauged, non-anomalous symmetry [12]. Neither ingredient is important for our purposes

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