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Higher-dimensional cosmological model with variable gravitational constant and bulk viscosity in Lyra geometry

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Abstract. We have studied five-dimensional homogeneous cosmological models with variable G and bulk viscosity in Lyra geometry. Exact solutions for the field equations have been obtained and physical properties of the models are discussed. It has been observed that the results of new models are well within the observational limit.

Keywords. Bulk viscosity; gravitational constant; Lyra geometry.

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1. Introduction

The possibility that space-time has more than four dimensions has attracted many researchers to the field of higher dimension [1]. Study of higher-dimensional space-time is also important because of the underlying idea that the cosmos at its early stage of evolution might have had a higher-dimensional era. The extra space reduced to a volume with the passage of time which is beyond the ability of experimental observation at the moment. Attempts have been made to explain why the universe presently appears to have only four space-time dimensions, if it is dynamically evolving $(4 + k)$-dimensional manifolds (k being the number of extra dimensions). It has been claimed that the solutions to Einstein's equation for $(4+k)$ dimension indicate that there is an expansion of four-dimensional space-time while fifth dimension contracts or remain constant [2]. Further, it has been reported that during contraction process, extra dimensions produce large amount of entropy which provides an alternative resolution to the flatness and horizon problem, as compared to usual inflationary scenario [3,4]. Marciano [5] has suggested that the experimental observation of fundamental constant with varying time could produce the evidence of extra dimensions.

One of the outstanding problem of standard cosmology is that of large entropy per baryon ratio. It has been widely discussed in the literature that during the evolution of the universe, bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation [6]. The possibility of bulk viscosity leading to inflationary-like solutions in general relativistic FRW models is discussed by Padmanabhan and Chitre [7]. Johri and Sudharsan [8] have pointed out that the bulk viscosity leads to inflationary solution in Brans–Dicke theory (BDT). The possibility of bulk viscosity-driven inflationary solutions of full Israel–Stewart theory in different cases are discussed by Zimdahl [9]. Many more efforts [10] have been made to obtain cosmological solutions for a fluid with bulk viscosity in BDT, because of the inflationary solutions due to the presence of bulk viscosity. Recently, Singh *et al* [11] have studied all the FRW (flat, open and closed) cosmological models with causal viscous fluid in BDT. In the context of open thermodynamic systems, Bianchi-type cosmological models with bulk viscosity and particle production have been studied in ref. [12].

Einstein's idea of geometrizing gravitational field in the form of general theory of relativity (GTR) motivated physicists to geometrize other physical fields. Weyl [13] suggested a non-Riemannian geometrical theory of gravitation and electromagnetism. But, because of the non-integrability of the length transfer of a vector under parallel transport, this theory was never studied seriously. Lyra [14] removes this non-integrability condition of the length of a vector under parallel transport by introducing a gauge function into the modified Riemannian geometry. Sen [15] proposed a new scalar–tensor theory of gravitation based on Lyra geometry. Subsequently, cosmological theory within the framework of Lyra geometry was studied by Halford [16], who had pointed out that the vector field ϕ_i in Lyra geometry plays similar role of cosmological term Λ in the GTR with a cosmological constant. The scalar–tensor treatment based on Lyra geometry [17] predicts the effects which are within the observational limits, as in Einstein's theory. Several authors [18] have studied cosmological models based on Lyra geometry with a constant displacement field without any prior reason. Soleng [19] has identified that the cosmology based on Lyra geometry with a constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology [20] or contain a special vacuum field which together with gauge vector may be considered as cosmological term. Singh and Singh [21] have discussed Bianchi Type I, III, Kantowshi Sachs and a new class of models with a time-dependent displacement field. They have made a comparative study of Robertson–Walker models with a constant deceleration parameter in Einstein's theory with a cosmological term and in the cosmological theory based on Lyra geometry (for a review of Lyra geometry and cosmology, please refer [22]). Isotropic and homogeneous FRW models of the universe have been studied in the presence of a bulk viscous fluid within the framework of Lyra geometry [23].

Motivated by the above investigations, we have considered the present study of higher-dimensional cosmological models with gravitational constant and bulk viscous fluid in Lyra geometry.

2. Field equations

We consider spatially flat five-dimensional space-time geometry of the universe with the line element

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(t)(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2) - B^2(t)\mathrm{d}y^2, \tag{1}$$

where $u^i = \delta_0^i, u^i = 0$, for $i = 1, 2, 3, 4$.

The Einstein's field equations based on Lyra geometry in normal gauge may be written as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G T_{ij}, \tag{2}$$

where ϕ_i is the displacement vector field and other symbols have their usual meaning as in Riemannian geometry. The displacement vector ϕ_i is defined as $\phi_i = (\beta, 0, 0, 0, 0)$.

The energy–momentum tensor has the form

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij}. \tag{3}$$

Here ρ, p, and u_i are respectively, the energy density, equilibrium pressure, and four velocities of the cosmic fluid distribution.

The field equations (2) together with (3) for the space-time metric (1) lead to the following set of equations:

$$3\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{B}}{AB}\right) = 8\pi G\rho + \frac{3}{4}\beta^2, \tag{4}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -8\pi G p - \frac{3}{4}\beta^2, \tag{5}$$

$$3\left(\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2}\right) = -8\pi G p - \frac{3}{4}\beta^2, \tag{6}$$

where overhead dot denotes differentiations with respect to t.

Equations (4)–(6) bear a close resemblance to equations of relativistic cosmology based on Riemannian geometry with $\frac{3}{4}\beta^2$ playing the role of the cosmological constant Λ.

3. Cosmological solutions

We have only three basic equations (4)–(6) and six unknowns, viz. A, B, G, ρ, p and β. In order to obtain exact solutions, we require three more physically reasonable relations (conditions) amongst the variables. In the following sections we consider, in turn, uniform energy density and bulk viscosity energy density law for further investigations. Subtracting eq. (6) from (5), we get

$$\frac{\ddot{A}}{A}+2\frac{\dot{A}^2}{A^2}-2\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{B}}{B}=0. \tag{7}$$

Equation (7) on integration yields the solution

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)A^3B=K_1, \tag{8}$$

where K_1 is a constant of integration.

Now, assuming a power law relation $A\sim B^n$ between metric coefficients A and B, eq. (8) suggests

$$A=A_0\left(\frac{t}{t_0}\right)^{n/(3n+1)}, \tag{9}$$

$$B=B_0\left(\frac{t}{t_0}\right)^{1/(3n+1)}, \tag{10}$$

where A_0 and B_0 are values of A and B at $t=t_0$.

Further, eqs (4)–(6) along with (9) and (10), give

$$8\pi G\rho+\frac{3}{4}\beta^2=8\pi Gp+\frac{3}{4}\beta^2=\frac{3n(n+1)}{(3n+1)^2}\,\frac{1}{t^2}. \tag{11}$$

It can be seen that eq. (11) yields Zeldovich condition $p=\rho$ for the superdense model of the universe.

Let us now turn our attention to variability of displacement vector field β^2 and gravitational constant G. A number of authors have studied cosmological models with ansatz $\Lambda\sim H^2$ (please refer [24] and references therein). It has been pointed out in the literature that $\beta^2=-(4/3)\Lambda$ gives complete equivalence between the cosmological models in Lyra geometry and general relativity [16,25]. Considering the above results, we assume

$$\beta^2=\frac{\beta_0^2}{H_0^2}H^2, \tag{12}$$

where H is defined as $H=(\dot{A}/A)+(1/3)(\dot{B}/B)$ and β_0, H_0 are representing present values of β and H respectively. Now by using eqs (9) and (10), eq. (12) reduces to

$$\beta^2=\beta_0^2\left(\frac{t_0}{t}\right)^2. \tag{13}$$

Further, we now assume a power law form for G as in the literature [26], of the type

$$G(t)=G_0\left(\frac{t}{t_0}\right)^m \tag{14}$$

with G_0 being the present value of G.

Again, from eqs (11)–(14), we obtain

$$\rho = \rho_0 \frac{1}{t^{2+m}}, \tag{15}$$

where

$$\rho_0 = \frac{t_0^m}{8\pi G_0}\left[\frac{3n(n+1)}{(3n+1)^2} - \frac{3}{4}\beta_0^2 t_0^2\right].$$

Equation (15) suggests that the condition on the energy density $\rho \geq 0$ requires $\beta_0^2 < 4n(n+1)/(3n+1)^2 t_0^2$. It can be easily checked from eqs (9) and (10), that in the first case where $n > 0$ the rate of expansion of A is faster than B while in the second case where $n < -1, A$ is expanding and B is contracting with the evolution of the universe.

Further, $m > -2$ ensures that energy density is a decreasing function of time. In this model, energy density (ρ) and displacement vector field β are decreasing whereas variable $G(t)$ is increasing with the evolution of the universe.

4. Cosmological model with bulk viscosity

In this case, we have considered the effect of bulk viscosity on the evolution of universe. The effect of bulk viscosity on the cosmological evolution of the universe has been discussed by many authors [27]. In fact it is the only dissipative mechanism that can be incorporated in an isotropic cosmological model. In the presence of bulk viscous stress the energy–momentum tensor takes the form

$$T_{ij} = (\rho + P_{\text{eff}})u_i u_j - P_{\text{eff}} g_{ij}, \tag{16}$$

where P_{eff} stands for effective pressure which may be defined as

$$P_{\text{eff}} = p + \Pi. \tag{17}$$

Here p is the equilibrium pressure and Π is the bulk viscous pressure.

In the presence of bulk viscosity, the set of field eqs (4)–(6) may be rewritten as

$$3\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{B}}{AB}\right) = 8\pi G\rho + \frac{3}{4}\beta^2, \tag{18}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -8\pi G(p + \Pi) - \frac{3}{4}\beta^2, \tag{19}$$

$$3\left(\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2}\right) = -8\pi G(p + \Pi) - \frac{3}{4}\beta^2. \tag{20}$$

Maartens [27] has suggested the causal evolution equation for the bulk viscous pressure.

$$\tau\dot{\Pi} + \Pi = -3\xi H - \frac{\varepsilon\tau\Pi}{2}\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right]. \tag{21}$$

It has been found that the solution for A and B are the same as the solution of the previous section. The set of equations (18)–(20) along with the equation of state

$$p = (\gamma - 1)\rho, \quad 1 \leq \gamma \leq 2 \tag{22}$$

suggests

$$\Pi = (2 - \gamma)\rho. \tag{23}$$

In the following subsections we shall consider, in turn, the behavior of bulk viscosity in truncated theory and full causal theory separately.

Case I: *Model in truncated causal theory*

In the truncated causal theory, second term on the right-hand side of eq. (21) is negligible and hence it reduces to

$$\tau\dot{\Pi} + \Pi = -3\xi H. \tag{24}$$

It is already suggested that the phenomenological relation

$$\tau = \frac{\xi}{\rho}, \tag{25}$$

is one way of ensuring that viscous signal do not exceed the speed of light in the truncated theory [27].

Equations (23)–(25) suggest

$$\xi = \xi_0 \frac{1}{t^{1+m}}, \tag{26}$$

where

$$\xi_0 = \frac{\rho_0(2-\gamma)}{2\gamma + m\gamma - 2m - 3}.$$

Equation (26) indicates that the bulk viscosity coefficient ξ is decreasing with evolution of the universe. Many authors have used the truncated causal theory in various types of investigations. However, truncated theory implies a drastic contradiction on temperature. On equating to zero, the second term on the right of eq. (21) gives

$$T = \frac{\tau}{\xi} R^3. \tag{27}$$

Equations (25) and (26) suggest $\xi \propto \theta^{1+m}$ and $\tau \propto H^{-1}$, i.e. the viscosity is determined by the expansion rate, τ is determined by the cosmic time. From eqs (25) and (27), we get $T \propto R^3/\rho$. This means that temperature rises during expansion of the universe, which violates the physical conditions. It has been suggested by Maartens [27] that in order to overcome the abovementioned problem, one should consider Israel–Stewart [28] full (non-truncated) causal thermodynamics, which is considered as the best currently available theory for analysing dissipative process in the universe.

Case II: *Model in full causal theory*

In this case we consider $\varepsilon = 1$ and hence eq. (21) may be rewritten as

$$\tau\dot{\Pi} + \Pi = -3\xi H - \frac{\tau\Pi}{2}\left[3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right]. \tag{28}$$

Maartens [29] has pointed out that the Gibbs integrability condition suggests that equation of state for the pressure and temperature are not independent. If the equation of state for pressure is barotropic then equation of state for temperature should be barotropic and may be written as

$$T \propto \exp\int\frac{\mathrm{d}p}{\rho + p}, \tag{29}$$

which with the help of eq. (22) yields

$$T = T_0\rho^{(\gamma-1)/\gamma}. \tag{30}$$

Here T_0 is the integration constant.

Now with the help of an expression of H, eqs (15), (23), (24) and (30), one can easily obtain ξ from eq. (28) as

$$\xi = \xi_1\frac{1}{t^{1+m}}, \tag{31}$$

where

$$\xi_1 = \frac{6(\gamma-2)\gamma\rho_0}{12\gamma - 3\gamma^2 - 3(2-\gamma)(2+m)}$$

which shows that the bulk viscosity coefficient ξ is decreasing with the expansion of the universe. Further, using eq. (15), the temperature T can be obtained from eq. (30) as

$$T = T_0\rho_0^{(\gamma-1)/\gamma}\frac{1}{t^{(\gamma-1)(2+m)/\gamma}}. \tag{32}$$

5. Discussion

In the present paper we have investigated the effect of bulk viscosity and time varying gravitational parameter G on the evolution of a five-dimensional model of the universe within the framework of Lyra geometry. The dimensional reduction is depending on the choice of the constant n. Equations (9) and (10) suggest that (i) for $n > 0$ the rate of expansion of the metric coefficient A is faster than B, (ii) for $n < -1/3$, A is expanding and B is contracting, (iii) for all values of n lying in the interval $(-1/2, -1/3)$, we get inflationary expansion in A while B is decreasing with time. This indicates that for this model extra dimension is either expanding at very slow rate or collapsing while three others continued to expand with evolution of the universe. The principle observational upper limits on $\dot{G}/G$ have been suggested by several authors based on different observations. Anderson *et al* [30] used Mariner 10 data and radar ranging to Mercury and Venus to obtain $\dot{G}/G < 0.0 \pm 2.0 \times 10^{-12}$ yr^{-1} (for detailed review of observational upper limits on $\dot{G}/G$, please refer Barrow and Parsons [31]). Equation (14) with the upper limits on $\dot{G}/G$ and $m > -2$ suggests that the gravitational parameter G can be considered as decreasing or increasing function of cosmic time t. In both the truncated and full causal theory the evolution equation of bulk viscosity coefficient is related to the gravitational parameter G. Assuming the age of the universe $t_0 \sim 10^{10}$ yr $\sim 3 \times 10^{17}$ s , $\gamma = 4/3$ and the estimated value of $m \leq 0 \pm 0.02$, eq. (32) suggests $T_0 \sim 10^{-9}$ MeV $\sim$1 K, which is in fair agreement with the present measured value of thermal radiation in the universe [32] .

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