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Realistic split fermion models

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Abstract. The standard model flavor structure can be explained in theories where the fermions are localized on different points in a compact extra dimension. We explain how models with two bulk scalars compactified on an orbifold can produce such separations in a natural way. We show that, generically, models of Gaussian overlaps are unnatural since they require very large Yukawa couplings between the fermions and the bulk scalars. We present a two-scalar model that accounts naturally for the quark flavor parameters and in particular yields order one CP violating phase.

The split fermion framework of Arkani-Hamed and Schmaltz (AS) [1] provides a possible explanation for the flavor puzzle. It is based on the idea that the standard model fermions are separated in one or more extra dimensions. Consequently, the four-dimensional (4D) Yukawa couplings between fermions are naturally suppressed by the overlap of their wave functions. Proton stability may be also accounted for in such a framework provided that the quarks and leptons are well-separated in the extra dimensions (see figure 1).

Despite the attractiveness of this framework, there is no complete realization of it. The two main requirements from any realization are chirality of the induced low energy 4D model and separation of the fermion wave functions in the extra dimensions. In [1], a five-dimensional (5D) model was presented. It uses domain wall fermions, namely, a bulk scalar field with non-trivial VEV that couples to the fermions. In addition, the fermions have different bulk masses in order to get the required separation between the fermion wave functions in the fifth dimension. A specific configuration for this model was presented in [2]. Assuming that the fermion wave functions are Gaussian, it produced the observed fermion masses and mixing angles. The model is chiral, however, only in the limit of an infinite extra dimension. This is not satisfactory since in reality the extra dimension must be finite. For a finite extra dimension a chiral theory can be obtained if the fifth dimension is an orbifold where the orbifold symmetry is used to project out one of the zero modes [3,4]. In such scenarios, however, the orbifold symmetry also forbids bulk masses. This is problematic since different bulk masses are needed in order to split the different fermions. Indeed, in that case the zero modes are localized at one of the orbifold fixed points, and the fermions' rich flavor structure cannot be naturally accounted for.

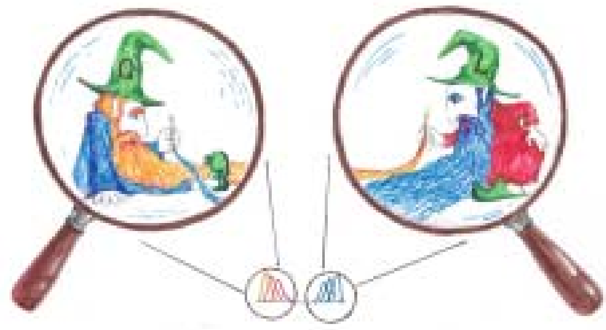

Figure 1. Configuration of the fifth-dimensional fermion wave functions in the AS model. Tiny overlaps between the quark (reddish curves on the LHS) and the leptons (bluish curves on the RHS) prevent proton decay. Small overlaps between the quarks yields the flavor hierarchy.

We consider a model with two scalar fields that couple to the fermions [5]. This rather minor modification of the ideas presented in [3,4] can produce fermion localization in the bulk. The advantage the two scalar models have over one scalar models is that the effective mass can vanish in the bulk. Intuitively the picture is as follows. With one bulk scalar the sign of the Yukawa coupling between the fermion and the scalar determines the boundary where the fermion is localized [3]. Once a second scalar with opposite sign Yukawa coupling is introduced, the picture is more complicated. The second scalar tends to localize the fermion on the other boundary. Sometimes, one scalar is dominant and the boundary where the fermion is localized is determined by the sign of the coupling between the fermion and the dominant scalar. In other cases, however, the tension between the two scalars results in a compromise: a configuration where the fermion is localized in the bulk.

Below we explain how two scalar models can naturally account for the quark masses, mixing angles and CP violating phase [5]. In particular, in such models the fermion wave functions are not of Gaussian shape. This implies that the resultant 4D mass matrices do not contain very small entries. The fact that there are no small entries ensures large CP violating phase, as required. This is in contrast to models where the fermion wave functions are assumed to be Gaussian [1,2,6] where the CP violation phase is very small. In fact, there are more problems for realistic models with Gaussian wave functions. We found that they are generically unnatural. For example, in the above framework proton stability requires fine tuning of $\mathcal{O}\left(10^{-4}\right)$.

We now move to present the model of [5]. The space-time of the model is described by an $M_4 \times S_1/Z_2$ orbifold. The physical region is defined as $0 \le u \le 1$ where $u \equiv x_5/L$ such that L is the size of the extra dimension. The model includes the standard model fermions ψ^j and two scalars, φ_i $(i = 1, 2)$ which acquire VEVs v_i. For each j one of the fermion components, $\psi^j_{\rm R}$ or $\psi^j_{\rm L}$, is even while the scalars and the other components of the fermions are odd under the orbifold symmetry. We assume that there is no interaction between the two scalars. In a scaling where $[\psi^j] = [v_i] = 3/2$ and $[\varphi_i] = [a_i] = [f] = [X] = 0$ the relevant part of the Lagrangian can be written as [5]

$$\mathcal{L}_{\rm int} = -\frac{1}{L^2}\left[f a_1 \bar{\psi}\left(\varphi_1 - X\varphi_2\right)\psi + \sum_i v_i^2 \frac{a_i^2}{2}\left(\varphi_i^2 - 1\right)^2\right], \qquad (1)$$

where we suppressed the flavor indices and ordered the scalars such that $a_2/a_1 > 1$. In the large a_i limit the scalar VEVs are approximated by [7]

$$h_i(u) \equiv \langle\varphi_i\rangle(u) = \tanh\left[a_i u\right]\tanh\left[a_i(1-u)\right]. \qquad (2)$$

The configuration of the fermions are determined by the following function:

$$g(u) = h_1(u) - X h_2(u), \qquad (3)$$

which plays the role of an effective bulk mass (see eq. (1)). The value of X is very important. When $X < a_1/a_2$ or $X > 1$ the function $g(u)$ vanishes only at the orbifold fixed points. For $a_1/a_2 < X < 1$, however, $g(u)$ has four roots where two of them are in the bulk. Since the fermions are localized around the roots of $g(u)$, they can be localized also in the bulk and not only at the orbifold fixed points. This property is important since it allows for separation between different fermions.

The wave function of the fermion zero mode (see e.g. [1]) is given by

$$y(u) \propto \exp\left[-f a_1 \int_0^u g(w)\,\mathrm{d}w\right]. \qquad (4)$$

The local maxima of $y(u)$ are at the roots of $g(u)$ which we denote by $u_{\max}$. In general there can be up to two maxima for $g(u)$. The dominant maximum can be at one of the orbifold fixed points or in the bulk. Moreover, there are cases where the two maxima are significant.

To analyze quark masses and mixing we need to restore the flavor indices of the model. Then, f and X are promoted to be flavor matrices. Based on the above analysis we constructed an example of a configuration which reproduces the correct quark masses mixing angles and CP violating phase [5].

Besides addressing the flavor problem, split fermion models can also be used to ensure long enough life-time for the proton. For that, the 5D wave functions of the quarks and leptons should be localized far away from each other with roughly Gaussian profiles [1]. This is translated to the requirement that the maximum of the wave function, $u_{\max}$, must be within the linear region of $h_i(u)$ (see eq. (2)) namely at $u_{\max} \ll 1/a_1$. For concreteness, we shall take $u_{\max} \sim 0.3/a_1$. The width of the wave function in the linear region is given by $\Gamma^{-1} \sim \sqrt{f a_1^2}$. In order that this model will correctly reproduce the quark flavor parameters the following relation should hold [2]:

$$\Gamma^{-1} u_{\max} \sim 0.3\sqrt{\frac{f}{2}} \sim 18 \implies f \sim 10^4. \qquad (5)$$

This result is independent of the size of the extra dimension, L. Thus, it is generic that in realistic models proton stability requires fine tuning of $\mathcal{O}\left(10^{-4}\right)$.

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