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Gauge coupling renormalization in AdS_5

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Abstract. We present the relation of the 4-dimensional low energy gauge coupling and the 5-dimensional fundamental gauge coupling of bulk gauge boson in a slice of AdS_5, which is orbifolded by $Z_2 \times Z_2'$. We calculate the full 1-loop corrections for the case of generic 5-dimensional scalar, Dirac fermion, and vector fields with arbitrary $Z_2 \times Z_2'$. For the supersymmetric case, we obtain the result more easily by using the 4-dimensional effective supergravity approach.

Keywords. Renormalization group; grand unification theory; field theories in higher dimension.

Recently, higher-dimensional field theories compactified on orbifold have been proposed as models providing an efficient mechanism for breaking GUT gauge symmetry. One of the most interesting ideas from higher-dimensional theories is generating large hierarchy via curved(warped) geometry, since it can naturally explain the large hierarchy between M_{Pl} and the weak scale. Warped geometry may also shed new light on higher-dimensional GUT. However, relatively rare works on GUT on AdS_5 space-time have been done compared with the flat case until now. Especially, a useful formula for gauge coupling is necessary to find out a model which does not lose the nice gauge unification properties of conventional 4D SUSY GUT.

In this talk, we present the one-loop low energy gauge coupling of the bulk gauge field living on an orbifolded AdS_5 space-time [1–3]. Our calculation deals with all possible cases of the gauge theory defined on a slice of AdS_5 orbifolded by $Z_2 \times Z_2'$ action. We consider contributions from fields with arbitrary spins and arbitrary orbifold parities. For supersymmetric case, the calculation can be carried over in the framework of 4D low energy effective supergravity. The 1-loop gauge couplings can be determined by the loop-induced axion couplings and the tree level properties of 4D effective SUGRA, and thus we do not have to do actual loop calculation. For non-supersymmetric case, we obtain results by doing actual loop calculation and regularizing the expression by dimensional regularization.

We consider the gauge theory on a slice of AdS_5 whose metric is given by $\mathrm{d}s^2 = \mathrm{e}^{-2kR|y|} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + R^2 \mathrm{d}y^2$. The orbifolding action Z_2 and Z_2' are reflection about $y = 0$ and $y = \pi$ axes, respectively. A bulk gauge field has its kinetic terms of the form

$$\int \mathrm{d}^4x\mathrm{d}y\sqrt{-G}\left(-\frac{1}{4g_{5a}^2}F^{aMN}F^a_{MN}-\frac{\delta(y)}{\sqrt{G_{55}}}\frac{1}{4g_{0a}^2}F^{a\mu\nu}F^{a\mu\nu}\right.$$
$$\left.-\frac{\delta(y-\pi)}{\sqrt{G_{55}}}\frac{1}{4g_{\pi a}^2}F^{a\mu\nu}F^a_{\mu\nu}\right), \tag{1}$$

where g_{5a}, g_{0a} and $g_{\pi a}$ are bulk gauge coupling, brane-localized gauge coupling at $y=0$ and $y=\pi$, respectively. Below the KK threshold scale (external momentum $p \ll M_{\mathrm{KK}} \sim k\exp(-\pi kR)$), the low energy gauge field is zero mode of higher-dimensional gauge field and low energy gauge coupling is given at tree level by $\frac{1}{g_a^2}=\frac{\pi R}{g_{5a}^2}+\frac{1}{g_{0a}^2}+\frac{1}{g_{\pi a}^2}$.

At one-loop level, Feynman diagrams contributing to the low energy gauge couplings contains the summation over all KK modes. By simple dimensional analysis, we note that $\frac{1}{g_{5a}^2}$ diverges depending linearly on cut-off Λ and $\frac{1}{g_{0a}^2}$ and $\frac{1}{g_{\pi a}^2}$ diverges logarithmically. One-loop correction to the low energy gauge coupling also contains conventional logarithmic running in 4D effective theory and calculable threshold corrections from matching 5D theory to 4D effective theory. We parametrize them by

$$\frac{1}{g_a^2}=\frac{\pi R}{g_{5a}^2(\Lambda)}+\frac{1}{g_{0a}^2(\Lambda)}+\frac{1}{g_{\pi a}^2}(\Lambda)+\frac{\gamma_a}{8\pi^3}\Lambda+\frac{1}{8\pi^2}\Delta_a(p,R,k,\Lambda). \tag{2}$$

Note that the definition of Δ_a is regularization-scheme independent. We obtain Δ_a for the supersymmetric case and the non-supersymmetric case.

For supersymmetric case, the low energy theory must be formulated in the framework of 4D SUGRA. The Wilsonian parameters which define the theory are Kähler potential K_0, Kähler metric Z_Φ for each chiral superfield Φ, superpotential P and holomorphic gauge coupling f_a. The relation between the physical gauge coupling and the Wilsonian parameters is given by [4]

$$\frac{1}{g_a^2}=\mathrm{Re}\ f_a+\frac{b_a}{16\pi^2}\ln\left(\frac{M_{\mathrm{Pl}}^2}{\mathrm{e}^{-K_0/3}p^2}\right)$$
$$-\sum_\Phi\frac{T_a(\Phi)}{8\pi^2}\ln(\mathrm{e}^{-K_0/3}Z_\Phi)+\frac{T_a(Adj)}{8\pi^2}\ln(\mathrm{Re}\ f_a), \tag{3}$$

and to obtain one-loop level gauge coupling, only tree level expression for every parameter except f_a is needed. Since we are only interested in the one-loop correction to the order of $\frac{1}{8\pi^2}$ times large log which is proportional to πkR in the setup, we investigate R dependence of f_a. By supersymmetry, R and B_5 compose of the scalar component of radion supermultiplet T. The one-loop contribution to the low energy holomorphic gauge coupling proportional to R must be the same as one-loop induced axion coupling $B_5F^{\mu\nu}\tilde{F}_{\mu\nu}$.

To find out one-loop induced axion coupling, we exploited $N=1$ superspace formulation of higher-dimensional supersymmetric theories [5,6]. We adopted the same action shown in [6], but redefined fields so that all T dependences in the action except for f_a appear in the combination of $T+T^*$. In such field basis, every loop diagram contributing to the axion coupling must be zero. However, field redefinition which is needed when migrating from the original field basis to the new field basis (especially, fermion phase rotation) should induce anomaly term which will contribute to the axion coupling.

We present the low energy gauge coupling for supersymmetric case:

$$\begin{aligned}\Delta_a = T_a(H_{++})&\left[\ln\left(\frac{k}{p}\right) - c_{++}\pi kR - \ln\left(\frac{\mathrm{e}^{(1-2c_{++})\pi kR}-1}{\pi(1-2c_{++})}\right)\right]\\ &-T_a(V_{++})\left[3\ln\left(\frac{\Lambda}{p}\right) - \frac{3}{2}\pi kR - \ln(\Lambda R)\right]\\ &+c_{+-}T_a(H_{+-})\pi kR - \frac{3}{2}T_a(V_{+-})\pi kR - c_{-+}T_a(H_{-+})\pi kR + \frac{3}{2}T_a(V_{+-})\pi kR\\ &+T_a(H_{--})\left[\ln\left(\frac{k}{p}\right) + c_{--}\pi kR - \ln\left(\frac{\mathrm{e}^{(1+2c_{--})\pi kR}-1}{\pi(1+2c_{--})}\right)\right]\\ &+T_a(V_{--})\left[\ln\frac{\Lambda}{p} + \ln\frac{\Lambda}{k} + \frac{1}{2}k\pi R + \ln\left(1-\mathrm{e}^{-2\pi kR}\right)\right],\end{aligned} \tag{4}$$

where $V_{zz'}$ and $H_{zz'}$ are vector multiplet and hypermultiplet with (z, z') parity of $Z_2 \times Z_2'$, respectively, and $c_{zz'}$ is the kink mass parameter for $H_{zz'}$. This result has the same cut-off dependence as the flat case. It is because the UV dynamics when $\Lambda \gg k$ must be ignorant of the existence of AdS curvature k.

For non-supersymmetric case, we explicitly calculate the diagram in the background field method regularized by dimensional regularization. In this regularization, power-law-like divergence for bulk gauge coupling is hidden, but only logarithmic divergences for brane couplings appear. To make the regularization consistent with locally 5D Lorentz invariant cut-off scheme, the subtraction scale for the brane gauge coupling $g_{\pi a}$ at $y = \pi$ is redshifted to $\Lambda\exp(-\pi kR)$. Besides the above subtlety, the calculation is straightforward. The results are summarized in [3]. Certainly, the non-supersymmetric results coincide with the supersymmetric results for supersymmetric particle contents.

In summary, we calculated the low energy gauge coupling from the gauge theory defined on a slice of AdS_5 with general classes of matter fields. Our results can be used for general GUT model building defined on a slice of AdS_5.

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