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Binary compact object inspiral: Detection expectations

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Abstract. We review the current estimates of binary compact object inspiral rates in particular in view of the recently discovered highly relativistic binary pulsar J0737-3039. One of the robust results is that, because of this discovery, the rate estimates for binary neutron stars have increased by a factor of 6–7 independent of any uncertainties related to the pulsar population properties. This rate increase has dramatic implications for gravitational wave detectors. For initial LIGO, the most probable detection rates for double neutron star (DNS) inspirals is 1 event/(5–250) yr; at 95% confidence we obtain rates up to 1/1.5 yr. For advanced LIGO, the most probable rates are 20–1000 events/yr. These predictions, for the first time, bring the expectations for DNS detections by initial LIGO to the astrophysically relevant regime. We also use our models to predict that the large-scale Parkes multibeam pulsar survey with acceleration searches could detect an average of three to four binary pulsars similar to those known at present. In comparison, rate estimates for binaries with black holes are derived based on binary evolution calculation, and based on the optimistic ends of the ranges, remain an important candidate for inspiral detection in the next few years.

We also consider another aspect of the detectability of binary inspiral: the effect of precession on the detection efficiency of astrophysically relevant binaries. Based on our current astrophysical expectations, large tilt angles are not favored. As a result the decrease in detection rate varies rather slowly with black hole spin magnitude and is within 20–30% of the maximum possible values.

Keywords. Compact objects; inspiral; event rates; precession.

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1. Introduction

Binary compact object inspirals are prime targets for gravitational wave detection by the ground-based interferometers LIGO [1], GEO [2], and VIRGO [3]. Event rate estimates are very important for the development of gravitational wave interferometers [4]. They are based on estimates of galactic rates and their extrapolation throughout a survey volume [5], given the source strength and instrument sensitivity. For double neutron star (DNS) binaries, galactic rate estimates have been obtained using two very different methods. One is purely theoretical and involves models of binary evolution calibrated usually to the observationally determined supernova rate for the galaxy. The other, more empirical, approach is based on the physical properties of the close DNS binaries known in the galactic field and modeling of radio pulsar survey selection effects. For a review and details of both these approaches, see [6] and references therein. The empirical method has generally provided us with better constraints on the coalescence rate [6], although the uncertainty still exceeds two orders of magnitude. This is primarily due to (i) the very small number (only two until recently) of close DNS known in the galactic field with merger times shorter than a Hubble time and (ii) the implicit assumption that this small sample is a good representation of the total galactic population [6].

Two recent developments make it appropriate to revisit the DNS inspiral rate calculations. Firstly, the discovery of the 2.4 h DNS binary PSR J0737-3039 in a large-area survey using the Parkes radio telescope [7] brings the number of known DNS systems to merge in the galactic field to three. With an orbital period of only 2.4 h, J0737-3039 will coalesce in only 85 Myr, a factor of 3.5 shorter than the merger time of PSR B1913+16. This immediately hints towards a possible significant increase of the coalescence rate [7]. Secondly, a novel statistical method has been developed by Kim and collaborators [8] that automatically takes into account statistical biases inherent in small-number samples, like the relativistic DNS binaries, and additionally allows us to quantify our expectation that the actual DNS binary coalescence rate has a particular value, given the current observations. Here we summarize the results presented in detail by Kim and collaborators [9].

Apart from the issue of event rates, successful inspiral detection also depends on the best possible understanding of the inspiral gravitational wave-form and the development of template families that can efficiently match the expected signals in the interferometer data streams. Relativistic spin–orbit and spin–spin couplings can cause inspiraling binary compact systems containing neutron stars or black holes to precess, provided that at least one of the two spins is of significant magnitude and misaligned with respect to the orbital angular momentum. This precession leads to a periodic change of the orbital plane orientation and therefore modifies the inspiral gravitational wave signal received by ground-based detectors.

The effect of this precession modulation also depends on the spin magnitude of the more massive object and the tilt angle of this spin with respect to the orbital angular momentum. The astrophysical origin of the compact object spin magnitudes and orientations and their expected probability distributions clearly affect any realistic probability statement about the detectability of precession-modulated signals. The physical origin of the kicks has been understood uniquely in the context of asymmetric compact object formation and the existence of supernova kicks [10–12]. Our current astrophysical understanding is well-developed and allows theoretical predictions for the distribution of tilt angles among different populations of binary compact objects. On the other hand, the evolution of compact object magnetic field strengths is not as well-understood, and therefore reliable probability distributions of spin magnitudes are difficult to calculate and the results are strongly model dependent.

Here we summarize the results presented by Grandclément and collaborators [13] who, for the first time, took into account the coupling between astrophysical predictions for the probability distributions of tilt angles with calculations of the reduction in signal-to-noise ratio if precession is ignored. As a result, we obtain astrophysically relevant results for the detection rate degradation as a function of the compact object spin magnitude.

2. Event rate calculation

2.1 *Statistical method for double neutron stars*

Until recently, estimates of DNS coalescence rates provided a range of possible values without any information on the likelihood of these values. Kim *et al* [8] presented a newly developed statistical analysis that allows the calculation of a probability distribution for the rate estimates and the determination of confidence intervals associated with the rate estimates. The method is described in detail in [8] but we briefly summarize the main elements here. The method involves the simulation of selection effects inherent in all the relevant radio pulsar surveys and a Bayesian statistical analysis for the probability distribution of the inspiral rate estimates. The small-number bias and the effect of the faint-end of the pulsar luminosity function, previously identified as the main sources of uncertainty in rate estimates [6], are implicitly included in this analysis.

For a model galactic pulsar population with an assumed spatial and luminosity distribution, we determine the fraction of the total population, which are actually detectable by the current large-scale pulsar surveys. In order to do this, we calculate the effective signal-to-noise ratio for each model pulsar in each survey and compare this with the corresponding detection threshold. Only those pulsars which are nominally above the threshold, count as detectable. After performing this process on the entire model pulsar population of size $N_{\rm tot}$, we are left with a sample of $N_{\rm obs}$ pulsars that are nominally detectable by the surveys. By repeating this process many times, we can determine the probability distribution of $N_{\rm obs}$, which we then use to constrain the population and with a Bayesian analysis derive the probability expectation that the actual galactic DNS inspiral rate takes on a particular value, given the observations.

When this method was first developed [8], it was shown that, although the shape of the probability distribution of rate estimates is very robust, the rate value at peak probability systematically depends primarily on the characteristics of the radio pulsar luminosity function: its slope and the physical minimum luminosity of pulsars. Both of these are constrained by the general pulsar population [14], but we explore the dependence of our results on the assumed values.

Here we consider the same set of pulsar population models as in [8], but we choose model 6 as our reference model in view of the recent discovery of very faint pulsars [15]. With the addition of the new DNS binary PSR J0737-3039, our calculations differ from those in [8] in two main ways: (i) the latest Parkes survey that led to the discovery of the new system [7] is included; (ii) we calculate and account for the effects of Doppler smearing for DNS binaries akin to J0737-3039 by creating fake time series for a variety of orbital phases (see [8] for details). Even for an approximately 4.5-min integration [7], this effect alone reduces the average signal-to-noise ratio of a 2.4-h DNS binary by 35%.

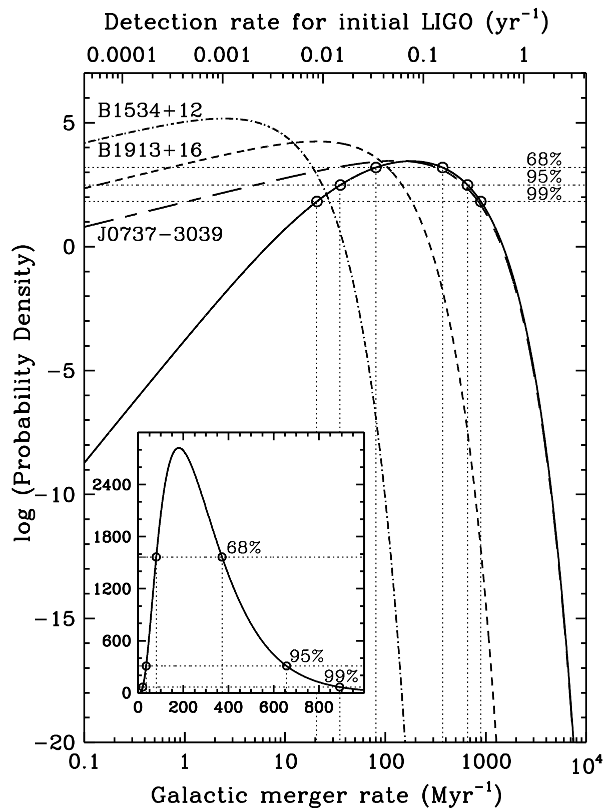

Figure 1. The probability density function that represents our expectation that the actual DNS binary merger rate in the galaxy (bottom axis) and the predicted initial LIGO rate (top axis) take on particular values, given the observations. The curves shown are calculated assuming our reference model parameters (see text). The solid line shows the total probability density along with those obtained for each of the three binary systems (dashed lines). Inset: The total probability density, and corresponding 68%, 95% and 99% confidence limits, shown in a linear scale.

The statistical analysis presented in [8] has been extended to account for three systems [16]. In our calculations we adopt a total lifetime for J0737-3039, defined as the sum of the current age and the remaining lifetime until the final coalescence, equal to $100 + 85 = 185$ Myr [7]. In the absence of detailed beam observations for the new binary, we adopt a beaming factor of $\simeq$6 equal to the average of the two observationally constrained beams of the previously known DNS binaries [6,7].

2.2 *Double neutron star rate estimates*

For our reference pulsar model (with a radio luminosity function consistent with current pulsar observations [14,15]), we find the most likely value of the total coalescence rate to be $\mathcal{R} = 180$ Myr^{-1}. The range of values at 68% and 95% confidence intervals are 80–370 and 40–660 Myr^{-1}, respectively. The width of these ranges are somewhat smaller than the previous estimates (the ratio between upper and lower limits at 68% confidence interval is 4.6, cf. 5.4 found by [8]) confirming the expectation that a bigger observed sample would reduce the uncertainty in the rate estimates [6]. The new value for $\mathcal{R}$ is a factor of 6.7 higher than that found by Kim *et al* [8]. From the resulting probability distribution shown in figure 1, it is clear that J0737-3039 dominates the total rate over the other two systems. This is due to two separate factors: (i) the estimated total number of DNS binaries similar to J0737-3039 (3800) is far higher than those of each of the other two systems (1300 for B1913+16 and 1100 for B1534+12). This is mainly due to the shorter pulsar spin and binary orbital period of J0737-3039 which results in a significant Doppler smearing and efficiently 'hides' them in the galaxy; (ii) the total lifetime of J0737-3039 (185 Myr) is significantly shorter than those of the other two (365 Myr for B1913+16 and 2.9 Gyr for B1534+12).

We have explored our results for all other models considered in [8]. Details on our main results are given in [9] where we have included a subset of models that reflect the widest variations of the rates (as shown in [8], variations in the space distribution of pulsars are not important). The main conclusions that can be easily drawn are: (i) the increase factor on the inspiral rate is highly robust against all systematic variations of the assumed pulsar models and is strongly constrained in the range 6–7; this is consistent with but somewhat lower than the simple estimate presented in [7]; (ii) the shape of the rate probability distribution also remains robust, but the rate value at peak probability depends on the model assumptions in the same way as described in detail in [8] (see figures 5–7 in [8]).

2.3 *Binary inspiral event rates: Conclusions*

We now consider the implications of the revised galactic DNS rate estimates for the detection of these events by LIGO and the other upcoming gravitational wave interferometers. Since these instruments can detect DNS inspirals out to $\simeq$20 Mpc for initial LIGO ($\simeq$ 350 Mpc for advanced LIGO) [5], it is necessary to extrapolate our galactic event rate out to the local group. Using the standard extrapolation of our reference model out to extragalactic distances [6,17], we find the most probable event rates for our reference model are 1/13 yr and 1/day, for initial and advanced LIGO, respectively. At the 95% confidence interval, the most optimistic predictions for the reference model are 1 event/4 yr and 4 events/day for initial and advanced LIGO, respectively. However, considering the full set of 27 models at 95% confidence interval indicates that the respective rates can reach up to 1 event/1.5 yr and 10 events/day, respectively. These results are quite encouraging, since, for initial LIGO in particular, this is the first time that DNS coalescence rate estimates are within an astrophysically relevant regime. Within a few years of LIGO operations it should be possible to directly test these predictions and, in turn, place better constraints on the properties of binary radio pulsars, the cosmic population, and the evolution of DNS binaries.

Rate estimates for inspiral detection rates for black hole binaries rely exclusively on our best current understanding of binary evolution and binary compact object formation [18]. These calculations do not provide us with rate predictions associated with a given statistical significance, but instead the quoted range directly depend on the extent of the model parameter studies in various investigations. At present, for initial LIGO, inspiral detection rates are as high as 1 in 3 years and 3 in 1 year, for BH–NS and BH–BH binaries, respectively. For advanced LIGO, the corresponding estimates reach 1,500 and 10,000 events per year respectively.

It is important to note that the uncertainties in the black hole inspiral rates could be reduced, if (i) the empirical DNS rate estimates and (ii) the absence of any BH–NS binaries detected as radio pulsars are used as constraints on the binary evolution models.

2.4 *Future detections of binary radio pulsars?*

As already mentioned, long integration times combined with very short binary orbital periods strongly select against the discovery of new binary pulsars. Specifically in the large-scale Parkes multibeam (PMB) survey [19] with an integration time of 35 min, the signal-to-noise ratio is severely reduced by Doppler smearing due to the pulsars' orbital motion. Acceleration searches in the current re-analysis of the PMB survey [20] should significantly improve the detection efficiency to DNS binaries.

Following [8], we calculate the probability distribution that represents our expectation that the actual number of DNS pulsars with merger times shorter than a Hubble time ($N_{\rm obs}$) that could be detected with the PMB survey takes on a particular value, given the current observations and assuming that the reduction in the flux due to the Doppler smearing is corrected perfectly. To illuminate the effect of the Doppler smearing we calculate the average number of expected new discoveries akin to each of the three known DNS binaries (see [9] for details). The result is shown for our reference model in figure 2. The average combined number is 3.6 and the discovery of up to 2 (or 4) DNS systems has a probability equal to $\simeq$20% ($\simeq$55%).

We conclude that, if the acceleration search can correct the Doppler smearing effect perfectly, then the PMB survey could be expected to detect an average of three to four DNS pulsars with pulse profile and orbital properties similar to any of the three already known systems. We also find that there is a significant probability (in excess of $\simeq$80%) that when acceleration searches of the PMB survey are completed more than two binary pulsars could be detected. The increase of the observed sample is very important for the reduction of the uncertainties associated with the inspiral rate estimates. We note, however, that the discovery of new systems that are similar to the three already known ones does not necessarily imply a significant increase in the rate estimates. Significant changes are expected in the case that new systems with pulse profiles or binary properties significantly different are discovered, as it is such systems that will reveal a new DNS sub-population in the galaxy.

3. Astrophysical predictions for precession and inspiral detection

3.1 *Astrophysical spin tilts and models*

Binary compact object formation is understood as the outcome of many different binary evolutionary sequences, all of which involve a long chain of phases of mass

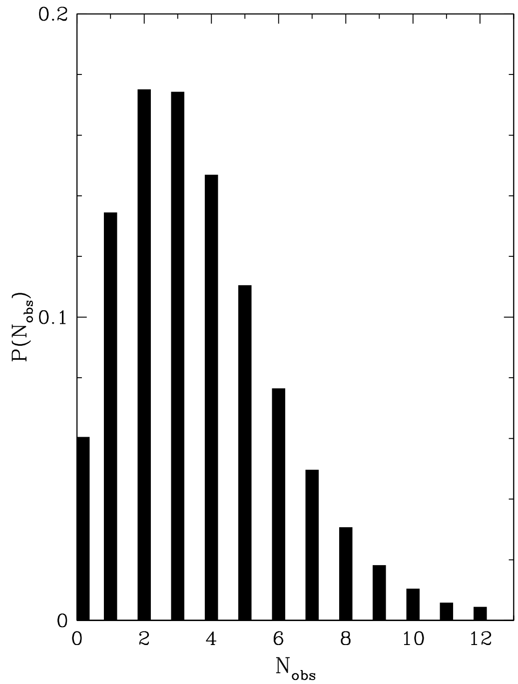

Figure 2. The probability density function of the predicted number of observed DNS binary systems $N_{\rm obs}$ for the PMB survey, for our reference model (model 6 in [8]).The mean value is estimated to be 3.6.

and angular momentum transfer between the binary components (the progenitors of the compact objects) as well as mass and angular momentum losses from the binaries (see [18]). Of all these evolutionary links there is only one that is believed to be the origin of spin–orbit misalignments: the asymmetric collapse of massive stars that imparts a randomly oriented recoil velocity (known as 'kick') to the nascent compact objects. This natal kick affects the binary orbital characteristics and depending on its magnitude and direction can cause the orbital plane to tilt from its pre-collapse orientation. Observational evidence in support of NS kicks has accumulated over the past two decades from a variety of sources, most important being the large measured space velocities of pulsars [21]. For a couple of binary pulsar systems (one of them being the Hulse–Taylor binary), careful timing and polarization pulsar measurements have provided evidence in support of a non-zero tilt angle between the pulsar spin axis and the orbital angular momentum axis. Such misalignments have been theoretically interpreted as the result of asymmetric kicks and have been used to provide quantitative constraints on kick magnitude and orientation [10,11,22].

This basic theoretical understanding has already been used in a preliminary study of the tilt angle distributions expected for BH–NS and BH–BH binaries and their general implications for the reduction in detection rate, if non-precessing templates are used in GW inspiral searches [23]. As discussed in [23] in more detail, the tilt-angle calculation is possible in part because of two main evolutionary factors:

- The complicated evolutionary history of binary compact objects includes multiple phases of strong tidal interactions and mass-transfer phases. The spins of the binary components are expected to be aligned with the orbital angular momentum axis just prior to the formation of the second compact object because of the exchanged angular momentum between the stars and the orbit through these tidal interactions. Therefore, even if, earlier in the evolution of the binary compact object progenitors, spins were misaligned, all momenta axes are expected to be aligned just prior to the second supernova explosion.
- In a BH–NS binary the BH is expected to form from the initially more massive star in the binary. Since more massive stars evolve faster, the BH is expected to be formed first and its spin is expected to be aligned with the orbit just prior to the second explosion. The NS formed in this second explosion is expected to receive a natal kick, which can result in the post-explosion orbital plane being tilted with respect to the pre-explosion orbital plane. Given the extremely small cross-section of the BH, its spin remains unaffected and aligned with the pre-explosion orbital angular momentum, and hence misaligned with respect to the orbital angular momentum axis of the newly formed double compact object. Similar considerations hold for a BH–BH binary, where the NS is typically replaced by the least massive of the two BHs.

In a recent analysis [13] we used the binary evolution code StarTrack that was originally developed by Belczynski *et al* [18] to calculate the expected distributions of tilt angles for binaries with black holes and neutron stars. In the past year this population synthesis code has been modified with special focus on the treatment of mass-transfer phases: a new, computationally efficient scheme has been developed that, for the first time allows the calculation of mass-transfer rates and binary evolution through mass-transfer phases that have been carefully tested against results of detailed stellar evolution codes and are in very good agreement [24]. Using this updated code we studied the properties of double compact objects using a subset of the set of models considered in [18]. We find that the results from the updated version are qualitatively identical to the early models (presented in detail in [18]) and, in terms of the tilt-angle distributions, quantitatively similar but with noticeable differences (typically the new models have a somewhat larger fraction of binaries formed with tilt angles in excess of 10°).

Taking into account detailed models for the full evolutionary history of close binary compact objects we calculate the probability distributions of tilt angles for different types of systems and find that the strong majority of tilts are smaller than $\simeq 60^\circ$. This is mainly due to the requirement that binaries remain bound after compact object formation and in a tight orbit (with coalescence times shorter than the Hubble time).

3.2 *Astrophysical relevance of precession in inspiral signals*

To investigate the astrophysical relevance of precession in the detection of inspiral signals we combine our calculations of spin tilt distributions with earlier

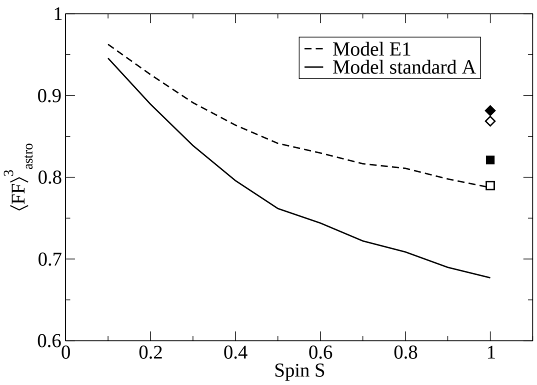

Figure 3. Reduced detection rate, given a probability distribution for κ. The system is composed of a black hole and a non-spinning NS. $\langle \mathrm{FF} \rangle^3_{\mathrm{astro}}$ is a measure of the detection volume and hence detection rate and is presented for both models, as a function of the spin of the black hole. The empty square (diamond) denotes the value for Apostolatos' templates, for model A (E1) and the filled square (diamond) the value using the 'spiky' templates for model A (E1).

calculations of the fitting factor in the presence of precession. The fitting factor FF is a quantitative measure of the reduction in signal-to-noise ratio if precession is important for the signal but it is not accounted for in the template database. Fitting factors can be calculated as a function of the spin magnitude S and spin tilt angle for inspiraling binaries [25–27]. We are able to derive 'astrophysically weighted' fitting factors $\langle \mathrm{FF} \rangle_{\mathrm{astro}}$ that depend only on the BH spin magnitude and are defined as:

$$\langle \mathrm{FF} \rangle_{\mathrm{astro}}(S) = \int_{-1}^{1} f(\kappa) \langle \mathrm{FF} \rangle(\kappa, S)\ \mathrm{d}\kappa, \tag{1}$$

where κ is the cosine of the tilt angle and $f(\kappa)$ is its distribution function.

In figure 3 we show the derived astrophysical $\langle \mathrm{FF} \rangle^3_{\mathrm{astro}}$ values (that are proportional to the rate reduction) as a function of the BH spin magnitude S, for two different models of binary evolution (models A and E1 from [18]). The variation across the range is moderate and $\langle \mathrm{FF} \rangle^3_{\mathrm{astro}}$ values are typically ≥ 0.7. They depend only moderately on S and on the details of the population models. This result is the outcome of a balance between the fact that the lowest $\langle \mathrm{FF} \rangle$ occur for $\kappa \sim -0.5$ and that there is only a low probability for BH–NS to have such large tilts. Figure 3 also shows the values obtained when using both Apostolatos' wave-forms (empty symbols) and the 'spiky' templates (filled symbols), for model A (the squares) and E1 (the diamonds). These suggested wave-forms attempt to account for the effects of precession in the template database used for inspiral searches. We note that such templates have also been suggested by a group led by Buonanno [28]. In conclusion we find that most probably Nature protects us from a large decrease of the inspiral detection rate in the case that non-precession templates are used: for a wide range of binary evolution models, the decrease factors of detection rates turn out to be rather insensitive to the spin magnitude and they remain rather well-restricted within 20–30% of the maximum possible.

The implication of our results for the astrophysical expectations of tilt angles is that precession effects may not be as important as previously thought for searches of gravitational wave inspirals. However, precession in principle can still be important, if larger tilt angles are more favored than currently thought. Given the fact that the current understanding of binary compact object formation is far from perfect and that the anticipated detection rates for first-generation interferometers may be low, it is probably best to still account for precession effects in the best way possible in gravitational wave data analysis.

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