

## Anisotropic Lyra cosmology

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**Abstract.** Anisotropic Bianchi Type-I cosmological models have been studied on the basis of Lyra's geometry. Two types of models, one with constant deceleration parameter and the other with variable deceleration parameter have been derived by considering a time-dependent displacement field.

**Keywords.** Cosmology; Lyra's geometry; anisotropic Bianchi Type-I models; constant and variable deceleration parameters.

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### 1. Introduction

*Einstein geometrized gravitation.* Weyl [1] was inspired by it and he was the first to try to unify gravitation and electromagnetism in a single space-time geometry. He showed how one can introduce a vector field in the Riemannian space-time with an intrinsic geometrical significance. But this theory was not accepted as it was based on non-integrability of length transfer.

Lyra [2] introduced a gauge function, i.e., a displacement vector in Riemannian space-time which removes the non-integrability condition of a vector under parallel transport. In this way Riemannian geometry was given a new modification by him and the modified geometry was named as Lyra's geometry.

Sen [3] and Sen and Dunn [4] proposed a new scalar-tensor theory of gravitation and constructed the field equations analogous to the Einstein's field equations, based on Lyra's geometry which in normal gauge may be written in the form

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G(t)T_{ij}, \quad (1)$$

where  $\phi_i$  is the displacement vector and other symbols have their usual meanings.

Halford [5] has pointed out that the constant vector displacement field  $\phi_i$  in Lyra's geometry plays the role of cosmological constant in the normal general relativistic

treatment. According to Halford [6] the scalar–tensor treatment based on Lyra’s geometry predicts the same effects, within observational limits, as far as the classical solar system tests are concerned, as well as tests based on the linearised form of field equations. In papers [7–12], the study of cosmology in Lyra’s geometry with a constant displacement field has been shown. Soleng [13] has pointed out that the constant displacement in Lyra’s geometry will either include a creation field and be equivalent to Hoyle’s creation field cosmology or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. Beesham [14] considered FRW models with a time-dependent field. Singh and Singh [15–18] have presented Bianchi Type-I, III and Kantowski–Sachs cosmological models with a time-dependent displacement field and have made a comparative study of the Robertson–Walker models with a constant deceleration parameter in Einstein’s theory with cosmological term and in the cosmological theory based on Lyra’s geometry. They have given a review of Lyra cosmological models. In a recent paper [19], Singh and Desikan have studied FRW models with a time-dependent displacement field with constant deceleration parameter.

Some very recent works done on Lyra’s geometry are given in [21,22], viz., Rahaman *et al* [21] have done investigations in cosmology within the framework of Lyra’s geometry. Rahaman *et al* [22] have studied string cosmology in five-dimensional space-time based on Lyra geometry and in one model they have shown that the gauge function is large in the beginning but decreases with the evolution of the model.

It can be mentioned here that the Brans–Dicke field is a scalar field  $\phi(t) = (1/G(t))$ . On the other hand, the Lyra field is a vector field, distinct from  $G(t)$ . This shows the difference between the Brans–Dicke field and the Lyra field.

The present universe is, by and large, isotropic conforming to FRW-metric with deviations from homogeneity  $\delta\rho/\rho \approx 10^{-5}$  and departures from isotropy  $\sigma^2/H_0 < 10^{-9}$ ,  $\sigma^2$  standing for the anisotropy [23].

Faber [24] and Guth [25] aver that at a very early stage, the universe might have been anisotropic. However, in the course of evolution the universe has developed isotropy as we observe today. We have been motivated by this conjecture to study the anisotropic models of the universe in this paper.

The purpose of this paper is to derive some Bianchi Type-I cosmological models with a time-dependent displacement field in terms of (i) constant and (ii) time-dependent deceleration parameters. The paper is organised as follows: We present field equations in §2, constant deceleration parameter models in §3 and variable deceleration models in §4. We conclude in §5.

## 2. Field equations

The energy–momentum conservation law is given by

$$T_{i;j}^j = 0. \tag{2}$$

The time-like displacement vector  $\phi_i$  in (1) is given by

$$\phi_i = (\beta(t), 0, 0, 0). \tag{3}$$

We consider the cosmic matter as a perfect fluid and write the energy–momentum tensor as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \quad (4)$$

where the terms indicate their usual meanings.

For the Bianchi Type-I metric

$$ds^2 = dt^2 - a_1^2(t)dx^2 - a_2^2(t)dy^2 - a_3^2(t)dz^2. \quad (5)$$

With (3)–(5), the field eq. (1) and the energy–momentum conservation equation (2) become

$$\frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2} \frac{\dot{a}_3}{a_3} + \frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} - \frac{3}{4}\beta^2 = \chi(t)\rho, \quad (6)$$

$$\frac{\dot{a}_2}{a_2} \frac{\dot{a}_3}{a_3} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{3}{4}\beta^2 = -\chi(t)p, \quad (7)$$

$$\frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{3}{4}\beta^2 = -\chi(t)p, \quad (8)$$

$$\frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{3}{4}\beta^2 = -\chi(t)p, \quad (9)$$

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (10)$$

where  $\chi(t) = 8\pi G(t)$  and  $H(\equiv \dot{R}/R) =$  Hubble's parameter with  $R(t) = (a_1 a_2 a_3)^{1/3}$ .

We take the anisotropy  $\sigma^2$  as

$$\sigma^2 \equiv 3H^2 - \left( \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} \right) \quad (\text{vide [20]}). \quad (11)$$

Now, eqs (6) and (11) lead to

$$3H^2 - \frac{3}{4}\beta^2 = \chi(t)\rho + \sigma^2. \quad (12)$$

From eqs (7)–(9) and (11), we have

$$2\dot{H} + 3H^2 + \frac{3}{4}\beta^2 = -\chi(t)p - \sigma^2. \quad (13)$$

From (6)–(9), we obtain the continuity equation

$$\chi\dot{\rho} + \dot{\chi}\rho + \frac{3}{2}\beta^2(\dot{\beta}/\beta + 3H) + \chi(\rho + p)3H = 0. \quad (14)$$

Equation (14) can also be obtained from (1) by directly using Bianchi identities (vide Appendix). From (10) and (14), we have

$$\frac{3}{2}\beta^2(\dot{\beta}/\beta + 3H) = -\dot{\chi}\rho. \quad (15)$$

Let us assume an equation of state

$$p = \gamma\rho, \quad 0 \leq \gamma \leq 1. \quad (16)$$

With (16), eqs (10), (13) and (14) become

$$\rho = \rho'_0 R^{-3(1+\gamma)}, \quad \rho'_0 = \text{constant of integration} \quad (17)$$

$$2\dot{H} + 3H^2 + \frac{3}{4}\beta^2 = -\chi(t)\gamma\rho - \sigma^2, \quad (18)$$

$$\chi\dot{\rho} + \dot{\chi}\rho + \frac{3}{2}\beta^2(\dot{\beta}/\beta + 3H) + \chi(1 + \gamma)\rho \cdot 3H = 0. \quad (19)$$

Adding (12) and (18), we have

$$2\dot{H} + 6H^2 = (1 - \gamma)\chi\rho. \quad (20)$$

$\beta^2$  in the above equations appears to play the role of a variable cosmological term  $\Lambda(t)$  in Einstein equation.

### 3. Constant deceleration parameter models

We have four independent equations above, viz. (12), (15), (17) and (20) for five unknowns, viz.  $(R(t), \chi(t), \rho(t), \beta(t), \sigma^2(t))$ . To solve them we take the following ansatz.

Ansatz (A):

$$R = R_0 t^n, \quad R_0 \text{ and } n \text{ are constants and } n > 0. \quad (21)$$

With (21), eq. (17) yields

$$\rho = \rho_0 t^{-3(1+\gamma)n}, \quad \rho_0 = \rho'_0 R_0^{-3(1+\gamma)} = \text{constant}. \quad (22)$$

Using (21) and (22) in (20), we obtain

$$\chi = \chi_0 t^{3n(1+\gamma)-2}, \quad \chi_0 = \frac{2n(3n-1)}{(1-\gamma)\rho_0}, \quad \gamma \neq 1. \quad (23)$$

Equation (15) with eqs (21)–(23) leads to

$$\beta^2 = \frac{C}{3(1-3n)} t^{-6n} - \frac{4n\{3n(1+\gamma)-2\}}{3(1-\gamma)} t^{-2}, \quad C = \text{constant}, \quad C > 0. \quad (24)$$

Using (21)–(24) in (12) we have

$$\sigma^2 = \frac{C}{2(3n-1)} t^{-6n}. \quad (25)$$

**Table 1.** Models for  $\gamma = 0, 1/3, 1/2$  for constant deceleration parameter.

Model no.	1	2	3
$\gamma$	0	$\frac{1}{3}$	$\frac{1}{2}$
$n$	$\frac{1}{2}$	$\frac{4}{9}$	$\frac{2}{5}$
$R$	$R_0 t^{1/2}$	$R_0 t^{4/9}$	$R_0 t^{2/5}$
$\rho$	$\rho_0 t^{-3/2}$	$\rho_0 t^{-16/9}$	$\rho_0 t^{-9/5}$
$\chi$	$\frac{1}{2\rho_0} t^{-1/2}$	$\frac{4}{9\rho_0} t^{-2/9}$	$\frac{8}{25\rho_0} t^{-1/5}$
$p$	0	$\frac{1}{3}\rho$	$\frac{1}{2}\rho$
$\sigma^2$	$Ct^{-3}$	$\frac{3C}{2} t^{-8/3}$	$\frac{5C}{2} t^{-12/5}$
$\beta^2$	$\frac{1}{3}t^{-2} - \frac{4C}{3}t^{-3}$	$\frac{16}{81}t^{-2} - 2Ct^{-8/3}$	$\frac{16}{75}t^{-2} - \frac{10C}{3}t^{-12/5}$

The deceleration parameter,  $q(= (-\ddot{R}/R)/(\dot{R}/R)^2) = (1 - n)/n = \text{constant}$ . Equations (22)–(25) show that for  $\chi, \sigma^2, \beta^2$  to be positive and  $\sigma^2$  to decay faster than  $\rho$ ,

$$\gamma < 1, \quad 1/3 < n < 2/(3(1 + \gamma))$$

and

$$t > \left[ \frac{C(1 - \gamma)}{2n(3n - 1)\{2 - 3n(1 + \gamma)\}} \right]^{1/(2(3n-1))}. \quad (26)$$

Equations (21), (23) and (26) indicate that this solution has only decelerating expansion with decaying  $G(t)$ .

For  $C(1 - \gamma) \ll [2n(3n - 1)\{2 - 3n(1 + \gamma)\}]$ ,  $\beta^2$  is seen to be negative at the very early stage after the birth of the universe. After that it becomes zero and then becomes maximum positive. Then the value of  $\beta^2(t)$  decreases with the evolution of the universe.

Some models for different values of  $\gamma$  and also for empty universe are presented below. Model 1 for  $\gamma = 0$  (dust model), Model 2 for  $\gamma = 1/3$  (radiation universe) and Model 3 for  $\gamma = 1/2$  (hard universe) are tabulated in table 1. Model 4 for empty universe ( $p = \rho = 0$ ) and Model 5 for stiff matter ( $\gamma = 1$ ) are presented later.

In table 1, all models have decelerating expansion with decaying  $G(t)$ .  $\sigma^2$  decays faster than  $\rho$  in each model. In the case of  $\beta^2(t)$ , we see that in all models at the time of birth of the universe it is infinitely large but negative, then its value increases and becomes positive and rises to a maximum value and then again decreases with the evolution of the universe. In Model 1,  $\beta^2 > 0$  for  $t > 4C$  and  $\beta^2 = (\beta^2)_{\text{max}}$  at  $t = 6C$ . In Model 2,  $\beta^2 > 0$  for  $t > (81C/8)^{3/2}$  and  $\beta^2 = (\beta^2)_{\text{max}}$  at  $t = (27C/2)^{3/2}$  and in Model 3,  $\beta^2 > 0$  for  $t > (125C/8)^{5/2}$  and  $\beta^2 = (\beta^2)_{\text{max}}$  at  $t = (75C/4)^{5/2}$ . Plots of the different functions in Model 1 of table 1 is shown in figure 1.

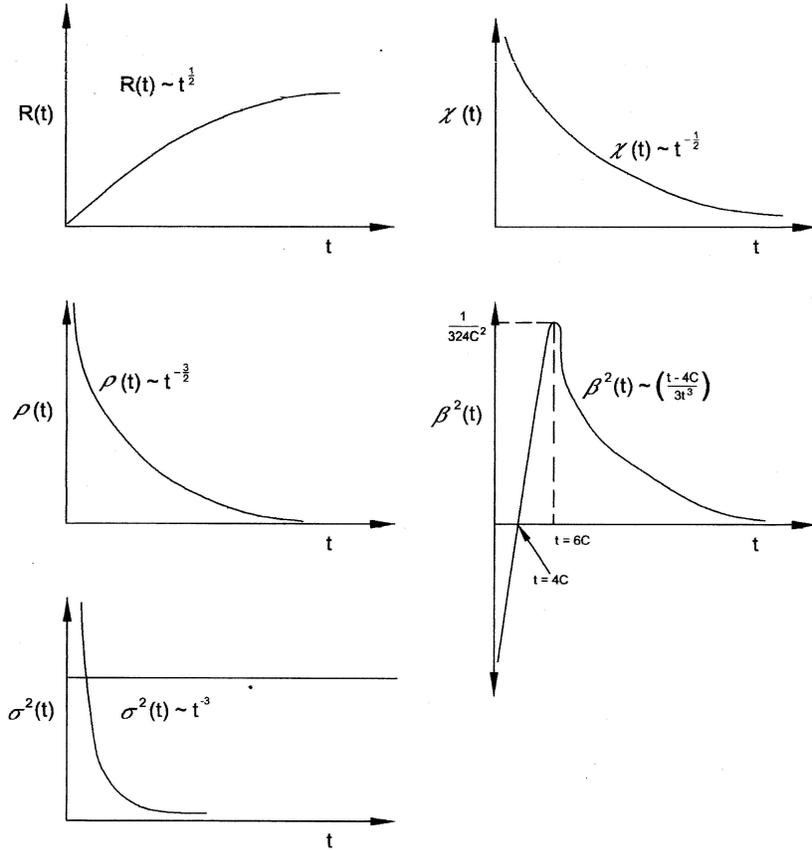


Figure 1. Plots of the different functions in Model 1 (of table 1).

Model 4. Empty universe ( $p = \rho = 0$ ).

With  $p = \rho = 0$ , eqs (12), (15) and (20) become

$$3H^2 = \frac{3}{4}\beta^2 + \sigma^2, \quad (27)$$

$$\left(\frac{3}{4}\beta^2\right) \cdot + \left(\frac{3}{4}\beta^2\right) \cdot 6H = 0, \quad (28)$$

$$\dot{H} + 3H^2 = 0. \quad (29)$$

From (29), we have

$$R = R_0 t^{1/3}, \quad R_0 = \text{constant}. \quad (30)$$

From (28) and (30) we obtain

$$\beta^2 = \frac{4D}{3}t^{-2}, \quad D = \text{constant}, \quad D > 0. \quad (31)$$

With (30) and (31), eq. (27) gives

$$\sigma^2 = \left(\frac{1}{3} - D\right)t^{-2}, \quad 0 < D < \frac{1}{3}. \quad (32)$$

As there is no  $\chi$  in eqs (27)–(29),  $\chi(t)$  may have any physically acceptable time-dependent or constant expression.

We see that the empty universe model is a model with decelerating expansion. Anisotropy and the gauge function decrease from a large value with the expansion of the universe.

*Model 5.  $\gamma = 1$  (stiff matter)*

With  $\gamma = 1$ , eqs (12), (15), (17) and (20) become

$$3H^2 = \chi(t)\rho + \frac{3}{4}\beta^2 + \sigma^2, \quad (33)$$

$$\left(\frac{3}{4}\beta^2\right)^\bullet + \left(\frac{3}{4}\beta^2\right)6H = -\dot{\chi}\rho, \quad (34)$$

$$\rho = \rho'_0 R^{-6} \quad (35)$$

and

$$\dot{H} + 3H^2 = 0. \quad (36)$$

From (36),

$$R = R_0 t^{1/3}, \quad R_0 = \text{constant}. \quad (37)$$

Using (37) in (35), we obtain

$$\rho = \rho_0 t^{-2}, \quad \rho_0 = \rho'_0 R_0^{-6}. \quad (38)$$

Now, using (37) and (38) in (34), we have

$$\beta^2 = \frac{4}{3} \left(\frac{B}{\rho_0} - \chi\right) \rho_0 t^{-2}, \quad B = \text{constant}, \quad B > 0. \quad (39)$$

From (33), (37)–(39), we obtain

$$\sigma^2 = \left(\frac{1}{3} - B\right)t^{-2}, \quad 0 < B < \frac{1}{3}. \quad (40)$$

Equation (16) gives

$$p = \rho. \tag{41}$$

$\beta^2$  and  $\chi$  cannot be determined uniquely.

Equations (38) and (40) show that  $\rho$  and  $\sigma^2$  decay at the same rate so that anisotropy is seen to exist with the existence of energy in the present universe. So, we do not get any physical stiff matter model with constant deceleration parameter in anisotropic Lyra cosmology.

*Model 6*

Ansatz (B):

$$R = R_0 e^{\alpha t}, \quad \alpha = \text{constant}, \quad \alpha > 0. \tag{42}$$

With (42), eq. (17) will give

$$\rho = \rho_0 e^{-\beta t}, \quad \rho_0 = \rho'_0 R_0^{-3(1+\gamma)}, \quad \beta = 3\alpha(1 + \gamma). \tag{43}$$

Equation (20) with (42) and (43) gives

$$\chi = \chi_0 e^{\beta t}, \quad \chi_0 = \frac{6\alpha^2}{\rho_0(1 - \gamma)}. \tag{44}$$

Using (42)–(44) in (12), we obtain

$$\sigma^2 + \frac{3}{4}\beta^2 = 3\alpha^2 - \chi_0\rho_0 = \text{constant}. \tag{45}$$

Equation (45) shows that sum of two positive quantities  $\sigma^2$  and  $(3/4)\beta^2$  both of which should decrease with time, has become constant. As this is an absurd case, we can say de Sitter expansion (inflationary) with deceleration parameter,  $q = -1$  is not allowed in anisotropic Lyra cosmology.

**4. Variable deceleration parameter models**

To solve eqs (12), (15), (17) and (20) for variable deceleration parameter we adopt the ansatz (C):

$$R = R_0 t^n (t + t_0)^n, \quad R_0, n, t_0 \text{ are constants.} \tag{46}$$

With (46), eq. (17) gives

$$\rho = \rho_0 t^{-3n(1+\gamma)} (t + t_0)^{-3n(1+\gamma)}, \quad \rho = \rho'_0 R_0^{-3(1+\gamma)}. \tag{47}$$

With (46) and (47), eq. (20) gives for  $n = 1/3$

$$\chi = \frac{4}{3(1 - \gamma)\rho_0} t^\gamma (t + t_0)^\gamma. \tag{48}$$

With  $n = 1/3$ , (46) and (47) turn into

**Table 2.** Models for  $\gamma = 0, \frac{1}{3}, \frac{1}{2}$  for variable deceleration parameter.

Model no.	1	2	3
$\gamma$	0	$\frac{1}{3}$	$\frac{1}{2}$
$R$	$R_0 t^{1/3} (t + t_0)^{1/3}$	$R_0 t^{1/3} (t + t_0)^{1/3}$	$R_0 t^{1/3} (t + t_0)^{1/3}$
$\chi$	$\frac{4}{3\rho_0}$	$\frac{2}{\rho_0} t^{1/3} (t + t_0)^{1/3}$	$\frac{8}{3\rho_0} t^{1/2} (t + t_0)^{1/2}$
$\rho$	$\rho_0 t^{-1} (t + t_0)^{-1}$	$\rho_0 t^{-4/3} (t + t_0)^{-4/3}$	$\rho_0 t^{-3/2} (t + t_0)^{-3/2}$
$\beta^2$	$\frac{4D}{3} t^{-2} (t + t_0)^{-2}$	$\frac{4}{3} t^{-2} (t + t_0)^{-2}$ $\times [D - \frac{2}{3} t (t + t_0)]$	$\frac{4}{3} t^{-2} (t + t_0)^{-2}$ $\times [D - \frac{4}{3} t (t + t_0)]$
$\sigma^2$	$\frac{(\frac{1}{3} t_0^2 - D)}{t^2 (t + t_0)^2}$	$\frac{(\frac{1}{3} t_0^2 - D)}{t^2 (t + t_0)^2}$	$\frac{(\frac{1}{3} t_0^2 - D)}{t^2 (t + t_0)^2}$

$$R = R_0 t^{1/3} (t + t_0)^{1/3} \tag{49}$$

and

$$\rho = \rho_0 t^{-(1+\gamma)} (t + t_0)^{-(1+\gamma)}. \tag{50}$$

Now, putting the values of  $\chi, R$  and  $\rho$  from (48)–(50) into eq. (15), we have

$$\beta^2 = \frac{4}{3} t^{-2} (t + t_0)^{-2} \left[ D - \frac{4\gamma}{3(1-\gamma)} t (t + t_0) \right], \tag{51}$$

where  $D =$  constant of integration and  $D > 0$ .

From (12), (48)–(51), we obtain

$$\sigma^2 = \frac{(\frac{1}{3} t_0^2 - D)}{t^2 (t + t_0)^2}. \tag{52}$$

Deceleration parameter obtained from (49) is

$$q = -\frac{\ddot{R}/R}{(\dot{R}/R)^2} = -6 \left[ \frac{t(t + t_0)}{(2t + t_0)^2} - \frac{1}{3} \right] \\ = \text{a function of time.} \tag{53}$$

From (51) and (52),

$$\frac{4\gamma t(t + t_0)}{3(1-\gamma)} < D < \frac{1}{3} t_0^2 \quad \text{for } \beta^2 > 0 \quad \text{and } \sigma^2 > 0. \tag{54}$$

Some models are presented below for variable deceleration parameter with different values of  $\gamma$ . Model 1 for  $\gamma = 0$  (dust model), Model 2 for  $\gamma = 1/3$  (radiation universe) and Model 3 for  $\gamma = 1/2$  (hard universe) are tabulated in table 2.

From table 2, we see that in Model 1, the universe has decelerating expansion with constant  $G(t)$ .  $\sigma^2$  decays faster than  $\rho$ .  $\beta^2 > 0$  and  $\sigma^2 > 0$  for  $(1/3)t_0^2 > D$ .

Gauge function  $\beta(t)$  is infinitely large in the early stage of the evolution of the universe and then decreases with time.

In Model 2, the universe expands in the same mode as in the dust model with  $G(t)$  growing at the same rate as  $R(t)$ .  $\sigma^2$  decays faster than  $\rho$ .

$$\sigma^2 > 0 \quad \text{for } (1/3)t_0^2 > D$$

and

$$\beta^2 > 0 \quad \text{for } t(t + t_0) < 3D/2.$$

This is a model valid for the early stage of the universe with  $\beta(t)$ , decreasing from a very large value, becomes imaginary at later times.

In Model 3, the universe expands in the same rate as those in dust and radiation universe with growing  $G(t)$ .  $\sigma^2$  decays faster than  $\rho$ .

$$\sigma^2 > 0 \quad \text{for } (1/3)t_0^2 > D$$

and

$$\beta^2 > 0 \quad \text{for } t(t + t_0) < 3D/4.$$

This is also a model valid for the early universe with  $\beta(t)$ , decreasing from a very large value in the early universe, becomes imaginary at later times. So, in all the three models, we see that  $R(t)$  and  $\sigma^2$  are independent of  $\gamma$  and they vary with time at the same rate. All the models are decelerating expansion models.

Equation (20) shows that no stiff matter model ( $\gamma = 1$ ) is allowed in variable deceleration parameter case. Equation (20) also shows that no time-dependant deceleration parameter model is allowed in vacuum ( $p = \rho = 0$ ) universe. Plots of the different functions from Model 1 (of table 2) and only of  $\beta^2(t)$  from Model 2 (of table 2) are shown in figure 2.

## 5. Concluding remarks

In this paper we have studied anisotropic Bianchi Type-I cosmological models in Lyra's geometry. We have found exact solutions of Sen equation for constant and variable decelerations. Singh and Desikan [19] have presented models with both positive and negative  $\beta^2(t)$ . We have shown the models with positive  $\beta^2(t)$  only because negative  $\beta^2$  will turn  $\beta(t)$  into an imaginary quantity which is not physical.

For both constant and variable deceleration parameter models we have taken  $\chi$  as a function of time.

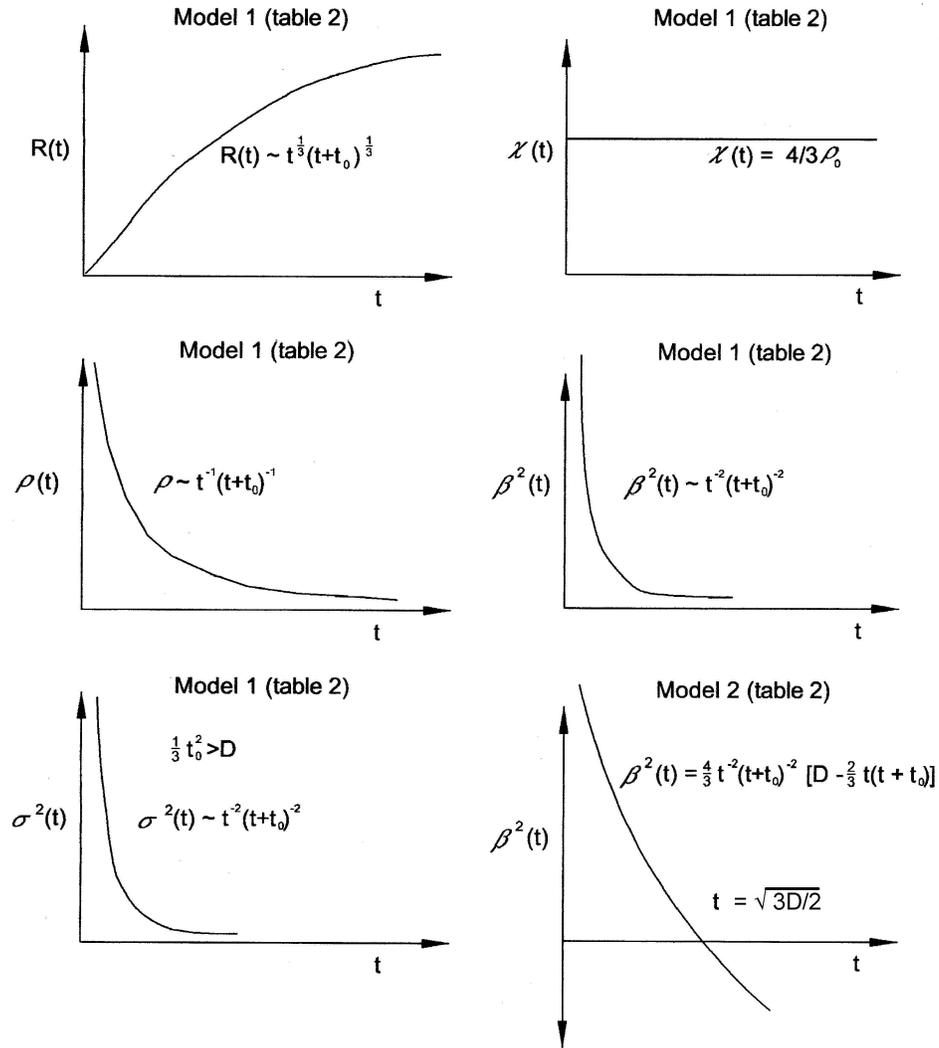
Table 1 shows the constant deceleration parameter models. For all models  $\sigma^2$  decays faster than  $\rho$ . In each model the universe evolves with decelerating expansion with decaying  $\chi(t)$ . In each model, assuming  $C$  to be very small, within the very small time after the birth of the universe  $\beta^2(t)$  starts from a very large negative value, rises to a positive maximum (large) value and after that begins to decrease with the evolution of the universe. In that era our universe remains in quantum state. As our theory is valid for classical era only, it is not surprising that  $\beta^2$  becomes negative or increasing in the quantum state. So, the imaginary and

increasing gauge function is not acceptable. Plots of the functions of Model 1 (from table 1) (figure 1) show the above-mentioned characters of the functions.

In Model 4, we see that empty universe allows decelerating expansion with independent nature of the function  $\chi(t)$ .  $\beta^2$  and  $\sigma^2$  decrease from large value with the expansion of the universe.

Model 5 shows that no stiff matter ( $\gamma = 1$ ) model is allowed in anisotropic Lyra cosmology.

Model 6 shows that we cannot obtain a de Sitter expansion in anisotropic Lyra universe.



**Figure 2.** Plots of the different functions from Model 1 (of table 2) and only of  $\beta^2(t)$  from Model 2 (of table 2).

Table 2 shows the three models with time-dependent deceleration parameter. All models show same rate of decelerating expansion and same rate of decrease in  $\sigma^2$ . Model 1 (dust model) shows constant  $G(t)$  and faster decay of  $\sigma^2$  than  $\rho$ .  $\beta^2$  decays from a large value with the expansion of universe. Model 2 and Model 3 show growing  $G(t)$  and faster decay of  $\sigma^2$  than  $\rho$ . In them  $\beta^2(t)$  starts from a large value, then decreases and becomes negative (but falling) after a time. So, these two models are valid for the early stage of the universe.

For vacuum universe and stiff matter ( $\gamma = 1$ ) universe no model is allowed for anisotropic Lyra cosmology.

## Appendix

Using Bianchi identities and the energy–momentum conservation law in eq. (1), we obtain

$$\begin{aligned}
 & -\frac{3}{2}(\phi_i\phi^j)_{;j} + \frac{3}{4}(g_i^j\phi_k\phi^k)_{;j} - 8\pi(G)_{;j}T_i^j = 0 \\
 \Rightarrow & -\frac{3}{2}(\phi_i\phi^j_{;j} + \phi^j\phi_{i;j}) + \frac{3}{4}(g_i^j\beta^2)_{;j} - 8\pi(G)_{;j}T_i^j = 0 \\
 \Rightarrow & -\frac{3}{2}\left[\phi_i\frac{1}{\sqrt{-g}}\frac{\partial(\phi^j\sqrt{-g})}{\partial x^j} + \phi^j\left(\frac{\partial\phi_i}{\partial x^j} - \Gamma_{ij}^p\phi_p\right)\right] \\
 & + \frac{3}{4}g_i^j(\beta^2)_{;j} - 8\pi(G)_{;j}T_i^j = 0, g = -(a_1a_2a_3)^2 \text{ and } g^j_{i;j} = 0. \\
 \Rightarrow & -\frac{3}{2}\left[\phi_0\frac{1}{\sqrt{-g}}\frac{\partial(\phi^0\sqrt{-g})}{\partial x^0} + \phi^0\left(\frac{\partial\phi_0}{\partial x^0} - \Gamma_{00}^0\phi_0\right)\right] \\
 & + \frac{3}{4}g_0^0\frac{\partial(\beta^2)}{\partial x^0} - 8\pi(G)_{;0}T_0^0 = 0, \quad (i, j = 1, 2, 3 \text{ give } 0 = 0.) \\
 \Rightarrow & -\frac{3}{2}\left[\beta^2\frac{1}{a_1a_2a_3}\frac{\partial(a_1a_2a_3)}{\partial t} + \beta\cdot\frac{\partial\beta}{\partial t} + \beta\frac{\partial\beta}{\partial t}\right] \\
 & + \frac{3}{4}\frac{\partial(\beta^2)}{\partial t} - 8\pi\dot{G}\rho = 0, \quad (\text{since } \Gamma_{00}^0 = 0) \\
 \Rightarrow & -\frac{3}{2}\left[\beta^2\cdot 3\frac{\dot{R}}{R} + 2\beta\dot{\beta}\right] + \frac{3}{2}\beta\dot{\beta} - 8\pi\dot{G}\rho = 0 \\
 \Rightarrow & -\frac{3}{2}\beta^2[\dot{\beta}/\beta + 3H] = 8\pi\dot{G}\rho.
 \end{aligned}$$

This is (15) in §2 of this paper.

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