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Higgs production at next-to-next-to-leading order

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Abstract. We describe the calculation of inclusive Higgs boson production at hadronic colliders at next-to-next-to-leading order (NNLO) in perturbative quantum chromodynamics. We have used the technique developed in ref. [4]. Our results agree with those published earlier in the literature.

Keywords. Higgs; quantum chromodynamics; next-to-next-to-leading order.

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1. Introduction

The discovery of the Higgs boson will shed light on the symmetry breaking mechanism of the standard model (SM). The experimental bound from the LEP experiments and precision studies within and beyond the standard model strongly suggest that hadron colliders such as the Tevatron and the LHC will see the Higgs boson if it exists. At these machines the dominant contribution to single Higgs boson production is the gluon–gluon fusion process through heavy quark loops. The reason for this is that the Higgs boson couples strongly to heavy quarks. In addition the large gluon flux at the LHC enhances the total inclusive cross-section substantially. The NLO corrections along with NLO parton distributions yield a large K factor and also show a strong scale dependence. Hence there is need for improved parton densities using NNLO splitting functions as well as inclusion of NNLO partonic cross-sections. The NNLO correction to Higgs boson production was first computed by an expansion technique in [1]. Exact results were obtained in [2] using Cutkosky rules. In our work [3], we have used a straightforward technique which was adopted in [4] to compute the NNLO corrections to Drell–Yan process. A clever choice of the integration variables in specific frames makes the computation manageable.

2. Method of computation

We use the effective Lagrangian approach which emerges from the SM in the heavy top quark limit ($m_t \to \infty$) and is found to be a good approximation at hadron colliders. For Higgs boson production at hadron colliders at the NNLO level one has to compute: (1) tree level $a+b \to c+d+H$, (2) one-loop corrected $a+b \to c+H$, and (3) two-loop corrected $g+g \to H$, where a, b, c, d are light partons such as quarks, antiquarks and gluons whose interactions are governed by QCD. These corrections involve the computation of $2 \to 3$ body phase-space integrals and two- and one-loop momentum integrals followed by $2 \to 1$ and $2 \to 2$ phase-space integrations respectively. We use dimensional regularization (space-time dimension is taken to be $n = 4 + \varepsilon$) to regulate both ultraviolet and infrared (soft and collinear) divergences. We first describe here how we have performed three-body phase-space integrals for $2 \to 3$ tree level matrix elements and the two-body phase-space integrals for one-loop corrected matrix elements. The $2 \to 3$ body processes involve two angular integrations (say θ, ϕ) and two parametric integrations (say z, y). Before we perform these integrations, it is important to classify the matrix elements in such a way that the phase-space integrations over them can be done in suitable frames. For example, when the Higgs boson is produced from the incoming partons, the center-of-mass (CM) frame of outgoing partons is the most suitable frame, because the massive propagators $1/(P_{15}^{\alpha} P_{25}^{\beta})$ $(\alpha, \beta \geq 1)$ will not involve angular dependence in this frame. Here $P_{i5} = (p_i + p_{\rm H})^2$, where $p_{\rm H}$ is the momentum of the Higgs boson, and p_i is the momentum of the massless parton. Similarly when the Higgs boson is produced from an outgoing parton, we choose the CM frame of incoming partons where $1/(P_{35}^{\alpha} P_{45}^{\beta})$ $(\alpha, \beta \geq 1)$ do not depend on the angles. Complications arise when we encounter processes where the Higgs boson is produced by both initial and final state partons, i.e., where interference terms of the form $1/(P_{15}^{\alpha} P_{45}^{\beta})$ $(\alpha, \beta \geq 0)$ appear. In this case we have chosen the CM frame of the fourth (or third) parton and the Higgs boson where the angular integrals and other parametric integrals are less difficult. In the CM frame of the incoming partons and the CM frame of the outgoing partons we perform the angular integrations exactly using the result given in [5].

$$\int_0^{\pi} \mathrm{d}\theta \int_0^{\pi} \mathrm{d}\phi \quad \sin^{n-3}\theta \sin^{n-4}\phi \, \mathcal{C}_{ij}(\theta, \phi, \chi) = 2^{1-i-j}\pi$$
$$\times \frac{\Gamma(\frac{1}{2}n - 1 - j)\Gamma(\frac{1}{2}n - 1 - i)\Gamma(n-3)}{\Gamma(n-2-i-j)\Gamma^2(\frac{1}{2}n - 1)} F_{1,2}\left(i, j, \frac{1}{2}n - 1; \cos^2(\chi/2)\right),$$

where $\mathcal{C}_{ij} = (1-\cos\theta)^{-i}(1 - \cos\chi\cos\theta - \sin\chi\cos\phi\sin\theta)^{-j}$. Here $\cos\chi$ is related to kinematical variables such as x $(= m_H^2/s$, s-CM energy) and the integration variables z and y, and $F_{1,2}$ is the hypergeometric function. In the CM frame of the fourth parton and Higgs boson, due to the complexity, we performed one angular integration (say ϕ) exactly and performed the remaining θ integration after expanding the integrands in powers of $\varepsilon = n - 4$. In all these frames, using various Kummer's relations, the hypergeometric functions are simplified to the form $F_{1,2}(\pm\varepsilon/2, \pm\varepsilon/2, 1 \pm \varepsilon/2; \mathcal{D}(x, y, z))$ which is the most suitable for integrations over z and y. The next hurdle in the computation is the appearance of terms with large powers in $1/(1-z)$ or $1/z$. We have reduced the higher powers of $1/(1-z)^{\alpha+\beta\varepsilon}$ or $1/z^{\alpha+\beta\varepsilon}$ where $\alpha > 1$ by successive integration by parts with exact hypergeometric functions until we arrive at $1/(1-z)^{1+\beta\varepsilon}$ or $1/z^{1+\beta\varepsilon}$ multiplied by regular functions. We have used the following identity to accomplish this:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}z}F_{1,2}\left(\frac{\varepsilon}{2},\frac{\varepsilon}{2},1+\frac{\varepsilon}{2};\mathcal{D}(z)\right) &= \frac{\varepsilon}{2\mathcal{D}(z)}\frac{\mathrm{d}\mathcal{D}(z)}{\mathrm{d}z}\Bigg((1-\mathcal{D}(z))^{-\varepsilon/2} \\ &\quad -F_{1,2}\left(\frac{\varepsilon}{2},\frac{\varepsilon}{2},1+\frac{\varepsilon}{2};\mathcal{D}(z)\right)\Bigg). \end{aligned} \tag{1}$$

In the end we are left with integrations of the form $\int_0^1 \mathrm{d}z z^{-1-\beta\varepsilon}f(z)$ and/or $\int_0^1 \mathrm{d}z(1-z)^{-1-\beta\varepsilon}f(z)$. Such integrals are simplified as follows:

$$\int_0^1 \mathrm{d}z z^{-1-\beta\varepsilon}f(z) = \int_0^\delta \mathrm{d}z\ z^{-1-\beta\varepsilon}f(z) + \int_\delta^1 \mathrm{d}z\ z^{-1-\beta\varepsilon}f(z), \quad \delta \ll 1. \tag{2}$$

The first term can be evaluated to be $f(0)([\beta\varepsilon]^{-1}+\log\delta+[\beta\varepsilon/2]\log^2\delta+\cdots)$. After expanding $z^{-\beta\varepsilon}$ in powers of ε in the second term the z integration can be performed exactly order-by-order in ε with non-zero δ. At the end the δ dependence cancels in each order in ε. Since the z integration over the hypergeometric functions is nontrivial due to their complicated arguments, we have expanded them in powers of ε prior to the z integration:

$$\begin{aligned}F_{1,2}\left(\frac{\varepsilon}{2},\frac{\varepsilon}{2},1+\frac{\varepsilon}{2};\mathcal{D}(z)\right) &= 1+\frac{\varepsilon^2}{4}\mathrm{Li}_2(\mathcal{D}(z))+\frac{\varepsilon^3}{8}(\mathrm{S}_{1,2}(\mathcal{D}(z))-\mathrm{Li}_3(\mathcal{D}(z))) \\ &\quad +\frac{\varepsilon^4}{16}(\mathrm{S}_{1,3}(\mathcal{D}(z))-\mathrm{S}_{2,2}(\mathcal{D}(z))+\mathrm{Li}_4(\mathcal{D}(z))),\end{aligned} \tag{3}$$

where $\mathrm{S}_{n,p}(z) = (-1)^{n+p-1}[(n-1)!p!]^{-1}\int_0^1 \mathrm{d}t[t]^{-1}\log^{n-1}(t)\log^p(1-zt)$ with $n,p \geq 1$ and $\mathrm{Li}_n(z)=\mathrm{S}_{n-1,1}(z)$ see [6]. We have repeated the same procedure to perform the remaining y integration. In addition to $2\to 3$ contributions, we encounter one-loop corrected $2\to 2$ processes at NNLO level. Here the one-loop tensorial integrals are reduced to scalar integrals using the Passarino–Veltman reduction procedure implemented in n dimensions. The resulting one-loop two- and three-point scalar integrals can be expressed in terms of kinematic invariants. In the case of the four-point function, the scalar integrals can be expressed only in terms of hypergeometric functions which increases the complexity of the two-body phase-space integrations. We follow the procedure adopted for the $2\to 3$ phase-space integrations to perform the two-body phase-space integrations. After performing all these integrals, we have removed all the ultraviolet divergences by strong coupling and operator renormalization constants. The remaining collinear divergences are removed by mass factorization. Then we are left with finite partonic cross sections which are folded with parton distribution functions to compute hadronic cross section for the inclusive Higgs boson production.

Using the method described above we have successfully computed the NNLO corrections to Higgs boson production at hadron colliders and found complete agreement with the results of [1,2]. We find that NNLO corrections improve the convergence of the perturbative result and decrease the scale ambiguities inherent in it.

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